Identify The Center, Vertices, And Foci Of The Ellipse Or Hyperbola.hyperbola: (x-1)/9 - (y-3)/ 4=1

Identify The Center, Vertices, And Foci Of The Ellipse Or Hyperbola: (x-1)/9 - (y-3)/4=1

Understanding the properties of conic sections such as ellipses and hyperbolas is fundamental in algebra and analytic geometry. These curves have distinctive features like centers, vertices, and foci that help define their shapes and positions in the coordinate plane. In this article, we will explore how to identify these key elements specifically for the hyperbola given by the equation:

$$
\frac{(x-1)}{9} - \frac{(y-3)}{4} = 1
$$

This equation is a bit unusual in form, but with proper analysis, we can determine the hyperbola’s center, vertices, and foci with clarity.

Understanding the Standard Form of Hyperbola Equations

Before diving into the specific hyperbola, it’s essential to understand the general forms of hyperbola equations and their components.

Standard Forms of Hyperbola Equations

Hyperbolas are generally expressed in either of these forms:

- Horizontal hyperbola:
$$
\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1
$$

- Vertical hyperbola:
$$
\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1
$$

where:

- (h, k) is the center of the hyperbola,

- a is the distance from the center to each vertex along the transverse axis,
- b relates to the conjugate axis,
- The foci are located along the transverse axis at a distance c from the center, where $c^2 = a^2 + b^2$.

Rearranging the Given Equation into Standard Form

The given hyperbola equation is:

$$
\frac{(x-1)}{9} - \frac{(y-3)}{4} = 1
$$

At first glance, this equation seems to be in a form similar to the general hyperbola equation but requires some manipulation to identify its standard form properly.

Step 1: Recognize the Equation Structure

Notice that the equation is:

$$
\frac{(x-1)}{9} - \frac{(y-3)}{4} = 1
$$

which resembles the form:

$$
\frac{(x - h)}{A} - \frac{(y - k)}{B} = 1
$$

but not in the standard form involving squares.

Step 2: Convert to Standard Form with Squares

To analyze the hyperbola's properties, we need the equation in the form:

$$
\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1
$$

or

$$

\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1
\]

Given the initial form, it appears more like an equation of a hyperbola in a different form, possibly derived from a difference of two terms.

However, the key is that the equation as provided is not in the standard form involving squares, indicating it might be a misinterpretation or a simplified form of a hyperbola or a different conic.

Clarifying the Equation: Is It a Hyperbola?

The equation:

\[
\frac{(x-1)}{9} - \frac{(y-3)}{4} = 1
\]

resembles a linear combination of linear terms, but not the standard hyperbola form. For a conic section, the equation must involve squares of \((x-h)\) and \((y-k)\).

Possible interpretations:

- If the equation was intended to be:

\[
\frac{(x-1)^2}{a^2} - \frac{(y-3)^2}{b^2} = 1
\]

then it would be a hyperbola centered at \((1, 3)\).

- Alternatively, if the equation was:

\[
\frac{(x-1)^2}{a^2} + \frac{(y-3)^2}{b^2} = 1
\]

it would be an ellipse.

Conclusion:
Given the initial form, it’s likely that the equation in the user prompt was a simplified or miswritten version. For the purpose of this article, we will assume the intended hyperbola is:

\[
\frac{(x-1)^2}{9} - \frac{(y-3)^2}{4} = 1
\]

which is a standard form of a hyperbola opening left and right, centered at \((1, 3)\).

Identifying the Center of the Hyperbola

In the standard form:

$$
\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1
$$

the center is located at (h, k).

Given the hyperbola:

$$
\frac{(x-1)^2}{9} - \frac{(y-3)^2}{4} = 1
$$

the center is:

$$
\boxed{(1, 3)}
$$

Determining the Vertices

Vertices are points on the hyperbola that are closest to or farthest from the center along the transverse axis.

For a hyperbola of the form $\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$:

- The transverse axis is horizontal.
- Vertices are located at $(h \pm a, k)$.

In our case:

- $a^2 = 9 \Rightarrow a = 3$
- $b^2 = 4 \Rightarrow b = 2$

Vertices are:

$$
(1 \pm 3, 3) \Rightarrow (-2, 3) \quad \text{and} \quad (4, 3)
$$

Summary:

- Vertices: $\boxed{(-2, 3)}$ and $\boxed{(4, 3)}$

These points are where the hyperbola intersects its transverse axis.

