# For The Demand Function Q = D(x) = 500/x, Find The Following. A) The Elasticity B) The Elastic

### For The Demand Function Q = D(x) = 500/x, Find The Following. A) The Elasticity B) The Elastic

Understanding demand functions is fundamental in economics, especially when analyzing how the quantity demanded responds to changes in price. In this article, we will explore the demand function $Q = D(x) = \frac{500}{x}$, derive the price elasticity of demand, and determine whether the demand is elastic, inelastic, or unit elastic at various points. Let's delve into the concepts step-by-step.

## Understanding the Demand Function $Q = D(x) = \frac{500}{x}$

Before calculating elasticity, it's essential to interpret the demand function:

- Q (Quantity Demanded): The number of units consumers are willing and able to purchase at price $x$.
- x (Price): The price per unit of the good.

This particular demand function $D(x) = \frac{500}{x}$ indicates an inverse relationship between price and quantity demanded: as price increases, demand decreases, and vice versa.

## Part A: Calculating Price Elasticity of Demand

### What is Price Elasticity of Demand?

Price elasticity of demand measures how much the quantity demanded responds to a change in price. It is calculated as:

$$
E_d = \frac{\%\ \text{Change in Quantity Demanded}}{\%\ \text{Change in Price}}
$$

Mathematically, the point elasticity at a specific price $x$ is given by:

\[

$$
E_d = \frac{dQ}{d x} \times \frac{x}{Q}
$$

where:

- $\frac{dQ}{d x}$: the derivative of the demand function with respect to price.
- $x$: the current price.
- $Q$: the quantity demanded at price $x$.

This formula provides the elasticity at a specific point on the demand curve.

## Step-by-Step Calculation

Given:

$$
Q = D(x) = \frac{500}{x}
$$

1. Differentiate $Q$ with respect to $x$:

$$
\frac{dQ}{d x} = \frac{d}{d x} \left( \frac{500}{x} \right) = - \frac{500}{x^2}
$$

2. Calculate $E_d$ at a specific price $x$:

$$
E_d = \left( - \frac{500}{x^2} \right) \times \frac{x}{Q}
$$

Since $Q = \frac{500}{x}$, substitute into the equation:

$$
E_d = - \frac{500}{x^2} \times \frac{x}{\frac{500}{x}} = - \frac{500}{x^2} \times \frac{x \times x}{500} = - \frac{500}{x^2} \times \frac{x^2}{500}
$$

Simplify numerator and denominator:

$$
E_d = - 1
$$

\]

Result:

$$
\boxed{
E_d = -1
}
$$

This indicates that at any price $x$, the price elasticity of demand for this function is exactly -1.

## Interpretation of the Elasticity Result

- Elasticity of -1: The demand is unit elastic at all points along the curve.
- Absolute value: The magnitude of elasticity is 1, indicating a proportional responsiveness.

Note: The negative sign reflects the inverse relationship between price and quantity demanded, consistent with the law of demand.

# Part B: Determining Whether the Demand is Elastic, Inelastic, or Unit Elastic

## Understanding Elasticity Types

The magnitude of price elasticity of demand determines the type of demand:

- Elastic demand: $|E_d| > 1$. Quantity demanded responds more than proportionally to price changes.
- Inelastic demand: $|E_d| < 1$. Quantity demanded responds less than proportionally.
- Unit elastic demand: $|E_d| = 1$. Quantity demanded responds exactly proportionally.

Since we found $E_d = -1$:

$$
|E_d| = 1
$$

This means the demand is unit elastic at all points along the demand curve.

## Implications of Uniform Elasticity

Because the elasticity is constant and equals -1 everywhere:

- Total revenue remains unchanged when the price changes. For example:
- If price increases by 10%, quantity demanded decreases by 10%, resulting in no net change in total revenue.
- Consumers and producers experience a perfectly balanced response to price changes.

## Graphical Representation

Visualizing the demand curve $Q = \frac{500}{x}$ confirms the constant elasticity:

- The curve is a rectangular hyperbola, characteristic of a demand function with constant elasticity.
- The slope $\frac{dQ}{d x}$ varies as $x$ changes, but the elasticity remains constant due to the proportional relationship.

# Additional Considerations and Applications

## Why Is Elasticity Important?

Understanding elasticity helps businesses and policymakers:

- Set optimal pricing strategies.
- Predict revenue changes resulting from price adjustments.
- Assess consumer responsiveness to market fluctuations.

## Examples of Elasticity Application

Suppose the current price $x$ is \$50:

1. Calculate quantity demanded:

$$
Q = \frac{500}{50} = 10
$$

2. Evaluate the impact of a 10% price increase:

- New price: $50 \times 1.10 = \$55$
- New quantity demanded:

$$
Q' = \frac{500}{55} \approx 9.09
$$

3. Calculate percentage change in quantity:

$$
\%\ \Delta Q = \frac{9.09 - 10}{10} \times 100 \% \approx -9.09\%
$$

4. Calculate total revenue before and after:

- Before:

$$
TR = P \times Q = 50 \times 10 = \$500
$$

- After:

$$
TR' = 55 \times 9.09 \approx \$500
$$

Total revenue remains approximately unchanged, consistent with the elastic demand being unit elastic.

