# Find Tn For The Geometric Sequence Where T3 = 24 And T9 = 1536. Why Are There 2 Answers? (4 Marks)

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When dealing with geometric sequences, a common problem involves finding the general term $T_n$ given specific terms. In this case, we're asked to find the general term $T_n$ of a geometric sequence where $T_3 = 24$ and $T_9 = 1536$. Interestingly, this problem can yield two different solutions, which might seem confusing at first. Understanding why there are two answers requires a clear grasp of the properties of geometric sequences and their common ratios. This article will guide you through the process of solving this problem, explaining why multiple solutions are possible, and providing tips to correctly determine the general term $T_n$.

## Understanding Geometric Sequences

### What Is a Geometric Sequence?

A geometric sequence is a sequence of numbers where each term is obtained by multiplying the previous term by a constant ratio, known as the common ratio $r$. The general form of a geometric sequence is:

- $T_n = T_1 \times r^{n-1}$, where $T_1$ is the first term and $r$ is the common ratio.

### Key Properties

- The ratio $r$ remains constant throughout the sequence.
- Given any two terms, you can find $r$ or $T_1$.
- The sequence can be increasing, decreasing, or oscillating depending on the value of $r$.

## Given Data and What Is Being Asked

### Given:

- $T_3 = 24$
- $T_9 = 1536$

### Objective:

Find the general term $T_n$ of the geometric sequence, which involves determining the first term $T_1$ and the common ratio $r$.

## Setting Up the Equations

### Expressing the Known Terms

Using the general formula:

- For $T_3$:
$$T_3 = T_1 \times r^{2} = 24$$
- For $T_9$:
$$T_9 = T_1 \times r^{8} = 1536$$

### Forming Equations to Find $r$ and $T_1$

Dividing the second equation by the first to eliminate $T_1$:

$$
\frac{T_9}{T_3} = \frac{T_1 \times r^{8}}{T_1 \times r^{2}} = r^{6} = \frac{1536}{24} = 64
$$

This simplifies to:

$$
r^{6} = 64
$$

Now, solving for $r$:

$$
r = \sqrt[6]{64}
$$

Since 64 is a perfect power of 2:

$$
64 = 2^{6}
$$

Thus,

$$
r^{6} = 2^{6} \Rightarrow r = \pm 2
$$

This is the crucial point: there are two possible values for the common ratio $r$: +2 and -2.

Now, find $T_1$ using $T_3 = 24$:

$$
T_1 = \frac{T_3}{r^{2}} = \frac{24}{(r)^2}
$$

- If $r = 2$:

$$
T_1 = \frac{24}{(2)^2} = \frac{24}{4} = 6
$$

- If $r = -2$:

$$
T_1 = \frac{24}{(-2)^2} = \frac{24}{4} = 6
$$

Interestingly, $T_1$ is the same in both cases: 6.

# Two Possible General Terms

Based on the two values of $r$, the general term $T_n$ can be written as:

- When $r = 2$:
  $$
  T_n = 6 \times 2^{n-1}
  $$

- When $r = -2$:
  $$
  T_n = 6 \times (-2)^{n-1}
  $$

These two different formulas produce different sequences, even though they share the same terms at $n=3$ and $n=9$. The reason is that the negative ratio introduces oscillation in the sequence, which still satisfies the given terms.

# Why Are There 2 Answers? (4 Marks)

## Mathematical Explanation

The existence of two solutions stems from the fact that when solving for $r$, the sixth root of 64 yields both a positive and a negative root:

- $r = 2$
- $r = -2$

Since both satisfy the original equations, both are valid solutions. This duality arises because raising a negative number to an even power results in a positive number, making both ratios consistent with the given terms.

## Implications for the Sequence

- The sequence with $r=2$:
$$
T_n = 6 \times 2^{n-1}
$$
produces a sequence that grows exponentially and remains positive for all $n$.
- The sequence with $r=-2$:
$$
T_n = 6 \times (-2)^{n-1}
$$
oscillates between positive and negative terms because of the negative ratio.

## Real-World Context

In practical applications, both solutions are mathematically valid, but understanding their differences is important:

- The positive ratio sequence is often used to model growth or decay processes without oscillations.
- The negative ratio sequence models phenomena with alternating behavior,

such as certain oscillations in physics or finance.

## Summary and Final Tips

- When solving for $T_n$ in a geometric sequence with given terms, always consider the possibility of multiple solutions for the ratio $r$.
- To find $r$, divide one known term by the previous term raised to the appropriate power, then take roots carefully, considering both positive and negative roots.
- Recognize that both solutions produce valid sequences that satisfy the initial given terms.
- Always check the sequence's behavior to understand which solution aligns with the context of the problem.

