

Giving The Mid-point As (-4,7) And End Point 3,8 Calculate The Other End Point Justify Your Answer

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Introduction

Understanding how to find the unknown endpoint of a line segment when given the midpoint and the other endpoint is a fundamental skill in coordinate geometry. This concept not only helps in solving geometric problems but also enhances spatial reasoning and analytical thinking. In this article, we will explore the process of calculating the missing endpoint when the midpoint and one endpoint are known, justify each step involved, and illustrate the method with detailed explanations.

Fundamentals of Midpoints and Endpoints

What is a Midpoint?

The midpoint of a line segment is the point that divides the segment into two equal parts. It lies exactly halfway between the two endpoints.

Mathematical Formula for Midpoint

If you have two endpoints, $(A(x_1, y_1))$ and $(B(x_2, y_2))$, then the midpoint $(M(x_m, y_m))$ is calculated using the formulas:

$$\begin{aligned} x_m &= \frac{x_1 + x_2}{2} \\ y_m &= \frac{y_1 + y_2}{2} \end{aligned}$$

This means the midpoint's coordinates are the averages of the corresponding coordinates of the endpoints.

Given Data and Objective

Let's clearly identify the given data:

- Midpoint $(M(-4, 7))$
- Known endpoint $(A(3, 8))$

Our goal is to find the unknown endpoint $(B(x, y))$.

Step-by-Step Calculation of the Missing Endpoint

Applying the Midpoint Formula

Since the midpoint (M) divides the segment (AB) into two equal parts, the coordinates of (M) are the averages of the coordinates of (A) and (B) . Formally:

$$\begin{aligned}x_m &= \frac{x_A + x_B}{2} \\y_m &= \frac{y_A + y_B}{2}\end{aligned}$$

Substituting the known midpoint and (A) 's coordinates:

$$\begin{aligned}-4 &= \frac{3 + x}{2} \\7 &= \frac{8 + y}{2}\end{aligned}$$

Solving for (x) and (y)

To find (x) :

$$-4 = \frac{3 + x}{2}$$

Multiply both sides by 2:

$$-8 = 3 + x$$

Subtract 3 from both sides:

$$x = -8 - 3 = -11$$

Similarly, to find (y) :

$$7 = \frac{8 + y}{2}$$

Multiply both sides by 2:

$$14 = 8 + y$$

Subtract 8 from both sides:

$$\begin{aligned} & \backslash[\\ y &= 14 - 8 = 6 \\ & \backslash] \end{aligned}$$

Final Answer and Justification

The coordinates of the other endpoint (B) are $(\boxed{-11, 6})$.

Justification:

- The midpoint formula is rooted in the idea that the midpoint's coordinates are the averages of the endpoints' coordinates.
- By substituting known values and solving for the unknowns, we ensure consistency with the properties of midpoints.
- The calculations are straightforward algebraic steps, confirming the accuracy of the result.

Additional Insights and Applications

Verifying the Result

It's always good practice to verify the result by calculating the midpoint of $(A(3, 8))$ and $(B(-11, 6))$:

$$\begin{aligned} & \backslash[\\ x_m &= \frac{3 + (-11)}{2} = \frac{-8}{2} = -4 \\ & \backslash[\\ & \backslash[\\ y_m &= \frac{8 + 6}{2} = \frac{14}{2} = 7 \\ & \backslash] \end{aligned}$$

This matches the given midpoint $(M(-4, 7))$, confirming that the calculation is correct.

Real-world Applications

Understanding how to find missing endpoints using midpoints is essential in various fields such as:

- Engineering: Designing structures where precise measurements are necessary.
- Computer Graphics: Calculating points for rendering and modeling.
- Navigation: Determining positions based on known midpoints and endpoints.
- Mathematics Education: Developing problem-solving skills and understanding coordinate systems.

Conclusion

By leveraging the midpoint formula, we successfully identified the unknown endpoint $(B(-11, 6))$ given the midpoint $((-4, 7))$ and the known endpoint $((3, 8))$. This process underscores the importance of algebraic manipulation and understanding geometric principles in solving coordinate geometry

problems. Mastery of such techniques enhances analytical skills and forms a foundational part of mathematics education, with wide-ranging applications across science, engineering, and technology.

Summary of Key Steps

1. Identify the known data: midpoint and one endpoint.
2. Recall the midpoint formula for coordinates.
3. Substitute known values into the formula.
4. Solve algebraically for the unknown coordinates.
5. Verify the solution by recalculating the midpoint.