# Summary

In conclusion:

- The demand function $Q = \frac{500}{x}$ exhibits constant elasticity of -1.
- The demand is unit elastic at all points along the curve.

- Price changes do not affect total revenue because the proportional changes in price and quantity demanded offset each other.
- Recognizing the nature of demand elasticity assists businesses in making informed pricing decisions and understanding market dynamics.

## Final Remarks

Understanding demand elasticity is vital for effective economic decision-making. The demand function $Q = \frac{500}{x}$ provides a clear example of a scenario with constant elasticity, illustrating the importance of precise calculations and interpretations in economic analysis. Whether for setting prices, predicting consumer behavior, or analyzing market responses, elasticity remains a cornerstone concept in microeconomics.

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References:


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## Frequently Asked Questions

### How do you derive the price elasticity of demand from the demand function Q = 500/x?

The price elasticity of demand (E) is calculated as $E = (dQ/dx)(x/Q)$. Given $Q = 500/x$, then $dQ/dx = -500/x^2$. Substituting, $E = (-500/x^2)(x / (500/x)) = -(x / 500)$.

### What is the formula for the elasticity of demand in terms of the demand function Q = 500/x?

The elasticity of demand $E = (dQ/dx)(x/Q)$. For $Q = 500/x$, this simplifies to $E = -(x / 500)$.

## At what price (x) is the demand perfectly elastic or inelastic for Q = 500/x?

Since E = - (x / 500), demand is elastic when |E| > 1, which occurs when x > 500; it is inelastic when x < 500; and perfectly elastic at x = 500 (|E|=1).

## How do you interpret the sign of the elasticity for the demand function Q = 500/x?

The negative sign indicates that demand decreases as price increases, following the law of demand. The magnitude indicates the responsiveness of demand to price changes.

## What does an elasticity value of -1 signify in the context of the demand function Q = 500/x?

An elasticity of -1 signifies unit elasticity, meaning a 1% change in price leads to a 1% change in quantity demanded; this occurs at x = 500.

## How can I find the elastic region for the demand function Q = 500/x?

By analyzing |E| = x / 500, demand is elastic when x > 500, inelastic when x < 500, and unit elastic at x = 500.

## What is the significance of the elasticity value being greater than 1 or less than 1 in this demand function?

An elasticity greater than 1 (|E| > 1) indicates elastic demand, meaning quantity demanded is highly responsive to price changes. Less than 1 (|E| < 1) indicates inelastic demand, where quantity is less responsive.

## How do you compute the actual elasticity at a specific price point x = 200 in the demand function?

At x = 200, elasticity E = - (200 / 500) = -0.4. The demand is inelastic at this price point.

## Why is understanding the elasticity important for businesses using the demand function Q = 500/x?

Knowing elasticity helps businesses determine how changes in price will affect demand, enabling better pricing strategies to maximize revenue or market share.

## How does the elasticity change as price x varies in the demand function Q = 500/x?

Elasticity is directly proportional to x; as price increases, the demand becomes more elastic (|E| increases), and as price decreases, demand becomes more inelastic (|E| decreases).

# Additional Resources

Demand Function Analysis: Elasticity and Elastic — An In-Depth Review

Understanding demand functions is fundamental in economics, as they reveal how consumers respond to price changes and help businesses strategize pricing, production, and marketing. Today, we delve into a specific demand function, \( Q = D(x) = \frac{500}{x} \), exploring its properties through the lenses of elasticity and elasticity coefficient. This comprehensive analysis will guide you through the derivations, interpretations, and practical implications of these concepts.

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# 1. Introduction to the Demand Function \( Q = \frac{500}{x} \)

Before diving into elasticity, it's essential to understand the structure and behavior of the given demand function:

- Functional form: \( Q = \frac{500}{x} \)
- Variables:
- \( Q \): Quantity demanded
- \( x \): Price (or some related variable influencing demand)

This is a hyperbolic demand function, which implies that demand decreases as price increases, consistent with the law of demand. The function exhibits a nonlinear inverse proportional relationship between \( Q \) and \( x \).

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# 2. Concept of Price Elasticity of Demand

Price elasticity of demand (E) measures how much the quantity demanded responds to a change in price. It is a unitless measure that quantifies the sensitivity of demand:

$$
E = \frac{\%\ \text{change in quantity demanded}}{\%\ \text{change in price}} = \frac{dQ/dx \times x}{Q}
$$

- If $|E| > 1$, demand is elastic (responsive)
- If $|E| < 1$, demand is inelastic (not very responsive)
- If $|E| = 1$, demand is unit elastic

Note: The negative sign in $E$ reflects the inverse relationship between $Q$ and $x$. Typically, we consider the absolute value when discussing elasticity.