Locating the Foci of the Hyperbola

Foci are special points that help define the shape of the hyperbola and are located along the transverse axis at a distance c from the center, where:

$$
c^2 = a^2 + b^2
$$

Calculating c:

$$
c^2 = 9 + 4 = 13 \Rightarrow c = \sqrt{13} \approx 3.605
$$

Since the hyperbola opens horizontally, the foci are positioned at:

$$
(h \pm c, k)
$$

which gives:

- Focus 1: $(1 + \sqrt{13}, 3) \approx (1 + 3.605, 3) \approx (4.605, 3)$
- Focus 2: $(1 - \sqrt{13}, 3) \approx (1 - 3.605, 3) \approx (-2.605, 3)$

Summary:

$$
\boxed{
\text{Foci:} \quad (4.605, 3) \quad \text{and} \quad (-2.605, 3)
}
$$

These points are crucial in the definition of hyperbola, as the difference of distances from any point on the hyperbola to these foci is constant.

Summary of Key Features

Element	Coordinates or Value	Explanation
Center	$(1, 3)$	The midpoint of the hyperbola
Vertices	$(-2, 3)$, $(4, 3)$	Located along the transverse axis $a = 3$ units from the center
Foci	$(-2.605, 3)$, $(4.605, 3)$	Located at $c \approx 3.605$ units along the transverse axis

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Visualizing the Hyperbola

To fully understand the hyperbola, plotting these key points helps:

- Draw the center at $(1, 3)$.
- Mark the vertices at $(-2, 3)$ and $(4, 3)$.
- Mark the foci at approximately $(-2.605, 3)$ and $(4.605, 3)$.
- Sketch the asymptotes, which pass through the center and are given by the lines:

$$
y - 3 = \pm \frac{b}{a}(
$$

Frequently Asked Questions

What is the center of the hyperbola given by the equation (x - 1)/9 - (y - 3)/4 = 1?

The given equation is not in the standard form of a hyperbola; it appears to be a different form. However, if it's intended as $(x - 1)^2/9 - (y - 3)^2/4 = 1$, then the center is at (1, 3).

How do you identify the vertices of the hyperbola (assuming it is in standard form)?

For the hyperbola in the form $(x - h)^2/a^2 - (y - k)^2/b^2 = 1$, the vertices lie at $(h \pm a, k)$. In this case, if the hyperbola is centered at (1, 3) with $a = 3$, the vertices are at $(1 \pm 3, 3)$.

What are the foci of the hyperbola with the equation (x - 1)^2/9 - (y - 3)^2/4 = 1?

The foci are located along the transverse axis at a distance c from the center, where $c^2 = a^2 +$

b^2. Here, $a=3$, $b=2$, so $c = \sqrt{9 + 4} = \sqrt{13} \approx 3.61$. The foci are at $(1 \pm c, 3)$.

How can you determine the center of the ellipse or hyperbola from its equation?

The center can be found by rewriting the equation in standard form and identifying the (h, k) coordinates, which are the values inside the (x - h) and (y - k) terms.

What is the significance of the vertices and foci in hyperbolas?

Vertices are the closest points on each branch to the center along the transverse axis, defining the shape's width, while foci are fixed points used to define the hyperbola's eccentricity and reflect the geometric focus property.

Is the given equation (x - 1)/9 - (y - 3)/4 = 1 in standard form for a hyperbola?

No, the equation appears to be in a different form. The standard form for a hyperbola is $(x - h)^2/a^2 - (y - k)^2/b^2 = 1$ or vice versa. If the equation is intended as $(x - 1)^2/9 - (y - 3)^2/4 = 1$, then it is in standard form.

How do you find the length of the transverse axis of the hyperbola (x - 1)^2/9 - (y - 3)^2/4 = 1?

The length of the transverse axis is 2a. Since $a^2 = 9$, $a = 3$, so the length is 2 3 = 6 units.

Additional Resources

Identify The Center, Vertices, And Foci Of The Ellipse Or Hyperbola: (x-1)/9 - (y-3)/4=1

Understanding the geometric properties of conic sections such as ellipses and hyperbolas is fundamental in various fields of mathematics, physics, engineering, and computer graphics. The equation provided, $\frac{x-1}{9} - \frac{y-3}{4} = 1$, is a hyperbola, and accurately identifying its center, vertices, and foci is crucial for graphing and analyzing its characteristics. In this comprehensive review, we will explore the methods to determine these key features, delve into the properties of hyperbolas, and discuss the significance of each component in understanding the shape and orientation of the conic.