## Conclusion

In summary, the problem of finding $T_n$ given $T_3=24$ and $T_9=1536$ reveals two possible solutions because the common ratio $r$ can be both $2$ and $-2$. Both satisfy the original equations due to the properties of exponents and roots. Knowing why multiple solutions exist helps develop a deeper understanding of geometric sequences and their applications. When solving similar problems, always consider the possibility of multiple roots and analyze the nature of the sequence to determine which solution makes sense in context.

Remember: In mathematics, multiple solutions are common, and recognizing them ensures a comprehensive understanding of the problem at hand.

## Frequently Asked Questions

### How do I find the general term Tn of a geometric sequence when T3 = 24 and T9 = 1536?

First, express T3 and T9 in terms of the first term a and common ratio r: T3 = $ar^2$ = 24 and T9 = $ar^8$ = 1536. Divide the second equation by the first to eliminate a: $(ar^8) / (ar^2) = r^6 = 1536 / 24 = 64$. Then, $r^6 = 64$, so $r = \pm 2$. Find a using T3: $a(2)^2 = 24$, so $a = 24 / 4 = 6$. Therefore, the two possible sequences have r = 2 and r = -2, with corresponding a values.

### Why are there two possible values for the common ratio r in this problem?

Because when solving $r^6 = 64$, taking the sixth root yields both positive and negative roots: r = 2 and r = -2. Both satisfy the original equations, leading to two valid geometric sequences.

### What are the two possible terms Tn based on the two different ratios r = 2 and r = -2?

Using a = 6: For r = 2, Tn = 6 2^{n-1}. For r = -2, Tn = 6 (-2)^{n-1}. Both formulas generate the sequence terms, but with different signs depending on n.

### How do the signs of the common ratio affect the sequence values?

If r = 2, the sequence terms are all positive and increase exponentially. If r = -2, the sequence alternates signs, with positive and negative terms alternating, affecting the pattern and sum of the sequence.

### Why is it important to consider both solutions for r in this type of problem?

Because both r = 2 and r = -2 satisfy the given terms, considering both ensures all valid sequences are identified. This reflects the fact that geometric sequences can have negative ratios, leading to different behaviors and applications.

## Additional Resources

Find Tn For The Geometric Sequence Where T3 = 24 And T9 = 1536. Why Are There 2 Answers? (4 Marks)

Understanding the behavior of geometric sequences is fundamental in mathematics, especially when dealing with problems that involve unknown terms. A common question involves determining the general term Tn of a sequence when certain terms are known. In this article, we explore a specific problem: finding the general term Tn of a geometric sequence given that T3 = 24 and T9 = 1536, and we delve into why this problem often yields two solutions. This exploration aims to clarify the methodology, illustrate the reasoning process, and shed light on the reasons behind multiple answers.

## Introduction to Geometric Sequences

Before addressing the specific problem, it’s essential to understand the foundational concepts of geometric sequences.

## What Is a Geometric Sequence?

A geometric sequence is a sequence of numbers where each term after the first is found by multiplying the previous term by a constant ratio, known as the common ratio (r).

Mathematically, the sequence can be expressed as:
- T1 = a (the first term)
- Tn = T1 r^(n-1), where n ≥ 1

Here:
- Tn is the nth term
- a is the first term
- r is the common ratio

For example, the sequence 3, 6, 12, 24, ... is geometric with a = 3 and r = 2.

## Why Is the Common Ratio Important?

The common ratio determines the growth or decay of the sequence:
- If |r| > 1, the sequence grows exponentially.
- If 0 < |r| < 1, the sequence diminishes.
- If r = 1, the sequence is constant.
- If r is negative, the sequence alternates signs.

Understanding r is crucial when solving for unknown terms, especially when only partial information about the sequence is provided.

# Given Data and Objective

The problem provides two specific terms:
- T3 = 24
- T9 = 1536

and asks to find the general term Tn, which is typically expressed as:
- Tn = a r^(n-1)

This involves determining the first term (a) and the common ratio (r). However, as we will see, the problem yields more than one valid solution, leading to the question: why are there two answers?

# Methodology for Finding Tn: Step-by-Step Approach

To find Tn, we need to determine the values of a and r. The process involves setting up equations based on the given terms.

