Find The Solution To The System Of Equations: X 2y + Z = 2 Y + 2z = 5 X + Y + 3z = 9 X = Y = Z =

Find The Solution To The System Of Equations: X 2y + Z = 2 Y + 2z = 5 X + Y + 3z = 9 X = Y = Z = is a classic problem in algebra that involves solving a system of linear equations to find the values of variables that satisfy all equations simultaneously. This type of problem is fundamental in mathematics, engineering, physics, and various applied sciences. Understanding how to approach and solve such systems not only enhances problem-solving skills but also provides insight into how variables interact within a set of equations. In this comprehensive guide, we will explore step-by-step methods to find the solution to this particular system, along with tips and tricks to handle similar problems efficiently.

Understanding the System of Equations

Before diving into solving the equations, it’s essential to understand the structure and what is being asked.

The Given System

The problem presents the following system:

1. \(X + 2y + z = 2 \)
2. \(2y + 2z = 5 \)
3. \(X + y + 3z = 9 \)

and indicates that

4. \(X = Y = Z \).

The last statement implies that all three variables are equal, which simplifies the system significantly.

Implication of \(X = Y = Z \)

Since \(X = Y = Z \), let’s denote this common value as \(t \). So,

\[
X = Y = Z = t
\]

This substitution reduces the problem from three variables to one, making the solution process more straightforward.

Step-by-Step Solution Process

Now, let's proceed with solving the system systematically.

Step 1: Substitute $X = Y = Z = t$ into the equations

Given the equalities, rewrite each equation:

- Equation 1:

$$
X + 2y + z = 2
$$

becomes

$$
t + 2t + t = 2
$$

- Equation 2:

$$
2y + 2z = 5
$$

becomes

$$
2t + 2t = 5
$$

- Equation 3:

$$
X + y + 3z = 9
$$

becomes

$$
t + t + 3t = 9
$$

Step 2: Simplify the equations

Simplify each:

- From Equation 1:

$$
t + 2t + t = 4t = 2
$$

- From Equation 2:

$$
2t + 2t = 4t = 5
$$

- From Equation 3:

$$
t + t + 3t = 5t = 9
$$

Step 3: Solve for t in each simplified equation

From the above, we have:

1. $4t = 2 \Rightarrow t = \frac{2}{4} = \frac{1}{2}$
2. $4t = 5 \Rightarrow t = \frac{5}{4}$
3. $5t = 9 \Rightarrow t = \frac{9}{5}$

Observation: The three equations give conflicting values for t:

$$
t = \frac{1}{2}, \quad t = \frac{5}{4}, \quad t = \frac{9}{5}
$$

This indicates an inconsistency in the system if all three variables are assumed equal, leading us to conclude that the initial assumption $X = Y = Z$ is incompatible with the equations as given.

Resolving the Inconsistency

Since the assumption $X = Y = Z$ yields conflicting solutions, the system as posed suggests that the variables are not all equal. Alternatively, perhaps the statement "X=Y=Z=" was meant to be an initial condition or a prompt to find the variables satisfying the system without that equality.

To clarify, let’s re-express the problem:

- Find the values of X, Y, Z satisfying the three equations:

$$
\begin{cases}
X + 2Y + Z = 2 \\
2Y + 2Z = 5 \\
X + Y + 3Z = 9
\end{cases}
$$

and then verify whether $X=Y=Z$ holds under those solutions, or if the problem is asking for the values of X, Y, Z that satisfy the system, possibly with the additional constraint $X=Y=Z$.

Clarifying the Problem

Given the conflicting results, the most logical interpretation is:

- The system is:

$$
\begin{cases}
X + 2Y + Z = 2 \quad (1) \\
2Y + 2Z = 5 \quad (2) \\
X + Y + 3Z = 9 \quad (3)
\end{cases}
$$

- The statement $X=Y=Z$ perhaps is an initial hypothesis to test, or an additional condition to impose.

Solving the System Without the Equality Constraint

Let’s proceed to solve the system for X, Y, Z without assuming $X=Y=Z$.

Step 1: Express Y and Z from Equation (2)

Equation (2):

$$
2Y + 2Z = 5 \Rightarrow Y + Z = \frac{5}{2}
$$

Express Y:

$$
Y = \frac{5}{2} - Z
$$

Step 2: Substitute Y into Equations (1) and (3)

