Find Three Consecutive Positive Integers Such That The Product Of The 1st And Second Is Equal To 20

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Understanding how to find three consecutive positive integers with specific properties is a common problem in mathematics, especially in algebra and number theory. One such intriguing problem asks: Find three consecutive positive integers such that the product of the first and second is equal to 20. This problem not only tests your algebraic skills but also enhances your problem-solving abilities by encouraging you to think logically and systematically. In this article, we will explore this problem in depth, providing step-by-step solutions, explanations, and related concepts to deepen your understanding of integers and algebraic equations.

Understanding the Problem: Key Concepts and Definitions

Before diving into solving the problem, it's essential to understand some fundamental concepts:

What Are Consecutive Positive Integers?

- Consecutive positive integers are integers that follow one after another in order.
- Examples: 1, 2, 3; 5, 6, 7; 10, 11, 12.
- They differ by exactly 1.

The Given Condition

- The product of the first and second integers should equal 20.
- If we denote the first integer as n, then the second is $n+1$, and the third is $n+2$.

Mathematical Representation

- The problem can be formulated as: Find n such that:

$$
n \times (n + 1) = 20
$$

- Once n is found, the three integers are n, $n+1$, and $n+2$.

Step-by-Step Solution to the Problem

Let's now solve the problem systematically.

Step 1: Set Up the Equation

- Based on the problem, the first two integers are n and $n+1$.
- The product of these two is 20:
$$
n(n + 1) = 20
$$

Step 2: Expand and Rearrange

- Expand the equation:
$$
n^2 + n = 20
$$
- Rearrange into standard quadratic form:
$$
n^2 + n - 20 = 0
$$

Step 3: Solve the Quadratic Equation

- Use the quadratic formula:
$$
n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$
where $a = 1$, $b = 1$, $c = -20$.

- Compute the discriminant:
$$
\Delta = b^2 - 4ac = 1^2 - 4 \times 1 \times (-20) = 1 + 80 = 81
$$

- Find the square root of the discriminant:
$$
\sqrt{81} = 9
$$

- Calculate the roots:
$$
n = \frac{-1 \pm 9}{2}
$$

- First root:

$$
n = \frac{-1 + 9}{2} = \frac{8}{2} = 4
$$

- Second root:

$$
n = \frac{-1 - 9}{2} = \frac{-10}{2} = -5
$$

Step 4: Interpret the Results

- Since we're looking for positive integers, discard $n = -5$.
- The valid solution is $n = 4$.

Step 5: Find the Three Consecutive Integers

- The integers are:

$$
4, 5, 6
$$

- Check the condition:

$$
4 \times 5 = 20
$$

- The condition is satisfied.

Additional Insights and Related Problems

This problem is a classic example of using algebra to solve for integers with specific properties. Let's explore some related concepts and variations.

Generalizing the Problem

- Instead of the product being 20, what if it was another number?
- For example, find three consecutive positive integers such that the product of the first and second equals a different constant k.

Sample Variations

- Variation 1: Find three consecutive integers where the product of the first and third equals 20.
- Variation 2: Find three consecutive integers such that their sum equals a specific number.

Different Approaches to Similar Problems

- Use inequalities for bounds.
- Graph the quadratic equations to visualize solutions.
- Use programming or computational tools for larger or more complex problems.

Practical Applications of Finding Consecutive Integers

Understanding these types of problems has real-world applications:

- Scheduling and Planning: Determining consecutive days or periods with specific constraints.
- Number Theory in Cryptography: Analyzing sequences and integer properties.
- Problem Solving in Puzzles and Games: Many puzzles involve consecutive numbers and their properties.

Tips for Solving Similar Problems Effectively

Here are some practical tips to approach and solve problems involving consecutive integers:

1. Define Variables Clearly: Assign variables to the integers to set up equations.
2. Translate Conditions into Equations: Convert the problem statement into algebraic form.
3. Solve the Equation Systematically: Use quadratic formulas, factoring, or completing the square.
4. Check for Validity: Ensure solutions satisfy the conditions (e.g., positivity).
5. Verify the Results: Substitute back into the original conditions to confirm.

Conclusion: Mastering the Art of Integer Problems

Finding three consecutive positive integers such that the product of the first and second equals 20 is a straightforward yet illustrative problem that demonstrates the power of algebra. By translating the problem into a quadratic equation and solving systematically, we identified the integers 4, 5, and 6 as the solution. Mastering such problems enhances your mathematical reasoning skills, which are invaluable in academic pursuits, competitive exams, and real-world applications. Whether you're tackling similar problems or exploring more complex number theory questions, the key lies in a structured approach, clear variable definitions, and thorough verification.

Further Resources for Learning About Consecutive Integers and Algebraic Equations

- Books:
- Elementary Number Theory by David M. Burton
- Algebra for Beginners by Charles P. McKeague

- Online Platforms:
- Khan Academy’s Algebra Courses
- Brilliant.org’s Number Theory Modules

- Practice Problems:
- Websites like Art of Problem Solving (AoPS)
- Math competition practice sets

Understanding how to find three consecutive positive integers with specific properties is a foundational skill in mathematics. By practicing such problems, you build a strong analytical mindset capable of tackling more complex challenges in mathematics and beyond.

Frequently Asked Questions

What are the three consecutive positive integers where the product of the first and second equals 20?

The three consecutive positive integers are 4, 5, and 6, since $4 \times 5 = 20$.