## Step 1: Write Equations for Known Terms

Using the general form:
- $T_3 = a r^{(3-1)} = a r^2 = 24$
- $T_9 = a r^{(9-1)} = a r^8 = 1536$

Thus, our equations are:
1. $a r^2 = 24$
2. $a r^8 = 1536$

## Step 2: Eliminate a to Find r

Divide equation (2) by equation (1):

$(a r^8) / (a r^2) = 1536 / 24$

Simplify:

$r^{(8 - 2)} = 64$

$r^6 = 64$

## Step 3: Solve for r

Now, solving $r^6 = 64$:

$r^6 = 64$

Since $64 = 2^6$, we get:

$r^6 = 2^6$

Thus:

$r^6 = (2)^6$

Taking the sixth root of both sides:

$r = \pm 2$

Why ±2? Because both 2 and -2 raised to the sixth power give 64:
- $2^6 = 64$
- $(-2)^6 = 64$ (since an even power makes the negative sign irrelevant)

Important Note: This step reveals that r can be either 2 or -2.

## Step 4: Find a for Each r

Using the value of r, find a from the first equation:

- For r = 2:

a (2)^2 = 24

a 4 = 24

a = 6

- For r = -2:

a (-2)^2 = 24

a 4 = 24

a = 6

Notice that in both cases, a = 6.

# Why Are There Two Answers? The Dual Solutions Explained

At first glance, both solutions seem identical: a = 6, with r = 2 or r = -2. But the question arises: is both valid? Why do we even consider the negative ratio? The answer lies in the nature of the sequence and the implications of negative ratios.

## Impact of the Sign of the Common Ratio

- Positive ratio (r = 2): The sequence is strictly increasing:
T1 = 6
T2 = 6 2 = 12
T3 = 24 (matches given)
T4 = 24 2 = 48
T5 = 96
...

- Negative ratio (r = -2): The sequence alternates signs:
T1 = 6
T2 = 6 (-2) = -12
T3 = -12 (-2) = 24 (matches given)
T4 = 24 (-2) = -48
T5 = 48
T6 = -96

T7 = 192
T8 = -384
T9 = 768 (but we want T9 = 1536, so wait...)

Let's check T9 in the negative ratio sequence:

T9 = a r^(8) = 6 (-2)^8

Since (-2)^8 = 256,

T9 = 6 256 = 1536

Which matches the given T9. So both ratios are valid solutions fitting the given data points.

## Summary of the Two Solutions

- Solution 1:

- a = 6
- r = 2

Sequence: 6, 12, 24, 48, 96, 192, 384, 768, 1536, ...

- Solution 2:

- a = 6
- r = -2

Sequence: 6, -12, 24, -48, 96, -192, 384, -768, 1536, ...

Both sequences satisfy the conditions T3 = 24 and T9 = 1536, hence both are valid solutions.

# Implications of Multiple Solutions in Geometric Sequences

The existence of two solutions stems from the mathematical property that an even power of a negative number yields a positive result, allowing both positive and negative ratios to produce the same terms at certain positions. This duality often appears in problems involving even exponents and roots.

Why Is This Important?

- It highlights the importance of considering the nature of the ratio, especially when the sequence involves even powers.
- It emphasizes the need to interpret context: in real-world scenarios,

negative ratios may or may not make sense depending on what the sequence models.
- It teaches students to recognize that algebraic solutions can have multiple valid answers, and understanding the context is key to selecting the most appropriate.

## Determining the General Term Tn in Both Cases

Now, having established both solutions, let's write the general term Tn for each case.

Case 1: r = 2

Tn = a r^(n-1) = 6 2^(n-1)

Case 2: r = -2

Tn = 6 (-2)^(n-1)

These formulas allow us to find any term in the sequence, given the initial parameters.

## Conclusion: Why Are There Two Answers? A Recap

This problem exemplifies a key aspect of algebra and sequences: the solutions are tied to the properties of exponents and the nature of the ratio. When solving for the common ratio in a geometric sequence, taking roots of even powers results in two possible values—positive and negative. Both satisfy the original equations, but the sequence's behavior differs depending on the sign of the ratio.

In practical terms, both solutions are mathematically valid, but in real-world applications, the context determines which ratio makes sense. For example, if modeling population growth, a negative ratio might be nonsensical, whereas in physics or finance, both might be plausible depending on the scenario.

Final Takeaway: When solving for the terms of a geometric sequence with partial data, always consider the possibility of multiple solutions resulting from taking roots of even powers. Recognize the implications of each solution, and select the most contextually appropriate one.

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In summary:

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- The general term Tn can be expressed as Tn = 6 2^(n-1) or Tn = 6
(-2)^(n-1).
- Both solutions satisfy the given terms T3 = 24 and T9 = 1536.
- Multiple answers
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## Find Tn For The Geometric Sequence Where T3 24 And T9 1536 Why Are There 2 Answers 4 Marks

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