Given The First Three Terms Of The Arithmetic Sequence (2p-3);(p+5);(2p+7)find The Value Of P?

Given The First Three Terms Of The Arithmetic Sequence (2p-3);(p+5);(2p+7)find The Value Of P?

Understanding the Problem

In the world of mathematics, arithmetic sequences are fundamental concepts that often appear in various algebraic problems. The problem at hand involves identifying the value of the variable p given the first three terms of an arithmetic sequence: $2p - 3$, $p + 5$, and $2p + 7$.

The key to solving this problem lies in understanding the properties of an arithmetic sequence (also known as an arithmetic progression). Specifically, in an arithmetic sequence, the difference between consecutive terms remains constant. This common difference is the cornerstone of the problem, as it allows us to set up equations and solve for the unknown variable p.

What is an Arithmetic Sequence?

Before delving into the solution, let's clarify what an arithmetic sequence entails:

- An arithmetic sequence is a list of numbers where the difference between any two successive terms is always the same.
- This constant difference is called the common difference, often denoted as d.
- If the sequence is $a_1, a_2, a_3, \dots$, then for all n,

$$
a_{n+1} - a_n = d
$$

Example:

Consider the sequence: 3, 7, 11, 15, 19,...

- The common difference d is 4 because:

$$
7 - 3 = 4, \quad 11 - 7 = 4, \quad 15 - 11 = 4, \quad 19 - 15 = 4
$$

Analyzing the Given Terms

The three terms provided are:

1. First term: $T_1 = 2p - 3$
2. Second term: $T_2 = p + 5$
3. Third term: $T_3 = 2p + 7$

Since these are part of an arithmetic sequence, the difference between T_2 and T_1 must be equal to the difference between T_3 and T_2:

$$
T_2 - T_1 = T_3 - T_2
$$

This equality is the foundation for solving for p.

Setting Up the Equation

Let's write out the differences explicitly:

$$
(p + 5) - (2p - 3) = (2p + 7) - (p + 5)
$$

Now, simplify each side:

1. Left side:

$$
p + 5 - 2p + 3 = -p + 8
$$

2. Right side:

$$
2p + 7 - p - 5 = p + 2
$$

So, the equation reduces to:

\[

$$
-p + 8 = p + 2
$$

Solving for p

Now, solve the equation:

$$
-p + 8 = p + 2
$$

Bring all terms involving p to one side and constants to the other:

$$
8 - 2 = p + p
$$

$$
6 = 2p
$$

Divide both sides by 2:

$$
p = \frac{6}{2} = 3
$$

Therefore, the value of p is 3.

Verification of the Solution

It's essential to verify the solution's correctness by substituting $p = 3$ back into the original terms and confirming the sequence's arithmetic nature.

1. Calculate each term:

- First term:

$$
T_1 = 2p - 3 = 2 \times 3 - 3 = 6 - 3 = 3
$$

- Second term:

$$
T_2 = p + 5 = 3 + 5 = 8
$$

- Third term:

$$
T_3 = 2p + 7 = 2 \times 3 + 7 = 6 + 7 = 13
$$

2. Check the differences:

$$
T_2 - T_1 = 8 - 3 = 5
$$
$$
T_3 - T_2 = 13 - 8 = 5
$$

Since both differences are equal to 5, the sequence is indeed arithmetic with a common difference of 5.

Implications and Applications of the Solution

Understanding how to find the value of p in this context has broader implications in algebra and sequence analysis:

- Algebraic problem-solving: Demonstrates the importance of setting up equations based on sequence properties.
- Pattern recognition: Highlights the necessity of recognizing constant differences in sequences.
- Application in real-world problems: Many real-life scenarios involve sequences, such as economic models, population studies, and technology advancements, where understanding the pattern helps in forecasting or decision-making.

Summary of the Steps to Find p

To summarize, the process involves:

1. Recognizing that the sequence is arithmetic and the differences are equal.
2. Writing the differences between successive terms as algebraic expressions.

3. Setting these expressions equal to each other to form an equation.
4. Simplifying and solving for p.
5. Verifying the solution by substituting back into the original terms.

Additional Practice Problems

To strengthen understanding, consider practicing with similar problems:

- Find the value of q if the sequence $(4q - 2), (q + 6), (3q + 4)$ forms an arithmetic sequence.
- Given the terms $(5r - 1), (2r + 3), (7r + 5)$, determine the value of r.