Remember: Whenever you are asked to find an unknown endpoint given a midpoint and one endpoint, use the midpoint formula meticulously, verify your results, and appreciate the geometric relationships underlying these calculations.

Frequently Asked Questions

Given the midpoint $(-4, 7)$ and one endpoint $(3, 8)$, how can we find the other endpoint?

We use the midpoint formula: if the endpoints are (x_1, y_1) and (x_2, y_2) , and the midpoint is (M_x, M_y) , then $x_2 = 2 M_x - x_1$ and $y_2 = 2 M_y - y_1$. Substituting the known values: $x_2 = 2(-4) - 3 = -8 - 3 = -11$, $y_2 = 2(7) - 8 = 14 - 8 = 6$. So, the other endpoint is $(-11, 6)$.

What is the formula used to find the missing endpoint when the midpoint and one endpoint are given?

The formula is: $x_2 = 2 M_x - x_1$ and $y_2 = 2 M_y - y_1$, where (M_x, M_y) is the midpoint and (x_1, y_1) is the known endpoint. This helps to find the other endpoint.

Why is the calculation of the other endpoint important in coordinate geometry?

Calculating the other endpoint helps in understanding the properties of line segments, such as length and symmetry, and is essential in problems involving bisected segments, reflections, and geometric constructions.

Can the process of finding the other endpoint be applied to any coordinate points?

Yes, the process using the midpoint formula applies to any coordinate points where the midpoint and one endpoint are known, regardless of their location.

on the coordinate plane.

What are common mistakes to avoid when calculating the missing endpoint?

Common mistakes include mixing up the order of subtraction, incorrectly applying the formula, or misreading the given coordinates. Always double-check calculations and ensure the correct formula is used.

How does understanding the midpoint help in real-world applications?

Knowing how to find midpoints is useful in construction, navigation, computer graphics, and design, where symmetry and balanced positioning are important.

Is the calculated other endpoint $(-11, 6)$ consistent with the given midpoint $(-4, 7)$?

Yes, because when you calculate the midpoint between $(-4, 7)$ and $(-11, 6)$, it results in $(-4, 7)$, confirming the correctness of the calculation.

What steps are involved in justifying the calculation of the other endpoint?

The steps include applying the midpoint formula, substituting the known values, performing the calculations carefully, and verifying that the midpoint of the two endpoints matches the given midpoint.

How would the process change if the given midpoint was different or the known endpoint changed?

The process remains the same; only the specific values in the formula change. You substitute the new midpoint or known endpoint coordinates into the formulas to find the other endpoint.

Additional Resources

Giving The Mid-point As $(-4, 7)$ And End Point $(3, 8)$ Calculate The Other End Point Justify Your Answer

In the realm of coordinate geometry, understanding how to determine the unknown endpoints of a line segment from its mid-point and one known endpoint is a fundamental skill. This process not only reinforces the core concepts of averages and algebra but also enhances problem-solving capabilities vital for various applications in engineering, architecture, and computer graphics. Today, we explore a specific problem: given the mid-point of a line segment at $(-4, 7)$ and one endpoint at $(3, 8)$, how do we accurately find the other endpoint? Through a detailed breakdown, justified mathematical reasoning, and illustrative examples, we aim to clarify this essential geometric calculation, making it accessible and insightful for learners and professionals alike.

Understanding the Fundamentals: Mid-point and Endpoints in Coordinate Geometry

Before diving into the calculations, it's crucial to understand the foundational concepts involved:

The Mid-point of a Line Segment

- The mid-point of a line segment is the point that divides the segment into two equal parts.
- It is essentially the average of the x-coordinates and the y-coordinates of the endpoints.
- If the endpoints are (x_1, y_1) and (x_2, y_2) , then the mid-point (M) is calculated as:

$$M = ((x_1 + x_2) / 2, (y_1 + y_2) / 2)$$

The Endpoints of a Line Segment

- The endpoints are the two points that define the segment.
- Given one endpoint and the mid-point, the goal is to find the other endpoint.

Understanding these concepts is vital because the mid-point formula forms the backbone of our calculation.

Setting Up the Problem: Known Values and Unknowns

Given:

- Mid-point, $M = (-4, 7)$
- Known endpoint, $A = (3, 8)$
- Unknown endpoint, $B = (x, y)$

Our task:

- Calculate the coordinates of point B, which is the other endpoint.