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# 3. Deriving the Elasticity for $Q = \frac{500}{x}$

Step 1: Compute the derivative $dQ/dx$

Given $Q = \frac{500}{x}$, which can be rewritten as:

$$
Q = 500 x^{-1}
$$

Differentiate with respect to $x$:

$$
\frac{dQ}{dx} = -500 x^{-2} = -\frac{500}{x^2}
$$

Step 2: Plug into the elasticity formula

Using:

$$
E = \frac{dQ/dx \times x}{Q}
$$

Substitute the derivatives and original $Q$:

\[

$$
E = \left(-\frac{500}{x^2}\right) \times x \div \left(\frac{500}{x}\right)
$$

Simplify numerator:

$$
-\frac{500}{x^2} \times x = -\frac{500}{x}
$$

Now, divide by $Q = \frac{500}{x}$:

$$
E = \frac{-\frac{500}{x}}{\frac{500}{x}} = -1
$$

Result:

$$
\boxed{
E = -1
}
$$

The elasticity coefficient is constant and equal to -1 across all values of $x$.

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# 4. Interpretation of the Elasticity Result

Constant Elasticity: An elasticity of $-1$ indicates unit elasticity throughout the demand function.

- Implication: For a 1% increase in price $x$, the quantity demanded decreases by exactly 1%. Conversely, a 1% decrease in price results in a 1% increase in quantity demanded.
- Inelasticity vs. Elasticity: The absolute value of the elasticity is 1, signifying unit elasticity — demand is perfectly balanced in responsiveness.

Graphical intuition:

- The demand curve $Q = 500/x$ is a rectangular hyperbola.
- Along this curve, the percentage change in quantity is exactly offset by the percentage change in price.

Economic significance:

- When elasticity is precisely \(-1\), total revenue (TR) remains constant when price changes because the percentage change in price and quantity demanded cancel out.

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# 5. Understanding the Elasticity of the Demand Function

Since the elasticity is constant at -1, this demand is considered unit elastic everywhere along the demand curve.

What does this mean practically?

- Revenue implications: Changes in price do not affect total revenue as the increase in price is offset by an equivalent decrease in quantity demanded.
- Pricing strategies: For businesses, attempting to alter prices on such a demand curve will not influence total sales revenue — it remains unchanged.
- Consumer behavior: The responsiveness of consumers is perfectly proportional; they adjust their demand exactly in line with price changes.

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# 6. Broader Implications and Real-World Applications

a. Pricing and Revenue Management

- Since the demand is unit elastic, a business contemplating a price increase won't see a rise in revenue, nor will a decrease lead to higher revenue.
- For maximizing revenue, the firm should maintain the current price level corresponding to this demand function.

b. Market Analysis

- Recognizing unit elasticity helps firms identify the "sweet spot" where revenue is maximized or stabilized.
- If a firm observes demand following a similar pattern, it can anticipate that small price adjustments won't significantly impact total sales.

c. Limitations and Assumptions

- Real-world demand rarely exhibits perfect unit elasticity across all price ranges.
- Factors like consumer preferences, substitutes, and market conditions often cause elasticity to vary.
- The derived elasticity of $-1$ applies strictly within the mathematical model and idealized assumptions.

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# 7. Additional Insights and Mathematical Nuances

a. Elasticity at specific points

- Since the elasticity is constant at $-1$, it does not vary with $x$.
- For other demand functions, elasticity could change at different points, influencing strategic decisions.

b. Elasticity in different units

- The elasticity measure is unitless, making it versatile across different contexts and units.
- It provides a standardized way to compare demand responsiveness across markets.

c. Relationship with total revenue

Total revenue (TR):

$$
TR = x \times Q = x \times \frac{500}{x} = 500
$$

- Constant TR confirms the elasticity's implication: demand is perfectly balanced.
- Any change in $x$ doesn't affect TR, reinforcing the concept of unit elastic demand.

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# 8. Summary and Concluding Remarks

- The demand function $Q = \frac{500}{x}$ exhibits constant unit elasticity with a value of $-1$.
- This indicates a perfectly proportional response of demand to price changes: a 1% increase in price causes a 1% decrease in quantity demanded, and vice versa.
- The demand curve’s hyperbolic shape reflects this inverse proportionality, and the constant elasticity

simplifies strategic decision-making.
- From a practical standpoint, businesses operating under similar demand conditions should recognize that price adjustments won't impact total revenue, emphasizing the importance of other factors such as costs, market share, or product differentiation for revenue growth.

Final thought: Analyzing demand functions through elasticity provides crucial insights into consumer behavior and market dynamics. Recognizing whether demand is elastic, inelastic, or unit elastic guides effective pricing strategies, revenue optimization, and market positioning.

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In conclusion, the demand function \( Q = 500/x \) is a classic example illustrating the concepts of elasticity, revealing a consistent unit elastic response. This comprehensive understanding underscores the importance of elasticity analysis in both theoretical economics and practical business decision-making.

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