How can I verify that 4, 5, and 6 are the consecutive integers satisfying the condition?

Check that the first and second integers are 4 and 5, and their product is $4 \times 5 = 20$, which matches the given condition. The three integers are then 4, 5, and 6.

Are there other sets of three consecutive positive integers that satisfy similar product conditions?

For this specific condition, only 4, 5, and 6 satisfy the product of the first two being 20. Different conditions may yield other sets, but under this one, this is the unique solution.

Can I extend this problem to find three consecutive integers where the product of the first and third equals a certain number?

Yes, you can set up an equation with the three consecutive integers n, $n+1$, $n+2$ and solve for the desired product, such as $n \times (n+2)$.

What is the general method to find three consecutive integers with a specific product condition?

Assign variables to the integers (e.g., n, n+1, n+2), set up an equation based on the condition (e.g., n × (n+1) = target number), and solve for n to find the integers.

Additional Resources

Find Three Consecutive Positive Integers Such That The Product Of The 1st And Second Is Equal To 20

When exploring the realm of basic algebra and number patterns, one interesting problem is to find three consecutive positive integers such that the product of the first and second equals 20. This problem not only tests understanding of variables and equations but also encourages logical reasoning and pattern recognition. Whether you're a student brushing up on algebraic skills or a math enthusiast curious about number puzzles, this guide will walk you through a comprehensive approach to solving such problems.

Understanding the Problem

Before diving into calculations, it's essential to carefully parse the problem statement:

- Goal: Find three consecutive positive integers.
- Condition: The product of the first and second integers equals 20.

Let's break down what this means:

- The integers are consecutive, so if the first is n, then the second is n + 1, and the third is n + 2.
- The positive integers imply n > 0.
- The key condition is that n multiplied by n + 1 equals 20.

Setting Up the Mathematical Equation

To formalize the problem, assign variables:

- Let n be the first positive integer.
- The second integer is n + 1.
- The third integer is n + 2.

The condition states:

> The product of the first and second integers is 20.

Expressed mathematically:

$n \times (n + 1) = 20$

This quadratic equation is the core of our problem, and solving it will lead us to the value(s) of n.

Solving the Equation

Step 1: Expand the Equation

Starting with:

$n(n + 1) = 20$

Expanding:

$n^2 + n = 20$

Step 2: Rearrange to Standard Quadratic Form

Bring all terms to one side:

$n^2 + n - 20 = 0$

Step 3: Factor or Use the Quadratic Formula

Since the quadratic doesn’t factor easily, apply the quadratic formula:

$n = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$

Here, $a = 1$, $b = 1$, $c = -20$

Calculating the discriminant:

$D = b^2 - 4ac = 1^2 - 4(1)(-20) = 1 + 80 = 81$

Since $\sqrt{81} = 9$:

$n = [-1 \pm 9] / 2$

Step 4: Find the Roots

Calculate both solutions:

- $n = (-1 + 9) / 2 = 8 / 2 = 4$
- $n = (-1 - 9) / 2 = -10 / 2 = -5$

Step 5: Interpret the Results

Remember the problem asks for positive integers:

- n = 4 (positive) → valid
- n = -5 (negative) → discard, since integers are positive

Identifying the Three Consecutive Integers

With n = 4, the three integers are:

- First: 4
- Second: 5 (4 + 1)
- Third: 6 (4 + 2)

Verification:

Check the condition:

- Product of first and second: 4 × 5 = 20 ▯

Thus, the three consecutive positive integers satisfying the condition are 4, 5, and 6.

Additional Analysis and Variations

While the above solution is straightforward, it opens avenues for further exploration:

1. What if the product condition changes?

Suppose the product of the first and second integers equals a different number, say k. The process remains similar:

- Set up the equation: n(n + 1) = k
- Solve for n using quadratic formula
- Check for positive integer solutions

This approach can be generalized for various target products.

2. Are there multiple solutions?

In this specific case, only n = 4 is valid because n = -5 is negative. However, for other values of k, multiple solutions may exist.

3. What about other types of sequences?

Instead of consecutive integers, consider:

- Consecutive even or odd integers
- Arithmetic sequences with different common differences
- Geometric sequences

Each variation involves setting up and solving different equations based on the sequence pattern.

Common Mistakes to Avoid

When tackling such problems, watch out for:

- Assuming negative integers are valid when the problem specifies positive integers.
- Mislabeling variables—ensure the variables correctly represent the integers in sequence.
- Incorrect algebraic manipulation—double-check expansion and factoring steps.
- Overlooking solutions—sometimes quadratic equations yield multiple roots; interpret them within the problem context.

Summary and Key Takeaways

- The problem of finding three consecutive positive integers such that the product of the first and second is 20 can be approached systematically using algebra.
- Assign variables to the integers, set up a quadratic equation, and solve using the quadratic formula.
- The solution reveals that the integers are 4, 5, and 6.
- This method can be adapted to similar problems with different conditions or sequences.

Final Thoughts

Number puzzles like this one serve as excellent practice for developing algebraic skills and logical reasoning. By translating word problems into equations, solving them systematically, and verifying solutions, you strengthen your ability to approach a wide array of mathematical challenges. Whether for academic purposes, competitive exams, or personal curiosity, understanding how to find three consecutive positive integers under various constraints is a valuable addition to your mathematical toolkit.

Happy problem-solving!

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