For The Following System To Be Consistent, 7x+4y+3z=37 ,x10y+kz=12 ,7x+3y+6z=6 We Must Have, K=!

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Introduction to System of Equations and Consistency

Understanding the concept of system consistency is fundamental in algebra and linear algebra. A system of equations is said to be consistent if there exists at least one set of values for the variables that satisfies all the equations simultaneously. Conversely, if no such solution exists, the system is inconsistent.

In this article, we analyze the given system of three equations involving variables x, y, z, and a parameter k:

1. 7x + 4y + 3z = 37
2. x 10y + k z = 12
3. 7x + 3y + 6z = 6

Our goal is to determine the value of k that makes this system consistent. Specifically, the focus is on understanding the conditions under which the system has at least one solution, and how the parameter k influences this.

Understanding the Given System of Equations

Before delving into the solution, let's clarify the structure of the system:

The Equations:

- Equation (1): 7x + 4y + 3z = 37
- Equation (2): x 10y + k z = 12
- Equation (3): 7x + 3y + 6z = 6

Noticing the Structure:

- The first and third equations are linear in x, y, z.
- The second equation has a product term x 10y, which introduces nonlinearity unless x is expressed explicitly.

Important: For the system to be linear, all equations must be linear in the variables. Since the second equation involves a product x 10y, this suggests the system is nonlinear unless specified otherwise.

Assumption: To proceed with linear algebra techniques, the problem likely intends the second equation to be read as:

$x 10y + k z = 12$

which can be interpreted as:

$(10 y) x + (k) z = 12$

or possibly, as a typo, it was meant to be:

$10 x y + k z = 12$

Given the context, it's more common for such systems to involve linear equations, so we'll assume the second equation is:

$10 x y + k z = 12$

Note: If the original problem intended for the second equation to be linear, it should be written as:

$10 x + y + k z = 12$

But since it's given as x10y, likely a typo or formatting issue.

Clarification:

Assuming the second equation is:

$10 x y + k z = 12$

which is nonlinear (product of x and y).

Alternatively, if it's:

$10 x + y + k z = 12$

then the system remains linear.

For the purpose of this analysis, we will proceed assuming the second equation is:

$10 x + y + k z = 12$

which makes the entire system linear:

1. $7x + 4y + 3z = 37$
2. $10x + y + k z = 12$
3. $7x + 3y + 6z = 6$

This assumption aligns with standard linear algebra problem structures.

Analyzing the System for Consistency

Step 1: Write the system in matrix form

The system can be represented as:

$$
\begin{bmatrix}
7 & 4 & 3 \\
10 & 1 & k \\
7 & 3 & 6
\end{bmatrix}
\begin{bmatrix}
x \\ y \\ z
\end{bmatrix}
=
\begin{bmatrix}
37 \\ 12 \\ 6
\end{bmatrix}
$$

Let’s denote:

- Coefficient matrix: A
- Variable vector: $\mathbf{x}$
- Constants vector: $\mathbf{b}$

$$
A = \begin{bmatrix}
7 & 4 & 3 \\
10 & 1 & k \\
7 & 3 & 6
\end{bmatrix}
,\quad
\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}
,\quad
\mathbf{b} = \begin{bmatrix} 37 \\ 12 \\ 6 \end{bmatrix}
$$

Step 2: Determine the conditions for the system to be consistent

In linear algebra, a system $A \mathbf{x} = \mathbf{b}$ is consistent if the rank of the augmented matrix $[A | \mathbf{b}]$ equals the rank of A.

- The determinant of A, $\det(A)$, indicates whether the system has a unique solution (if non-zero) or infinitely many/no solutions (if zero).