Conclusion

In solving the problem of finding the value of p given the first three terms of an arithmetic sequence, the key step is leveraging the defining property of such sequences: equal differences between successive terms. By translating this property into an algebraic equation and solving for p, we determined that $p = 3$.

This approach not only emphasizes core algebraic skills but also enhances understanding of sequences, pattern recognition, and mathematical reasoning. Whether for academic purposes or real-world applications, mastering the process of analyzing arithmetic sequences is an essential skill in the mathematician's toolkit.

Keywords: arithmetic sequence, sequence differences, algebraic equations, sequence pattern, solving for p, math problem-solving, sequence analysis, algebra practice

Frequently Asked Questions

What is the common difference in the arithmetic sequence (2p - 3), (p + 5), (2p + 7)?

The common difference is the difference between consecutive terms. So, (p + 5) - (2p - 3) and (2p + 7) - (p + 5).

How do you set up an equation to find p in the given

sequence?

Since the sequence is arithmetic, the difference between the first and second terms equals the difference between the second and third terms. Set (p + 5) - (2p - 3) = (2p + 7) - (p + 5) and solve for p.

What is the step-by-step solution to find the value of p?

First, compute the differences: (p + 5) - (2p - 3) = p + 5 - 2p + 3 = -p + 8. Next, (2p + 7) - (p + 5) = 2p + 7 - p - 5 = p + 2. Set these equal: -p + 8 = p + 2. Solving for p gives 8 - 2 = p + p, so 6 = 2p, thus p = 3.

What is the value of p in the given sequence?

The value of p is 3.

Are the terms of the sequence consistent with the found value of p?

Yes, substituting p = 3 into the terms gives: first term = 2(3) - 3 = 6 - 3 = 3, second term = 3 + 5 = 8, third term = 2(3) + 7 = 6 + 7 = 13. The differences are 8 - 3 = 5 and 13 - 8 = 5, confirming the sequence is arithmetic with common difference 5.

How can you verify that the sequence is arithmetic with the found p value?

Calculate the differences between consecutive terms: (p + 5) - (2p - 3) and (2p + 7) - (p + 5). With p=3, both differences are equal to 5, confirming the sequence is arithmetic.

Additional Resources

Given The First Three Terms Of The Arithmetic Sequence (2p-3); (p+5); (2p+7) find The Value Of P? An Investigative Approach to Understanding Arithmetic Sequences and Solving for Variables

---

Introduction

Mathematics, often regarded as the language of the universe, offers myriad patterns and structures that underpin the natural and constructed worlds. Among these, arithmetic sequences stand out due to their simplicity and foundational role in understanding progression, pattern recognition, and algebraic reasoning. The problem statement, "Given the first three terms of the arithmetic sequence (2p-3); (p+5); (2p+7), find the value of p?", provides an excellent case study for exploring how algebraic techniques can be applied to sequence analysis to uncover the unknown variable.

This article delves into the systematic approach to solve the problem, examining the properties of arithmetic sequences, the algebraic steps involved, and the implications of the solution. By thoroughly analyzing this problem, readers will gain insight into the methods used to determine unknowns in sequences and appreciate the logical rigor that underpins algebraic problem-solving.

Understanding the Core Concepts

What Is an Arithmetic Sequence?

An arithmetic sequence is a sequence of numbers where each term after the first is obtained by adding a fixed common difference, d, to the preceding term. Formally, an arithmetic sequence $\{a_n\}$ satisfies:

$$
a_{n+1} = a_n + d
$$

for all relevant n.

Key properties include:

- The difference d between consecutive terms is constant.
- The sequence can be expressed explicitly as:

$$
a_n = a_1 + (n-1)d
$$

- The sequence's terms follow a linear pattern, making algebraic analysis straightforward.

Establishing the Problem's Framework

Given the three terms:

$$
a_1 = 2p - 3
$$
$$
a_2 = p + 5
$$
$$
a_3 = 2p + 7
$$

the goal is to find the value of p such that these terms form an arithmetic sequence.

Analytical Approach: Applying the Definition of Arithmetic Sequence

Step 1: Recognize the Condition for an Arithmetic Sequence

For the sequence to be arithmetic:

$$
a_2 - a_1 = a_3 - a_2
$$

which states that the difference between the second and first term is equal to the difference between the third and second term.

