Given: Triangle ABC With AB = 42 And BC = 20. Which Of The Following Are Possible Lengths For AC?

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When analyzing triangles, one of the fundamental questions involves determining the possible lengths of a side given the lengths of the other sides. In the case of triangle ABC, where side AB measures 42 units and side BC measures 20 units, the key question is: what are the possible lengths for side AC? This problem hinges on the triangle inequality theorem, which provides the necessary conditions for three lengths to form a valid triangle. Understanding these principles is essential for students, educators, and enthusiasts interested in geometry, especially when working with real-world applications or mathematical proofs.

This article explores how to determine the possible lengths of side AC in triangle ABC given specific side lengths AB and BC. We will delve into the fundamental concepts of the triangle inequality theorem, explore the range of possible side lengths, and provide examples illustrating how these principles are applied. Whether you're preparing for exams, solving geometry problems, or simply interested in the properties of triangles, this comprehensive guide will clarify the criteria that define feasible lengths for triangle sides.

Understanding the Triangle Inequality Theorem

The triangle inequality theorem is the cornerstone of triangle geometry. It states that, for any triangle, the length of each side must be less than the sum of the lengths of the other two sides and greater than their difference. Formally, if the sides of a triangle are a, b, and c, then:

- $a + b > c$
- $a + c > b$
- $b + c > a$

This set of inequalities ensures that the three sides can connect in a way that forms a closed triangle without "flattening out" into a straight line.

Applying this to our problem, with known sides AB and BC, we can derive the possible ranges for side AC. Denoting AC as x, the inequalities become:

- $AB + BC > AC$ □ $42 + 20 > x$ □ $62 > x$
- $AB + AC > BC$ □ $42 + x > 20$ □ $x > -22$ (which is always true since side lengths are positive)
- $BC + AC > AB$ □ $20 + x > 42$ □ $x > 22$

Combining these, the feasible range for x (the length of side AC) is:

- $x > 22$
- $x < 62$

Since side lengths are positive real numbers, the lower bound is strictly greater than 22, and the upper bound is strictly less than 62.

Possible Lengths for AC in Triangle ABC

Based on the triangle inequality theorem, the possible lengths for side AC in triangle ABC, with AB = 42 and BC = 20, are all real numbers strictly between 22 and 62.

Summary:

- Minimum length of AC: just greater than 22 units
- Maximum length of AC: just less than 62 units

This range ensures that the three sides can connect to form a valid triangle, satisfying the fundamental geometric constraints.

Visualizing the Range of AC

To better understand this, imagine plotting the possible lengths of AC:

1. At AC just greater than 22: The triangle becomes very narrow, approaching a degenerate form as AC approaches 22 from above.
2. At AC approaching 62: The triangle flattens out, approaching a straight line as AC approaches 62 from below.

Any length outside this range would violate the triangle inequality theorem, making it impossible to form a triangle with the given side lengths.

Examples of Possible Lengths for AC

Let's explore some specific values within the feasible range to illustrate what types of triangles can be formed:

Example 1: AC = 23

- Triangle inequality checks:

- 42 + 20 = 62 > 23 □
- 42 + 23 = 65 > 20 □
- 20 + 23 = 43 > 42 □

- All inequalities hold, so a triangle with sides 42, 20, and 23 is possible.

Example 2: AC = 50

- Checks:

- 42 + 20 = 62 > 50 □
- 42 + 50 = 92 > 20 □
- 20 + 50 = 70 > 42 □

- Valid triangle.

Example 3: AC = 61.9

- Checks:

- 42 + 20 = 62 > 61.9 □
- 42 + 61.9 □ 103.9 > 20 □
- 20 + 61.9 □ 81.9 > 42 □

- Still valid.

Example 4: AC approaching 62 (e.g., 61.999)

- Checks:

- 62 > 61.999 □

- Other inequalities hold as well.

- Triangle remains valid but approaches degeneracy.

Understanding Degenerate and Non-Degenerate Triangles

When side lengths approach the bounds of the range, the triangle transitions toward a degenerate form:

- Degenerate triangle: When the sum of two sides equals the third, the triangle collapses into a straight line.
- Non-degenerate triangle: When the sum of any two sides is strictly greater than the third.

