Find The Value Of K So That The Point A Lies On The Line Given By The Equation. A(2,1), 3x+ky=8

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Introduction

In coordinate geometry, one of the fundamental concepts is understanding how points relate to lines. Determining whether a specific point lies on a given line involves substituting the point's coordinates into the line's equation and verifying whether the equation holds true. This process is essential in numerous mathematical applications, including solving geometric problems, analyzing linear relationships, and in fields like physics and engineering where spatial positioning is critical.

In this article, we focus on a specific problem: given a point A(2,1) and a line described by the equation 3x + ky = 8, how do we find the value of the parameter k such that point A lies on this line? This problem not only reinforces our understanding of the basic principles of coordinate geometry but also demonstrates how parameters within equations influence the location of points relative to lines.

Throughout this discussion, we'll explore the theoretical background, step-by-step solution methods, and practical implications of such problems. Whether you're a student preparing for an exam, a teacher designing exercises, or someone interested in the intricacies of algebra and geometry, this comprehensive guide aims to clarify the process and enhance your understanding.

Understanding the Line Equation and Coordinates

Before diving into calculations, it’s important to clarify the components involved:

- Point A(2,1): This is a point in the Cartesian plane, with x-coordinate 2 and y-coordinate 1.
- Line Equation 3x + ky = 8: A linear equation where k is a parameter that can vary, affecting the slope and position of the line.

The goal is to determine the value of k such that the point A satisfies this line equation, i.e., when substituting x=2 and y=1 into the equation, it holds true.

The Concept of Points Lying on a Line

A point (x_0, y_0) lies on a line described by the equation $Ax + By + C = 0$ if and only if substituting (x_0, y_0) into the equation results in a true statement.

For the line $3x + ky = 8$, the point A(2,1) lies on the line if:

$$3(2) + k(1) = 8$$

which simplifies to:

$$6 + k = 8$$

This is a simple linear equation in k, which we can solve directly.

Step-by-Step Solution

Let's proceed with the detailed steps to find the value of k:

Step 1: Substitute the point's coordinates into the line equation

Given point A(2,1), substitute $x=2$ and $y=1$ into $3x + ky = 8$:

$$3(2) + k(1) = 8$$

which simplifies to:

$$6 + k = 8$$

Step 2: Solve for k

Subtract 6 from both sides:

$$k = 8 - 6$$

$$k = 2$$

Thus, the value of k that makes point A lie on the line is $k = 2$.

Step 3: Verify the solution

Substitute $k=2$ back into the line equation:

$$3x + 2y = 8$$

Check if point A(2,1) satisfies the equation:

$$ 3(2) + 2(1) = 6 + 2 = 8 $$

which confirms the point lies on the line when k=2.

Geometric Interpretation of the Result

Understanding the geometric significance of the value of k is crucial:

- When k=2, the line becomes:

$$ 3x + 2y = 8 $$

- The slope of the line can be expressed in the form y = mx + c:

$$ 2y = 8 - 3x $$
$$ y = \frac{8 - 3x}{2} $$
$$ y = -\frac{3}{2}x + 4 $$

- The slope (m) is -3/2, indicating the line's inclination, and the y-intercept is 4.

- The point A(2,1) lies exactly on this line, confirming the correctness of the value of k.

Extending the Concept: General Approach

While this problem involves a specific point and a line with a parameter, the methodology applies broadly to similar problems:

1. Identify the point and line equations

- Point: (x_0, y_0)
- Line: Ax + By + C = 0 or similar form

2. Substitute the point into the line equation

- Plug x_0 and y_0 into the equation.

3. Solve for unknown parameters

- If the line contains parameters (like k), solve for them to satisfy the point-line condition.

4. Verify the solution

- Re-substitute the found parameter into the line equation and confirm the point satisfies it.

