Give A Rough Sketch Of The Following Function G(x) = Cotx = Cosx/sinx . Find Domain And Range.

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Introduction to the Cotangent Function

The cotangent function, denoted as G(x) = Cotx, is a fundamental trigonometric function that describes the ratio of the cosine to the sine of an angle x. Like other trigonometric functions, it exhibits periodicity, symmetry, and specific domain and range characteristics. Understanding these properties is essential for graphing the function accurately and applying it effectively in various mathematical contexts.

In this article, we will explore the definition of G(x) = cotx, analyze its domain and range, and provide a conceptual sketch of its graph, highlighting key features such as asymptotes, zeros, and periodic behavior.

Definition and Basic Properties of G(x) = Cotx

Mathematical Definition

The cotangent function is defined as:

- G(x) = Cotx = Cosx / Sinx

where Sinx and Cosx are the sine and cosine functions, respectively.

Key Properties

- Periodicity: Cotx has a fundamental period of π (180°). This means the graph repeats every π units.
- Symmetry: The cotangent function is an odd function, satisfying G(-x) = -G(x). Its graph is symmetric with respect to the origin.
- Discontinuities: Points where Sinx = 0 lead to undefined values of cotx, producing vertical asymptotes.
- Range: G(x) can take all real values, from negative infinity to positive infinity, wherever it is defined.

Finding the Domain of $G(x) = \cot x$

Understanding the Definition Constraints

Since $G(x) = \cos x / \sin x$, the function is undefined wherever the denominator $\sin x = 0$. This occurs at specific points on the x-axis.

Points of Discontinuity

- $\sin x = 0$ at $x = n\pi$, where n is any integer ($n \in \mathbb{Z}$).
- Therefore, the points $x = \ldots, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi, \ldots$ are where $G(x)$ is undefined.

Expressing the Domain

- The domain includes all real numbers except the points where $\sin x = 0$.
- Mathematically, the domain D is:

- $D = \mathbb{R} \setminus \{x \mid x = n\pi, n \in \mathbb{Z}\}$

This can be expressed as a union of open intervals:

- $(-\infty, -\pi), (-\pi, 0), (0, \pi), (\pi, 2\pi), (2\pi, 3\pi), \ldots$

or more generally,

- For each integer n, the domain includes $(n\pi, (n+1)\pi)$, excluding the endpoints where the function is undefined.

Analyzing the Range of $G(x) = \cot x$

Behavior of the Cotangent Function

- As x approaches points where $\sin x = 0$ from the left or right, $\cot x$ tends to $\pm\infty$.
- Between these asymptotes, $\cot x$ is continuous and monotonically decreasing from $+\infty$ to $-\infty$ over each interval.

Range Determination

- Since cotx approaches infinity or negative infinity near the vertical asymptotes and takes all real values in between, the range covers all real numbers.

Summary of Range

- Range: $\mathbb{R}$ (all real numbers)

Graphical Features of G(x) = Cotx

Vertical Asymptotes

- Occur at points where Sinx = 0, i.e., $x = n\pi$.
- The graph approaches infinity or negative infinity as it nears these points.

Zeros of the Function

- Cotx = 0 where Cosx = 0 and Sinx $\neq$ 0.
- Cosx = 0 at $x = (\pi/2) + n\pi$.
- Therefore, zeros of cotx occur at:

- $x = (\pi/2) + n\pi$, where $n \in \mathbb{Z}$.

Behavior Between Asymptotes

- On each interval between two asymptotes, the graph is a smooth, decreasing curve.
- It extends from $+\infty$ to $-\infty$ or vice versa, depending on the interval.

Periodicity and Symmetry

- The graph repeats every π.
- It is symmetric about the origin, reflecting its odd nature.

Rough Sketch of the Graph

Step-by-Step Visualization

1. Draw vertical dashed lines at $x = n\pi$ to indicate asymptotes.
2. Mark zeros at $x = (\pi/2) + n\pi$.
3. On each interval between asymptotes:
- Start from $+\infty$ near one asymptote.
- Pass through zero at the midpoint (where Cosx=0).
- Descend towards $-\infty$ near the next asymptote.
4. Repeat this pattern periodically, maintaining symmetry and the decreasing trend.

Key Points to Remember

- The graph has a series of decreasing curves between asymptotes.
- It crosses the x-axis at $x = (\pi/2) + n\pi$.
- Asymptotes are vertical lines at $x = n\pi$.

Applications and Significance of G(x) = Cotx

Applications in Mathematics

- Solving trigonometric equations involving cotangent.
- Integration and differentiation in calculus.
- Analyzing periodic phenomena, such as wave behavior.