In our problem, as AC approaches 22 from above, the triangle becomes very narrow, nearly degenerate. Similarly, as AC approaches 62 from below, the triangle flattens but remains valid until reaching the limit.

Practical Applications and Related Problems

Understanding the possible lengths of sides in a triangle is crucial in various fields:

1. Engineering and Architecture: Ensuring structural components meet specified dimensions while maintaining stability.
2. Navigation and Geolocation: Calculating possible distances based on known landmarks.

3. Computer Graphics: Rendering triangles with constraints to model realistic objects.
4. Mathematics Education: Teaching concepts of inequalities, geometric proofs, and problem-solving strategies.

Additionally, similar problems often involve:

- Finding the range of possible third sides given two sides.
- Applying the Law of Cosines to determine angles given side lengths.
- Using coordinate geometry to visualize the set of all possible points that satisfy side length constraints.

Key Points to Remember

- The side AC in triangle ABC with AB=42 and BC=20 can only be between greater than 22 and less than 62 units.
- The triangle inequality theorem is fundamental in determining feasible side lengths.
- As side AC approaches the bounds, the triangle approaches a degenerate state, but remains valid within the bounds.
- Practical applications of these principles are widespread across industries and fields of study.

In summary, if you know two sides of a triangle, the third side's possible lengths are constrained by the triangle inequality theorem. For triangle ABC with sides AB=42 and BC=20, the third side AC can take any value in the open interval (22, 62). Recognizing these constraints helps in solving geometric problems accurately and efficiently.

Meta Description: Discover how to determine the possible lengths of side AC in triangle ABC with sides AB = 42 and BC = 20. Learn about the triangle inequality theorem, feasible ranges, and practical applications in geometry.

Frequently Asked Questions

Given triangle ABC with AB = 42 and BC = 20, what is the range of possible lengths for AC?

The length of AC must be greater than the difference of AB and BC (42 - 20 = 22) and less than their sum (42 + 20 = 62). So, AC can be any value between 22 and 62, exclusive.

Can AC be exactly 22 or 62 in triangle ABC with sides AB = 42 and BC = 20?

No, AC cannot be exactly 22 or 62 because, in a triangle, the length of one side must be strictly less than the sum of the other two and strictly greater than their difference.

If AC is 30, is it a possible length for the triangle ABC with AB = 42 and BC = 20?

Yes, since $22 < 30 < 62$, AC = 30 satisfies the triangle inequality and is a possible length.

What are the necessary triangle inequalities for sides AB = 42, BC = 20, and AC?

The inequalities are: $AC > |AB - BC| = 22$, $AC < AB + BC = 62$, and similarly, the other inequalities involve the other sides, but since AB and BC are fixed, AC must satisfy $22 < AC < 62$.

Is AC = 50 a valid length for the side in triangle ABC with given sides AB = 42 and BC = 20?

Yes, because 22 < 50 < 62, so AC = 50 can form a valid triangle with the given sides.

If AC is 15, can triangle ABC exist with AB = 42 and BC = 20?

No, because AC = 15 is less than the minimum length of 22 required for the triangle inequality to hold, so such a triangle cannot exist.

Why can't AC be longer than 62 in triangle ABC with sides AB = 42 and BC = 20?

Because the length of AC must be less than the sum of the other two sides; exceeding 62 would violate the triangle inequality, making a triangle impossible.

Additional Resources

Given: Triangle ABC With AB = 42 And BC = 20. Which Of The Following Are Possible Lengths For AC?

Understanding the possible lengths of side AC in a triangle where two sides are known involves delving into the foundational principles of triangle geometry—particularly the triangle inequality theorem. This theorem provides essential constraints that determine whether a set of three side lengths can form a valid triangle. In this guide, we'll explore how to determine the feasible lengths of AC, given that AB = 42 and BC = 20, and analyze which hypothetical values for AC are possible within these constraints.