Equation (1):

$$
X + 2Y + Z = 2
$$

Substitute Y:

$$
X + 2 \left(\frac{5}{2} - Z \right) + Z = 2
$$

Simplify:

$$
X + 2 \times \frac{5}{2} - 2Z + Z = 2
$$

$$
X + 5 - Z = 2
$$

$$
X = 2 - 5 + Z = -3 + Z
$$

Equation (3):

$$
X + Y + 3Z = 9
$$

Substitute $X = -3 + Z$ and $Y = \frac{5}{2} - Z$:

$$
(-3 + Z) + \left(\frac{5}{2} - Z \right) + 3Z = 9
$$

Simplify:

$$
-3 + Z + \frac{5}{2} - Z + 3Z = 9
$$

Combine like terms:

$$
(-3 + \frac{5}{2}) + (Z - Z + 3Z) = 9
$$

$$
\left(-\frac{6}{2} + \frac{5}{2} \right) + 3Z = 9
$$

$$
-\frac{1}{2} + 3Z = 9
$$

Solve for Z:

$$
3Z = 9 + \frac{1}{2} = \frac{18}{2} + \frac{1}{2} = \frac{19}{2}
$$

$$
Z = \frac{19}{6}
$$

Now, find Y:

$$
Y = \frac{5}{2} - Z = \frac{5}{2} - \frac{19}{6}
$$

Express both with denominator 6:

$$
\frac{15}{6} - \frac{19}{6} = -\frac{4}{6} = -\frac{2}{3}
$$

And X:

$$
X = -3 + Z = -3 + \frac{19}{6}
$$

Express -3 as $\frac{-18}{6}$:

$$
\frac{-18}{6} + \frac{19}{6} = \frac{1}{6}
$$

Final Solution:

\[
\boxed{
\begin{cases}
X = \frac{1}{6} \\
Y = -\frac{2}{3} \\
Z = \frac{19}{6}
\end{cases}
}
\]

Validating the Solution

Let's verify these values satisfy all original equations:

- Equation (1):

\[
X + 2Y + Z = \frac{1}{6} + 2 \times \left(-\frac{2}{3}\right) + \frac{19}{6}
= \frac{1}{6} - \frac{4}{3} + \frac{19}{6}
\]

Express all with denominator 6:

\[
\frac{1}{6} - \frac{8}{6} + \frac{19}{6} = \frac{1 - 8 + 19}{6} =
\frac{12}{6} = 2
\]

Correct.

- Equation (2):

\[
2Y + 2Z = 2 \times \left(-\frac{

Frequently Asked Questions

How do I solve the system of equations: 2y + z = 2, y + 2z = 5, and x + y + 3z = 9?

First, express y and z from the first two equations and substitute into the third to find x. For example, from 2y + z = 2, express z = 2 - 2y. Substitute into y + 2z = 5: y + 2(2 - 2y) = 5. Simplify and solve for y, then find z, and finally compute x using the third equation.

What is the step-by-step method to find values of x, y, and z in this system?

Step 1: Solve one of the equations for one variable (e.g., $z = 2 - 2y$). Step 2: Substitute into the second equation ($y + 2z = 5$) to solve for y. Step 3: Use y to find z. Step 4: Substitute y and z into the third equation ($x + y + 3z = 9$) to find x.

Can I use matrix methods to solve this system of equations?

Yes, you can write the system in matrix form and use methods like Gaussian elimination or matrix inversion to find the solutions for x, y, and z.

What are the possible solutions if the system is inconsistent?

If the equations lead to a contradiction (e.g., $0 = 5$), then the system has no solution. Otherwise, if equations are dependent, there may be infinitely many solutions.

How do I verify the solution once I find x, y, and z?

Plug the found values back into all the original equations to ensure each equation is satisfied. If all are true, the solution is correct.

Are there alternative methods to solve such a system besides substitution?

Yes, methods like elimination or using matrices and determinants (Cramer's rule) can also be employed to solve the system efficiently.

Additional Resources

Finding solutions to systems of equations is a fundamental task in algebra, with applications spanning engineering, computer science, economics, and everyday problem-solving. The particular system under consideration — consisting of multiple linear equations involving three variables — offers a classic example of how algebraic methods can be employed to uncover relationships and determine specific values for the variables involved. In this article, we will explore the process of solving the system:

\[
\begin{cases}
**X + 2y + Z = 2 **

**Y + 2z = 5 **
X + Y + 3z = 9
\end{cases}
\]

and analyze how to find the values of \(X\), \(Y\), and \(Z\). We will delve into the methods, step-by-step procedures, and the implications of the solutions, providing a comprehensive understanding of this classic problem.

Understanding the System of Equations

The Nature of Linear Systems

A system of linear equations involves multiple equations where each term is either a constant or a variable raised to the first power. The goal is to find the set of variable values that simultaneously satisfy all equations in the system.

In our case, we have three equations with three unknowns:

1. \(X + 2y + Z = 2\)
2. \(Y + 2z = 5\)
3. \(X + Y + 3z = 9\)

Note that the variables are \(X\), \(Y\), and \(Z\), and the equations are linear with respect to these variables.

Rearranging and Simplifying the System

Expressing Variables in Terms of Others

The first step in solving a system of equations is to express some variables in terms of others to reduce the complexity. Let's analyze each equation:

- Equation (2): \(Y + 2z = 5\)

This can be rearranged to:

\[

Y = 5 - 2z
\]

- Equation (3): \(X + Y + 3z = 9\)

Substituting the expression for \(Y\):

\[
X + (5 - 2z) + 3z = 9
\]

which simplifies to:

\[
X + 5 + z = 9
\]

and further to:

\[
X + z = 4
\]

Thus,

\[
X = 4 - z
\]

- Equation (1): \(X + 2y + Z = 2\)

Recall that \(X = 4 - z\). Now, notice that \(Y = 5 - 2z\).