Real-World Contexts

- Engineering: signal processing, oscillations.
- Physics: describing angles of projection and wave interactions.
- Geometry: in problems involving right triangles and circle properties.

Conclusion

The cotangent function, G(x) = Cosx / Sinx, is a pivotal element of trigonometry with distinctive features. Its domain excludes points where Sinx = 0, i.e., multiples of π, leading to vertical asymptotes at these points. The range encompasses all real numbers due to the function's unbounded behavior between asymptotes. Its graph is characterized by periodic decreasing curves, zeros at specific points where Cosx = 0, and symmetry about the origin. Understanding these properties enables mathematicians and students to sketch the graph accurately and apply the cotangent function effectively in various analytical contexts.

This comprehensive overview provides the foundational knowledge necessary to visualize and interpret G(x) = cotx, highlighting its fundamental properties and behaviors.

Frequently Asked Questions

What is the function G(x) = Cot x, and how is it defined?

The function G(x) = Cot x is the cotangent function, defined as the ratio of cosine to sine: Cot x = Cos x / Sin x.

What is the domain of G(x) = Cot x?

The domain of Cot x is all real numbers except where Sin x = 0, i.e., $x \neq n\pi$, where n is an integer.

What is the range of the cotangent function?

The range of Cot x is all real numbers, $(-\infty, \infty)$, as it can take any real value.

How does the graph of G(x) = Cot x look like? Can you give a rough sketch?

The graph of Cot x is a repeating, periodic curve with vertical asymptotes at $x = n\pi$. It decreases from $+\infty$ to $-\infty$ within each period $(0, \pi)$, creating a series of downward-sloping curves.

What are the key features of G(x) = Cot x in terms of asymptotes and intercepts?

Vertical asymptotes occur at $x = n\pi$ where sine is zero. The function crosses the x-axis at $x = \pi/2 + n\pi$, where cosine equals zero.

How is the period of G(x) = Cot x determined?

The period of Cot x is π, meaning the pattern repeats every π units along the x-axis.

Are there any special points or zeros of the function G(x) = Cot x?

Yes, G(x) crosses the x-axis (has zeros) at $x = \pi/2 + n\pi$, where cosine is zero and cotangent is zero.

In what contexts is the function G(x) = Cot x commonly used?

Cotangent functions are used in trigonometry, calculus, wave analysis, and in solving problems involving angles and periodic phenomena.

Additional Resources

Give A Rough Sketch Of The Following Function G(x) = Cotx = Cosx / Sinx. Find Domain And Range.

When exploring the function $G(x) = \cot x = \frac{\cos x}{\sin x}$, understanding its fundamental properties is essential for graphing and analyzing its behavior. The cotangent function is a classic example of a reciprocal trigonometric function, closely related to the tangent function but with distinctive characteristics. Its graph exhibits unique features such as vertical asymptotes, periodicity, and a range that excludes zero, all of which are crucial for mathematicians, students, and professionals working with trigonometric models. This article provides a comprehensive overview of $G(x)$, including its domain, range, key features, and a rough sketching guide.

Understanding the Function $G(x) = \cot x$

The cotangent function is defined as the ratio of the cosine to the sine of an angle x:

$$
G(x) = \cot x = \frac{\cos x}{\sin x}
$$

This ratio describes how the cosine and sine functions relate to each other for a given x. Since both sine and cosine are fundamental trigonometric functions with well-understood behaviors, their ratio inherits many interesting properties that are crucial in various applications such as physics, engineering, and mathematics.

Domain of $G(x) = \cot x$

The domain of a function refers to all the input values x for which the function is defined. For $G(x) = \frac{\cos x}{\sin x}$, the main restriction arises from the denominator:

- The sine function, $\sin x$, must not be zero because division by zero is undefined.
- The solutions to $\sin x = 0$ occur at specific points:

$$
x = n\pi, \quad n \in \mathbb{Z}
$$

where n is any integer.

Therefore, the domain of $G(x)$ is:

\[

\boxed{
x \in \mathbb{R} \setminus \{ n\pi \mid n \in \mathbb{Z} \}
}
\]

In words, the domain includes all real numbers except integer multiples of \(\pi \).

Summary of the domain:

- Excludes points where \(\sin x = 0 \).
- Consists of intervals between these points, such as \((-\frac{\pi}{2}, \frac{\pi}{2}) \), \((\pi, 2\pi) \), \((-2\pi, -\pi) \), etc.

Range of \(G(x) = \cot x \)

The range describes all possible output values of \(G(x) \) for the inputs within its domain.

- Because \(G(x) \) is the ratio of \(\cos x \) to \(\sin x \), and both are bounded between -1 and 1, the values of \(\cot x \) can vary widely, but with some restrictions.