Variations and Additional Considerations

The problem can be extended or modified in various ways:

a) Point Not on the Line

If the point does not lie on the line initially, solving for the parameter can help determine how to shift or rotate the line to include the point.

b) Line Passing Through Multiple Points

Given multiple points, parameters can be determined to find a line passing through all points, often involving systems of equations.

c) Parameter Dependent Lines

Lines with parameters like k often represent families of lines. Finding the specific value of k for a particular point helps in understanding the family's geometry.

Practical Applications of Finding k

Understanding how to find the value of k is not purely academic; it has practical applications in various fields:

- Engineering: Designing structures where specific points must lie on certain lines or curves.
- Computer Graphics: Calculating parameters for lines passing through given points.
- Physics: Determining parameters of motion paths or force lines.
- Data Analysis: Fitting lines to data points with adjustable parameters.

Summary of Key Steps

Here's a concise summary for solving similar problems:

1. Write down the point's coordinates.
2. Substitute the coordinates into the line equation.
3. Solve the resulting equation for the unknown parameter(s).
4. Verify the solution by substituting back into the original equation.
5. Understand the geometric implications of the parameter value.

Conclusion

Finding the value of a parameter such as k so that a specific point lies on a given line is a fundamental skill in coordinate geometry. It involves straightforward substitution and algebraic manipulation, but it also deepens understanding of the relationship between points and lines in the Cartesian plane.

In our specific problem, we determined that $k = 2$ ensures the point $A(2,1)$ lies on the line $3x + ky = 8$. This process underscores the importance of algebraic methods in geometric contexts and offers a foundation for tackling more complex problems involving lines, points, and parameters.

Mastering these techniques enhances problem-solving skills and provides a solid basis for exploring more advanced topics in geometry, calculus, and applied mathematics.

Additional Resources

- Coordinate Geometry Textbooks
- Online Geometry Calculators
- Math Practice Problems and Worksheets
- Video Tutorials on Line Equations and Point-Line Relationships

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Remember: The key to mastering such problems is understanding the principles behind substitution and solving equations, which serve as building blocks for more advanced mathematical concepts.

Frequently Asked Questions

What is the method to find the value of k so that a point A(2,1) lies on the line 3x + ky = 8?

Substitute the coordinates of point A into the line equation and solve for k: $3(2) + k(1) = 8$, which simplifies to $6 + k = 8$, so $k = 2$.

How do you verify if a point lies on a given line with variable coefficients?

By substituting the point's x and y coordinates into the line equation and checking if the resulting equation holds true. If it does, the point lies on the line.

What is the value of k when point A(2,1) lies on the line 3x + ky = 8?

The value of k is 2, obtained by substituting x=2 and y=1 into the equation: 32 + k1 = 8 → 6 + k = 8 → k = 2.

Can the same method be used for any point and line equation to find the missing parameter?

Yes, the same substitution method applies universally: substitute the point's coordinates into the line equation and solve for the unknown parameter.

What are common mistakes to avoid when finding the value of k for a point on a line?

Common mistakes include incorrect substitution of coordinates, forgetting to simplify correctly, or making algebraic errors when solving for k.

If a point does not satisfy the line equation after substitution, what does that imply?

It implies that the point does not lie on the line for the given value of k. You may need to adjust k or verify the point's coordinates.

Is the process of finding k from the point and line equation applicable in three-dimensional geometry?

Yes, but in 3D, you would have an equation of a plane or line, and the process involves substituting coordinates into the equation to find unknown parameters accordingly.

Additional Resources

Find The Value Of K So That The Point A Lies On The Line Given By The Equation. A(2,1), 3x + ky = 8

Understanding the relationship between points and lines is a fundamental aspect of coordinate geometry. Whether you're a student tackling algebra homework or a professional working on geometric modeling, grasping how to determine the parameters that define a line is essential. Today’s focus is on a classic problem: finding the value of the parameter \(k\) so that a specific point lies on a given line. Specifically, we are asked to find the value of \(k\) such that the point \(A(2,1)\) satisfies the line equation \(3x + ky = 8\).