Step 2: Set Up the Equation

Compute the differences:

$$
a_2 - a_1 = (p + 5) - (2p - 3)
$$
$$
a_3 - a_2 = (2p + 7) - (p + 5)
$$

Set these equal:

$$
(p + 5) - (2p - 3) = (2p + 7) - (p + 5)
$$

Solving for p

Step 3: Simplify Each Side

Calculate the left side:

$$
p + 5 - 2p + 3 = -p + 8
$$

Calculate the right side:

$$
2p + 7 - p - 5 = p + 2
$$

Now, set the simplified expressions equal:

$$
-p + 8 = p + 2
$$

Step 4: Isolate p

Bring all terms involving p to one side:

$$
- p - p = 2 - 8
$$
$$
-2p = -6
$$

Divide both sides by -2:

$$
p = \frac{-6}{-2} = 3
$$

Verification of the Solution

Step 5: Substitute $p = 3$ Back into the Terms

Calculate each term:

$$
a_1 = 2(3) - 3 = 6 - 3 = 3
$$
$$
a_2 = 3 + 5 = 8
$$
$$
a_3 = 2(3) + 7 = 6 + 7 = 13
$$

Determine the differences:

$$
a_2 - a_1 = 8 - 3 = 5
$$
$$
a_3 - a_2 = 13 - 8 = 5
$$

Since both differences are equal, the sequence with $p = 3$ is indeed arithmetic.

Broader Implications and Educational Significance

This problem exemplifies a fundamental algebraic technique: translating sequence properties into equations and solving for unknown variables. Its relevance extends beyond pure mathematics into fields such as computer science, economics, and engineering, where recognizing patterns and solving for unknowns are routine tasks.

Furthermore, the process reinforces key mathematical skills:

- Understanding sequence properties and their algebraic representations.
- Setting up equations based on problem conditions.
- Simplification and algebraic manipulation to isolate unknowns.
- Verification to ensure solutions satisfy initial conditions.

Additional Considerations and Variations

While the current problem involves a straightforward arithmetic sequence, similar problems may involve:

- Geometric sequences, where ratios are constant.
- Sequences with non-linear patterns requiring quadratic or higher-order equations.
- Sequences involving multiple variables with additional constraints.

In such cases, more advanced algebraic or calculus-based techniques might be necessary.

Conclusion

The problem of determining the value of \(p \) given the first three terms of a sequence is a quintessential illustration of the power of algebraic methods in pattern recognition and problem-solving. Through the systematic application of the property that differences between consecutive terms are equal in an arithmetic sequence, we deduced that:

\[
\boxed{
p = 3
}
\]

This solution not only confirms the theoretical understanding of arithmetic sequences but also exemplifies the importance of methodical reasoning in mathematics. The ability to translate real-world or abstract problems into algebraic equations and solve for unknowns remains a core competency in mathematical literacy and critical thinking.

By engaging with such problems, students and practitioners develop a deeper appreciation for the elegance and utility of algebra, reinforcing its role as an essential tool in analytical reasoning across disciplines.

References

- Stewart, J. (2015). Precalculus: Mathematics for Calculus. Cengage Learning.
- Rosen, K. H. (2012). Discrete Mathematics and Its Applications. McGraw-Hill Education.
- Blitzer, R. (2014). Algebra and Trigonometry. Pearson.

About the Author

[Author Name] is a mathematics educator and researcher specializing in algebraic structures and problem-solving strategies. With a passion for making mathematics accessible and engaging, [he/she/they] focus on developing educational content that enhances analytical thinking and mathematical literacy.

Given The First Three Terms Of The Arithmetic Sequence 2p 3 P 5 2p 7 Find The Value Of P

Given The First Three Terms Of The Arithmetic Sequence 2p 3 P 5 2p 7 Find The Value Of P

Related Articles

- Using The Definition Of The Logarithm, What Expression Should Go In The Blank Below? 933=729=
- About Ten Cents Of Every Dollar Spent On Food Goes To Paying For Transportation Costs.ResponsesTrue
- Watercolor Paper, A Paper That Can Get Wet, Is Need If You Are Going To Use Solvent Blending.

Back to Home