But the variable \(Y\) is not directly in equation (1). Since \(Y\) appears in equation (3), and we've expressed \(X\) and \(Y\) in terms of \(z\), we can now attempt to express \(Z\) in terms of \(z\) as well.

Rearranged, equation (1) becomes:

\[
(4 - z) + 2y + Z = 2
\]

To proceed, notice that the variable \(y\) appears in the first equation as \(2y\). But we haven't yet expressed \(y\) in terms of \(z\).

Important note: The variable notation uses uppercase \(X, Y, Z\) in the equations, but in the problem statement, the variables are sometimes denoted as \(x, y, z\). Clarifying this, the system seems to be:

\[

\begin{cases}
X + 2y + Z = 2 \\
Y + 2z = 5 \\
X + Y + 3z = 9
\end{cases}
\]

with variables \(X, Y, Z, y, z\). It appears there's a notation inconsistency, but assuming the intended variables are:

- \(X\),
- \(Y\),
- \(Z\),
- and that the lowercase \(y, z\) are the same as the uppercase \(Y, Z\), or perhaps that the variables are \(X, y, z\).

Given the notation confusion, let's clarify and standardize:

Assumption: The variables are \(X, Y, Z\), and the equations are:

1. \(X + 2Y + Z = 2\)
2. \(Y + 2Z = 5\)
3. \(X + Y + 3Z = 9\)

This is a logical interpretation because the initial problem statement appears to mix uppercase and lowercase variables.

Proceeding with this assumption, the system is:

\[
\begin{cases}
X + 2Y + Z = 2 \quad (1) \\
Y + 2Z = 5 \quad (2) \\
X + Y + 3Z = 9 \quad (3)
\end{cases}
\]

Now, this is a standard system with three variables and three equations, suitable for solving via substitution or elimination.

Step-by-Step Solution Process

Step 1: Solve Equation (2) for \(Y\)

Equation (2):

$$
Y + 2Z = 5
$$

Rearranged:

$$
Y = 5 - 2Z
$$

Step 2: Substitute Y into Equations (1) and (3)

- Equation (1):

$$
X + 2Y + Z = 2
$$

Substitute $Y = 5 - 2Z$:

$$
X + 2(5 - 2Z) + Z = 2
$$

Simplify:

$$
X + 10 - 4Z + Z = 2
$$

$$
X + 10 - 3Z = 2
$$

Rearranged:

$$
X = 2 - 10 + 3Z = -8 + 3Z
$$

- Equation (3):

$$
X + Y + 3Z = 9
$$

Substitute $Y = 5 - 2Z$, and $X = -8 + 3Z$:

$$
(-8 + 3Z) + (5 - 2Z) + 3Z = 9
$$

Simplify:

$$
-8 + 3Z + 5 - 2Z + 3Z = 9
$$

Combine like terms:

$$
(-8 + 5) + (3Z - 2Z + 3Z) = 9
$$

$$
-3 + (4Z) = 9
$$

Rearranged:

$$
4Z = 9 + 3 = 12
$$

$$
Z = \frac{12}{4} = 3
$$

Step 3: Find Y and X using $Z = 3$

- Calculate Y:

$$
Y = 5 - 2Z = 5 - 2(3) = 5 - 6 = -1
$$

- Calculate X:

$$
X = -8 + 3Z = -8 + 3(3) = -8 + 9 = 1
$$

Final Solution and Verification

The solution set for the system is:

$$
\boxed{
X = 1, \quad Y = -1, \quad Z = 3
}
$$

To ensure accuracy, verify the solutions by substituting back into the original equations:

- Equation (1):

$$
X + 2Y + Z = 1 + 2(-1) + 3 = 1 - 2 + 3 = 2 \quad \checkmark
$$

- Equation (2):

$$
Y + 2Z = -1 + 2(3) = -1 + 6 = 5 \quad \checkmark
$$

- Equation (3):

$$
X + Y + 3Z = 1 - 1 + 3(3) = 0 + 9 = 9 \quad \checkmark
$$

All equations are satisfied, confirming the correctness of the solution.

Discussion and Implications of the Solution

Understanding the Nature of the Solution

The solution indicates a unique set of values for the variables X, Y, Z that satisfy all three equations simultaneously. This indicates that the system is consistent and independent — a hallmark of a well-posed linear system with a unique solution.

In real-world applications, such solutions can represent specific conditions or states in physical systems, economic models, or engineering designs. For example, in a problem modeling resource allocation, the variables could denote quantities of different resources, and the equations could represent constraints or equations governing their relationships.

Sensitivity and Stability

Linear systems are sensitive to small changes in coefficients or constants, potentially leading to different solutions or no solutions at all. The stability of the solution can be analyzed by examining the coefficient matrix's properties, such as its determinant or condition number.

In our case, the coefficient matrix:

```
\[
\begin{bmatrix}
1 & 2 &
```

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Find The Solution To The System Of Equations X 2y Z 2 Y 2z 5 X Y 3z 9 X Y Z

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