- The key is to analyze the behavior of \(\cot x \) as \(x \) approaches the points where \(\sin x = 0 \), i.e., the vertical asymptotes.

Behavior near asymptotes:

- As \(x \to n\pi^- \) (approaching from the left), \(\sin x \to 0^+ \), so \(\cot x = \frac{\cos x}{\sin x} \to +\infty \) or \(-\infty \) depending on the cosine's sign.
- As \(x \to n\pi^+ \) (approaching from the right), \(\sin x \to 0^- \), and \(\cot x \to -\infty \) or \(+\infty \).

Implication:

- The cotangent function can take any real value except zero.

- The function \(\cot x \) has no zero outputs because:

\[
\cot x = 0 \implies \cos x = 0
\]

which occurs at \(x = \frac{\pi}{2} + n\pi \). But at these points, \(\sin x \neq 0 \), so the function is defined there, but the ratio is zero only at points where the cosine is zero, i.e., \(\cos x = 0 \). Since the cotangent is undefined at \(\sin x=0 \), and zero occurs when \(\cos x=0 \), which is within the domain (excluding \(n\pi \)), the function can actually approach zero but not take it.

Therefore, the range is:

\[
\boxed{

(-\infty, \infty) \quad \text{excluding } 0
}
\]

or, more precisely,

\[
\boxed{
\mathbb{R} \setminus \{ 0 \}
}
\]

Summary:

- The range of \(G(x) \) is all real numbers except zero.
- The function can produce arbitrarily large positive or negative values.

Key Features of the Graph of \(G(x) = \cot x \)

Understanding the shape and features of the cotangent graph is essential for sketching it accurately. Here are the main features:

Periodicity
- The cotangent function is periodic with a period of \(\pi \):

\[
\cot (x + \pi) = \cot x
\]

- This means the graph repeats every \(\pi \) units.

Asymptotes
- Vertical asymptotes occur at the zeros of \(\sin x \):

\[
x = n\pi, \quad n \in \mathbb{Z}
\]

- At these points, \(\cot x \to \pm \infty \).

Behavior on each interval
- On the interval \((0, \pi) \), \(\cot x \) decreases monotonically from \(+\infty \) to \(-\infty \).
- Similarly, in other intervals between asymptotes, the function exhibits the same decreasing trend.

Symmetry
- The cotangent function is odd:

\[
\cot(-x) = -\cot x
\]

- This implies symmetry with respect to the origin.

Key points
- At $x = \frac{\pi}{2}$, $\cot x = 0$.
- As $x \to n\pi^-$, $\cot x \to +\infty$.
- As $x \to n\pi^+$, $\cot x \to -\infty$.

Rough Sketching of $G(x) = \cot x$

When sketching $\cot x$, follow these steps for clarity:

1. Identify asymptotes at $x = n\pi$, where the function is undefined.
2. Plot key points:
- At $x = \frac{\pi}{2} + n\pi$, $\cot x = 0$.
- At $x = (n + \frac{1}{2})\pi$, the function approaches infinity (positive or negative).
3. Draw the curves:
- For each interval between asymptotes, draw a decreasing curve from $+\infty$ to $-\infty$.
- Use the symmetry property to mirror the pattern in each interval.
4. Note the periodicity:
- Repeat the pattern every π units.

Pros and Cons of the Cotangent Function

Pros:

- Periodic behavior makes it useful in wave and oscillation modeling.
- Monotonic segments within each period make it predictable and easy to analyze locally.
- Symmetry simplifies understanding of its overall shape.

Cons:

- Discontinuities at asymptotes can complicate integration or differentiation.
- Limited range (excluding zero) may restrict its use in some applications.
- Undefined points require careful handling in calculations and graphing.

Applications of the Cotangent Function

- Signal processing: Modeling waveforms with phase shifts.
- Physics: Describing certain angular relationships in mechanics.
- Mathematics: Solving differential equations involving cotangent.
- Engineering: Analyzing periodic signals and resonances.

Conclusion

The function $G(x) = \cot x = \frac{\cos x}{\sin x}$ is a fundamental trigonometric function with distinct features, including a periodic nature, vertical asymptotes at multiples of π, and a range covering all real numbers except zero. Its domain excludes points where $\sin x = 0$, and understanding these restrictions is critical for graphing and application. The typical graph showcases an infinite sequence of decreasing curves between asymptotes, exhibiting symmetry and predictable behavior within each period. Mastery of the cotangent function's properties enhances one's ability to analyze oscillatory phenomena, solve trigonometric equations, and interpret mathematical models across various scientific disciplines.

In summary:

- Domain: \(x \in \mathbb{

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