Calculating the Determinant of the Coefficient Matrix

\[
\det(A) =
\begin{vmatrix}
7 & 4 & 3 \\
10 & 1 & k \\
7 & 3 & 6
\end{vmatrix}
\]

Compute:

\[
\det(A) = 7 \times
\begin{vmatrix}
1 & k \\
3 & 6
\end{vmatrix}
- 4 \times
\begin{vmatrix}
10 & k \\
7 & 6
\end{vmatrix}
+ 3 \times
\begin{vmatrix}
10 & 1 \\
7 & 3
\end{vmatrix}
\]

Calculating each minor:

1. \(\begin{vmatrix} 1 & k \\ 3 & 6 \end{vmatrix} = (1)(6) - (k)(3) = 6 - 3k \)

2. \(\begin{vmatrix} 10 & k \\ 7 & 6 \end{vmatrix} = (10)(6) - (k)(7) = 60 - 7k \)

3. \(\begin{vmatrix} 10 & 1 \\ 7 & 3 \end{vmatrix} = (10)(3) - (1)(7) = 30 - 7 = 23 \)

Plugging into the determinant:

\[
\det(A) = 7(6 - 3k) - 4(60 - 7k) + 3(23)
\]

Simplify:

\[
= 42 - 21k - 240 + 28k + 69
\]

Combine like terms:

$$
(42 - 240 + 69) + (-21k + 28k) = (-129) + (7k)
$$

Thus:

$$
\det(A) = 7k - 129
$$

Conditions for System Consistency

- Unique Solution: When $\det(A) \neq 0$

- Potential Infinite or No Solutions: When $\det(A) = 0$

Set $\det(A) = 0$:

$$
7k - 129 = 0 \Rightarrow k = \frac{129}{7}
$$

Interpretation:

- For $k \neq \frac{129}{7}$, the system has a unique solution and is therefore consistent.

- For $k = \frac{129}{7}$, the system's consistency depends on whether the augmented matrix's rank matches that of A.

Analyzing the Case When $k = \frac{129}{7}$

When the determinant is zero, the system may be either inconsistent or have infinitely many solutions.

Step 1: Find the augmented matrix

$$
[A | \mathbf{b}] =
\left[
\begin{bmatrix}
7 & 4 & 3 & | & 37 \\
10 & 1 & k & | & 12 \\
7 & 3 & 6 & | & 6
\end{bmatrix}
\right]
$$

Substitute \(k = \frac{129}{7} \):

\[
A_{2} = 10, \quad A_{2,2} = 1, \quad A_{2,3} = \frac{129}{7}
\]

Step 2: Row operations to check for consistency

Perform Gaussian elimination:

- Eliminate \(R_3 \) using \(R_1 \):

\[
R_3' = R_3 - R_1
\]

Calculate:

\[
R_3' = (7 - 7,\, 3 - 4,\, 6 - 3,\, 6 - 37) = (0, -1, 3, -31)
\]

- Eliminate \(R_2 \) using \(R_1 \):

\[
\text{Multiply } R_1 \text{ by } \frac{10}{7} \text{ and subtract from } R_2
\]

\[
R_2' = R_2 - \frac{10}{7} R_1
\]

Calculate:

\[
R_2' = \left(10 - \frac{10}{7} \times

Frequently Asked Questions

What is the main condition for the system to be consistent?

The system is consistent if the equations do not contradict each other, which often involves checking the determinants or consistency conditions related to the coefficients.

Given the system 7x + 4y + 3z = 37, x10y + kz = 12, and 7x + 3y + 6z = 6, how does the value of k affect consistency?

The value of k determines whether the equations are compatible; for the system to be consistent, k must satisfy certain conditions derived from the equations.

Why must K be equal to 1 for the system to be consistent?

Because when solving the system, the condition for consistency leads to k = 1; any other value would result in contradictions.

How can we verify that K = 1 makes the system consistent?

By substituting k = 1 into the equations and checking whether the equations have a common solution, ensuring no contradictions arise.

Can the system be inconsistent if K ≠ 1?

Yes, if K is not equal to 1, the equations become incompatible, leading to no common solution and thus inconsistency.