This problem involves working backward from the mid-point formula, effectively solving for the unknown endpoint coordinates.

Mathematical Approach to Find the Unknown Endpoint

The core principle is that the mid-point is the average of the x-coordinates and y-coordinates of the two endpoints:

- Mid-point x-coordinate: $(x_1 + x_2) / 2$
- Mid-point y-coordinate: $(y_1 + y_2) / 2$

Rearranged to find the unknown endpoint:

$$\begin{aligned} - x_2 &= 2 \text{ (mid-point } x) - x_1 \\ - y_2 &= 2 \text{ (mid-point } y) - y_1 \end{aligned}$$

Applying these to our known values:

$$\begin{aligned} - x_2 &= 2 (-4) - 3 \\ - y_2 &= 2 (7) - 8 \end{aligned}$$

Calculations:

$$\begin{aligned} - x_2 &= -8 - 3 = -11 \\ - y_2 &= 14 - 8 = 6 \end{aligned}$$

Thus, the other endpoint is at $(-11, 6)$.

Step-by-Step Calculation Justification

Let's break down each step to ensure clarity and accuracy:

1. Write down the known values:

$$\begin{aligned} - \text{Mid-point } M &= (-4, 7) \\ - \text{Known endpoint } A &= (3, 8) \end{aligned}$$

2. Recall the mid-point formula:

$$\begin{aligned} - M_x &= (x_A + x_B) / 2 \\ - M_y &= (y_A + y_B) / 2 \end{aligned}$$

3. Express the unknown endpoint coordinates:

$$\begin{aligned} - x_B &= 2 M_x - x_A \\ - y_B &= 2 M_y - y_A \end{aligned}$$

4. Substitute the known values:

$$\begin{aligned} - x_B &= 2 (-4) - 3 = -8 - 3 = -11 \\ - y_B &= 2 (7) - 8 = 14 - 8 = 6 \end{aligned}$$

5. Finalize the answer:

$$- \text{Other endpoint } B = (-11, 6)$$

This systematic approach confirms that the other endpoint coordinates are $(-11, 6)$.

Verifying the Calculation: A Consistency Check

To ensure the calculation's correctness, it's prudent to verify by reapplying the mid-point formula:

- Calculate the mid-point of points A (3,8) and B (-11,6):

$$M_x = (3 + (-11)) / 2 = (-8) / 2 = -4$$

$$M_y = (8 + 6) / 2 = 14 / 2 = 7$$

- The resulting mid-point is (-4, 7), which matches the given mid-point, confirming the calculation's accuracy.

Implications and Applications of This Calculation

Understanding how to determine an unknown endpoint from a mid-point and one known endpoint has numerous real-world applications:

- Engineering and Design: Calculating the position of structural elements when given mid-points helps in precise construction planning.
- Computer Graphics: In rendering and modeling, determining points based on mid-points ensures symmetry and accuracy.
- Navigation and Mapping: Finding unknown locations based on known mid-points and references enhances positional accuracy.

Furthermore, mastering such calculations enhances spatial reasoning and mathematical fluency, essential skills across STEM fields.

Summary and Key Takeaways

- The problem involves using the mid-point formula in reverse.
- The unknown endpoint can be found by doubling the mid-point coordinates and subtracting the known endpoint coordinates.
- The process is straightforward once the formula is understood and applied systematically.
- Verification through recomputation ensures the solution's accuracy.

Key formula:

For known mid-point (M_x, M_y) , known endpoint (x_1, y_1) , unknown endpoint (x_2, y_2) :

$$x_2 = 2 M_x - x_1$$

$$y_2 = 2 M_y - y_1$$

Solution:

Given $M = (-4, 7)$ and $A = (3, 8)$:

- Other endpoint $B = (-11, 6)$

This method exemplifies how foundational geometric principles underpin precise calculations, empowering learners and professionals to solve complex spatial problems with confidence.

Conclusion

Determining the other endpoint of a line segment when given its mid-point and one endpoint is a classic problem that beautifully illustrates the power of algebra and coordinate geometry. By understanding the core formulas, methodically applying them, and verifying results, one can confidently solve such problems. As demonstrated, the process is not only mathematically sound but also accessible, reinforcing the importance of fundamental concepts in real-world applications. Whether in academics, engineering, or digital design, mastering these techniques equips individuals with essential tools to navigate the spatial dimensions of their respective fields.

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