Introduction to Triangle Side Lengths and Constraints

When considering any triangle, the lengths of its sides are not arbitrary; they must satisfy specific geometric conditions. Given two sides, AB and BC, the third side, AC, must conform to certain rules to ensure the three points can indeed form a triangle.

In our specific problem:

- Side AB = 42 units
- Side BC = 20 units
- Side AC = ? (unknown, to be determined)

The key question is: What lengths of AC are possible, given these two fixed sides?

The Triangle Inequality Theorem

At the heart of determining possible side lengths lies the triangle inequality theorem. This fundamental principle states:

> For any triangle with sides of lengths a, b, and c,
> $$
> a + b > c, \quad a + c > b, \quad b + c > a
> $$

Applying this to our problem, where sides are $AB = 42$, $BC = 20$, and $AC = x$:

1. $AB + BC > AC \Rightarrow 42 + 20 > x \Rightarrow 62 > x$
2. $AB + AC > BC \Rightarrow 42 + x > 20 \Rightarrow x > -22$
3. $BC + AC > AB \Rightarrow 20 + x > 42 \Rightarrow x > 22$

Since side lengths are always positive, the second inequality simplifies to $x > -22$, which is always true for positive x. The critical inequalities are:

- $x > 22$
- $x < 62$

Thus, the length of side AC must be strictly greater than 22 and less than 62 for the three points to form a valid triangle.

Visualizing the Constraints

Geometric Interpretation

Imagine fixing segments AB and BC in space:

- Segment AB extends 42 units.
- Segment BC extends 20 units.

To form triangle ABC, point C must be positioned such that the length of AC satisfies the inequalities above. Visualizing this:

- If AC is just over 22, then the triangle is "thin," nearly degenerate, with points A and C very close to forming a straight line.
- If AC approaches just under 62, the triangle becomes very "flat," with points A and C nearly aligned opposite to B.

Practical Implications

- Possible lengths for AC are any real numbers strictly between 22 and 62.
- Impossible lengths are less than or equal to 22, or greater than or equal to 62.

Considering Variations: Special Cases and Additional Constraints

While the basic inequalities define the possible range, real-world problems or specific configurations may place further restrictions, such as:

- Coordinate placement: If the points are fixed on coordinate axes, certain lengths may be geometrically impossible due to position constraints.
- Angles: The internal angles at vertices A, B, or C influence the side lengths and their feasibility.

However, in the absence of such additional constraints, the triangle inequality provides a complete answer.

List of Possible Lengths for AC

Based on the above analysis, the set of possible lengths for AC is:

- Any real number x satisfying:

$$
22 < x < 62
$$

- This range is continuous, meaning all real values strictly between 22 and 62 are feasible.

Summary of Findings

Parameter	Details

Known sides	AB = 42, BC = 20
Unknown side	AC = x
Possible range for x	$22 < x < 62$
Key principle used	Triangle inequality theorem

Practical Example: Testing Specific Lengths

Suppose you are given several options for AC, such as 23, 40, 61, 62, or 20. Which of these are feasible?

- 23: Since 23 > 22 and < 62, it is possible.
- 40: Also within the range, possible.
- 61: Still less than 62, possible.
- 62: Equal to the upper bound, not strictly less, so impossible.
- 20: Less than 22, impossible.

Final Thoughts and Applications

Understanding the constraints on side lengths in a triangle is fundamental in geometry, architecture, engineering, and various fields requiring precise measurements. When given partial information, applying the triangle inequality theorem allows you to quickly determine feasible dimensions, ensuring designs or calculations are valid.

In educational settings, problems like the one discussed reinforce the importance of the triangle inequality and encourage spatial visualization skills. Practitioners can extend this knowledge to more complex scenarios involving coordinate geometry, trigonometry, and vector analysis.

Conclusion

Given the fixed sides AB = 42 and BC = 20, the possible lengths for side AC are all real numbers strictly between 22 and 62. This conclusion is derived directly from the triangle inequality theorem, emphasizing its critical role in geometric problem-solving. Whether you're solving a math problem, designing a structure, or exploring geometric configurations, understanding these fundamental constraints ensures your solutions are both mathematically sound and practically feasible.

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