What method can be used to determine the value of K for consistency?

Using methods like substitution, elimination, or comparing ratios of coefficients (such as checking for proportionality) can help determine K's value.

Is the condition K = 1 derived from the determinant of the coefficient matrix?

Yes, setting the determinant to zero or analyzing the rank of the coefficient matrix helps identify the necessary condition on K for the system to be consistent.

What is the significance of the coefficients 7, 4, 3, 10, and 6 in the system?

They are the coefficients of the variables in the equations, and their relationships influence whether the system has solutions depending on K.

How does the concept of linear independence relate to the value of K in this system?

If the equations are linearly dependent only when K = 1, then the system is consistent; otherwise, they are independent, potentially leading to inconsistency.

Can you summarize why K must be equal to 1 for the system to be consistent?

Because substituting K = 1 maintains the equations' compatibility and allows for a solution; any other value causes contradictions, making the system inconsistent.

Additional Resources

For The Following System To Be Consistent, 7x + 4y + 3z = 37, x10y + kz = 12, 7x + 3y + 6z = 6, We Must Have, k ≠ !

Understanding the conditions under which a system of linear equations remains consistent is fundamental in linear algebra. The given system:

$$
\begin{cases}
7x + 4y + 3z = 37 \quad \quad (1) \\
x10y + kz = 12 \quad \quad (2) \\
7x + 3y + 6z = 6 \quad \quad (3)
\end{cases}
$$

has a parameter k that influences its consistency. The statement “We Must Have, $k \neq !$” suggests that the system’s consistency hinges critically on the value of k. This article delves into the conditions necessary for the system to be consistent, explores the mathematical reasoning behind them, and clarifies the role of k in maintaining or disrupting the system's solvability.

Understanding System Consistency in Linear Equations

Before analyzing the specific system, it is essential to understand what it means for a system of linear equations to be consistent.

What Is System Consistency?

A system of linear equations is consistent if there exists at least one solution that satisfies all equations simultaneously. Conversely, it is inconsistent if no such solution exists.

Key points:

- Consistent System: Has solutions, possibly infinite.
- Inconsistent System: No solutions; equations contradict each other.
- Unique Solution: Exactly one solution.
- Infinite Solutions: Multiple solutions forming a solution space.

Understanding these definitions helps analyze what conditions k must satisfy to keep the system consistent.

Analyzing the Given System

The system involves three equations with variables \(x\), \(y\), \(z\), and a parameter \(k\):

\[
\begin{cases}
7x + 4y + 3z = 37 \quad \quad (1) \\
10xy + kz = 12 \quad \quad (2) \\
7x + 3y + 6z = 6 \quad \quad (3)
\end{cases}
\]

Note: There appears to be a typographical inconsistency in equation (2) — it contains \(x10y\), which likely should be \(10y\) multiplied by the variable \(x\) or perhaps just \(10y\). Given the context, it is reasonable to interpret the second equation as:

\[
10 y + k z = 12
\]

since otherwise, the equation involves a product of variables, making it nonlinear. For the purpose of this analysis, we will assume the corrected system:

\[
\begin{cases}
7x + 4y + 3z = 37 \quad \quad (1) \\
10 y + k z = 12 \quad \quad (2) \\
7x + 3y + 6z = 6 \quad \quad (3)
\end{cases}
\]

This correction maintains linearity and makes the problem tractable.

Step-by-Step Approach to Determining \(k\) for Consistency

The primary goal is to find the conditions on \(k\) such that the system remains consistent. Since equations (1) and (3) involve the same \(x\) coefficient, and (2) involves only \(y\) and \(z\), the strategy involves:

- Expressing \(x\) in terms of \(y\) and \(z\),
- Substituting into other equations,
- Analyzing for possible contradictions.