This problem, while seemingly straightforward, involves a deep understanding of how points relate to line equations. It combines algebraic substitution with an understanding of linear equations in two variables. Let’s explore this problem in detail, breaking down the steps involved to arrive at the solution, and discussing the underlying principles along the way.

Understanding the Problem: Points and Line Equations

Before diving into calculations, it’s important to clearly understand what the problem is asking. The key elements are:

- The Point $A(2,1)$: This is a specific coordinate point on the XY-plane, with an x-coordinate of 2 and a y-coordinate of 1.

- The Line Equation $3x + ky = 8$: This is a linear equation involving two variables, x and y, with an unknown parameter k.

The question is: What value of k makes the point $A(2,1)$ lie on this line?

In other words, when we substitute $x=2$ and $y=1$ into the line equation, the equation should hold true — meaning it should be satisfied exactly.

Methodology: Substituting the Point into the Line Equation

The core approach to this problem involves substitution. If a point (x_0, y_0) lies on the line $ax + by = c$, then plugging x_0 and y_0 into the equation should make it true.

Steps to determine k:

1. Write down the point coordinates: $x=2$, $y=1$.
2. Substitute into the line equation: Replace x with 2, and y with 1.
3. Solve for k: Rearrange the resulting equation to find the value of k.

Let’s perform these steps explicitly.

Step-by-Step Calculation

Given the line equation:

$$
3x + ky = 8
$$

and the point $A(2,1)$, we substitute:

$$
x = 2, \quad y = 1
$$

which yields:

$$
3 \times 2 + k \times 1 = 8
$$

Simplifying:

$$
6 + k = 8
$$

Solving for k:

$$
k = 8 - 6
$$

$$
k = 2
$$

Result: The value of k is 2.

Verification: Confirming the Solution

To ensure our solution is correct, we can double-check by substituting $k=2$ back into the line equation:

$$
3x + 2y = 8
$$

Substituting the point $(2,1)$:

$$
3 \times 2 + 2 \times 1 = 6 + 2 = 8
$$

which matches the right side of the equation. Since the equality holds, the point indeed lies on the line when $k=2$.

Implications of the Result

The process of finding k reveals several insights:

- Parameter Dependence: The value of k depends on the specific point in question. If $A(2,1)$ were different, the value of k would change accordingly.
- Line-Point Relationship: The calculation confirms that the line's slope and intercepts are flexible depending on the parameter k. When $k=2$, the line passes through the given point.
- Geometric Interpretation: Graphically, with $k=2$, the line $3x + 2y = 8$ passes through point $A(2,1)$. Any change in k would shift the line so that the point no longer lies on it.

Broader Context: Why Is This Important?

While this problem is straightforward, it exemplifies a common approach in analytical geometry: verifying whether a point is on a line, and determining parameters that define geometric figures.

Applications include:

- Design and Engineering: Ensuring specific points lie on designed curves or lines.
- Computer Graphics: Calculating line parameters for rendering objects.
- Navigation and Mapping: Confirming waypoints on routes defined by linear equations.
- Mathematical Modelling: Adjusting parameters to fit models to data points.

Understanding how to manipulate and interpret line equations with parameters like k is fundamental for solving more complex problems involving geometric transformations, intersections, and optimizations.

Conclusion: The Key Takeaways

- The problem of finding the value of \(k\) such that a point lies on a line boils down to substitution and solving a simple algebraic equation.
- In our specific case, substituting \(A(2,1)\) into \(3x + ky = 8\) directly led us to determine that \(k=2\).
- Confirming the solution by plugging \(k=2\) back into the equation ensures accuracy.
- This method can be generalized to any point and line, making it a powerful tool in coordinate geometry.

By mastering these fundamental steps, students and professionals alike enhance their ability to navigate the geometric landscape, ensuring they can handle similar problems efficiently and confidently. Whether in academic settings or real-world applications, understanding the relationship between points and lines remains a cornerstone of mathematical literacy.

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