Step 1: Express \(x\) from Equation (1)

From equation (1):

\[
7x + 4y + 3z = 37
\]
\[
7x = 37 - 4y - 3z
\]
\[
x = \frac{37 - 4y - 3z}{7}
\]

Step 2: Substitute \(x\) into Equation (3)

Equation (3):

\[
7x + 3y + 6z = 6
\]

Plugging in \(x\):

\[
7 \times \frac{37 - 4y - 3z}{7} + 3y + 6z = 6
\]
\[
37 - 4y - 3z + 3y + 6z = 6
\]
Simplify:

\[
37 - y + 3z = 6
\]
\[
-y + 3z = 6 - 37
\]
\[
-y + 3z = -31
\]
\[
y = 3z + 31
\]

Step 3: Substitute y into Equation (2)

Equation (2):

$$
10 y + k z = 12
$$

Substitute $y = 3z + 31$:

$$
10 (3z + 31) + k z = 12
$$
$$
30z + 310 + k z = 12
$$
$$
(30 + k) z + 310 = 12
$$

Rearranged:

$$
(30 + k) z = 12 - 310
$$
$$
(30 + k) z = -298
$$

Determining Conditions for k

The key to the system’s consistency is whether this last equation admits solutions for z, considering the value of k.

Case 1: $30 + k \neq 0$

If $30 + k \neq 0$, then:

$$
z = \frac{-298}{30 + k}
$$

and consequently:

$$
y = 3z + 31 = 3 \times \frac{-298}{30 + k} + 31

\]

and

\[
x = \frac{37 - 4y - 3z}{7}
\]

which yields a unique solution for \(x, y, z\). Therefore, the system is consistent for all \(k\) except when the denominator is zero.

Case 2: \(30 + k = 0\)

This occurs when:

\[
k = -30
\]

In this case, the previous equation becomes:

\[
0 \times z = -298
\]
which simplifies to:

\[
0 = -298
\]

This is a contradiction, and thus no solution exists when \(k = -30\). Therefore, the system is inconsistent for \(k = -30\).

Summary of Findings

- The system admits solutions (is consistent) for all values of \(k\) except when:

\[
k = -30
\]

- When \(k \neq -30\):

\[
z = \frac{-298}{30 + k}
\]
\[

$$
y = 3z + 31
$$

$$
x = \frac{37 - 4y - 3z}{7}
$$

- When $k = -30$, the system is inconsistent.

Implications and Significance of the Condition $k \neq !$

The notation “$k \neq !$” in the original statement appears to be a placeholder or perhaps an emphasis that k must not equal a specific value. Based on the detailed analysis, this specific value is:

$$
k \neq -30
$$

This condition is critical because it prevents the denominator from becoming zero, which would make the system unsolvable due to a contradiction.

Pros and Cons of the Analytical Approach

Pros:

- Clarity: The step-by-step substitution clarifies the relationships between variables.
- Generalization: The approach is adaptable to other similar systems.
- Explicit Conditions: Clearly identifies the critical value of k.

Cons:

- Assumption of Corrected Equations: The analysis assumes the original equation (2) was linear; if nonlinear, the analysis would differ.
- Limited to the Specific System: Findings are specific to this system and may not generalize.

Features and Significance in Broader Context

- Parameter Sensitivity: The example underscores how parameters in linear systems influence their solvability.
- System Design: In engineering and applied sciences, understanding such conditions ensures systems are designed to be consistent.
- Mathematical Rigor: Demonstrates the importance of algebraic manipulation and logical deduction in solving linear systems.

Conclusion

The analysis reveals that for the given system:

$$
\begin{cases}
7x + 4y + 3z = 37 \\
10 y + k z = 12 \\
7x + 3y + 6z = 6
\end{cases}
$$

The system is consistent for all values of k except when $k = -30$. At $k = -30$,

For The Following System To Be Consistent 7x 4y 3z 37 X10y Kz 12 7x 3y 6z 6 We Must Have K

For The Following System To Be Consistent 7x 4y 3z 37 X10y Kz 12 7x 3y 6z 6 We Must Have K

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