Understanding the Hyperbola Equation

The given equation:

$$
\frac{x-1}{9} - \frac{y-3}{4} = 1
$$

initially appears to be a hyperbola, but to classify it correctly, it's essential to rewrite it in its standard form. Typically, the standard form of a hyperbola centered at (h, k) with horizontal transverse axis is:

$$
\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1
$$

Similarly, for a hyperbola with a vertical transverse axis:

$$
\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1
$$

Given the original equation involves linear denominators, it's likely a misinterpretation or perhaps a simplified form. Since the form involves subtraction, the standard hyperbola form is plausible. To clarify, we should manipulate the equation:

$$
\frac{x-1}{9} - \frac{y-3}{4} = 1
$$

but as it stands, this is a linear combination, not a quadratic one. Typically, the equations for hyperbolas involve squared terms. Therefore, perhaps the original equation is a typo or misstatement. Assuming the intended equation is:

$$
\frac{(x - 1)^2}{9} - \frac{(y - 3)^2}{4} = 1
$$

which is a standard hyperbola with a horizontal transverse axis centered at $(1, 3)$.

Identifying the Center of the Hyperbola

Definition of the Center

The center of a hyperbola is the point equidistant from its vertices and foci, serving as the midpoint of the transverse and conjugate axes. It is the point around which the hyperbola is symmetric.

How to Find the Center

For hyperbolas in standard form:

$$
\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1
$$

the center is at (h, k).

Given the assumed form:

$$
\frac{(x - 1)^2}{9} - \frac{(y - 3)^2}{4} = 1
$$

the center is at:

$$
\boxed{(1, 3)}
$$

Features:
- The center is the geometric midpoint between the vertices and foci.
- It acts as the origin for the hyperbola's axes.

Pros:
- Easily identifiable in the standard form.
- Serves as a reference point for locating other key features.

Cons:
- Requires the equation to be in the correct standard form; not directly observable in general form.

Vertices of the Hyperbola

Definition of Vertices

Vertices are the points on the hyperbola where it intersects its transverse axis. They are the closest points to the center along the axis of symmetry.

How to Find the Vertices

In the standard form:

$$
\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1
$$

- The transverse axis is horizontal.
- The vertices are located at $(h \pm a, k)$.

For our assumed equation:

$$
\frac{(x - 1)^2}{9} - \frac{(y - 3)^2}{4} = 1
$$

- $a^2 = 9 \Rightarrow a = 3$
- $b^2 = 4 \Rightarrow b = 2$

Therefore,

$$
\textbf{Vertices:} \quad (h \pm a, k) = (1 \pm 3, 3)
$$

which yields:

$$
\boxed{
\text{Vertices at } (-2, 3) \text{ and } (4, 3)
}
$$

Features:
- Located along the x-axis, symmetric about the center.
- Define the width of the hyperbola at its narrowest points.

Pros:
- Easy to calculate once a is known.
- Useful in graphing and understanding the hyperbola's shape.

Cons:
- Only applicable for hyperbolas aligned along the x-axis; different for vertical hyperbolas.

Foci of the Hyperbola

Definition of Foci

Foci are two fixed points located along the transverse axis, positioned such that the difference of the distances from any point on the hyperbola to these foci is constant. They are critical in defining the shape and eccentricity of the hyperbola.

How to Find the Foci

In standard form:

$$
\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1
$$

- The foci are at $(h \pm c, k)$ where:

$$
c = \sqrt{a^2 + b^2}
$$

Calculating c:

$$
c = \sqrt{9 + 4} = \sqrt{13} \approx 3.605
$$

Thus, the foci are at:

$$
(1 \pm c, 3) = (1 \pm \sqrt{13}, 3)
$$

which gives:

$$
\boxed{
\text{Foci at } \left(1 - \sqrt{13}, 3\right) \text{ and } \left(1 + \sqrt{13}, 3\right)
}
$$

Features:
- Located along the same line as the vertices.
- The distance c from the center indicates the hyperbola's eccentricity.

Pros:
- Vital for understanding the hyperbola's asymptotic behavior.
- Used in applications involving optics and satellite communications.

Cons:
- Requires calculation of c, which depends on a and b.

Additional Features and Considerations

Asymptotes

The asymptotes of the hyperbola are straight lines passing through the center, given by:

$$
y - k = \pm \frac{b}{a}(x - h)
$$

In our example:

$$
y - 3 = \pm \frac{2}{3} (x - 1)
$$

These lines guide the shape of the hyperbola at large distances from the center.

Graphing and Visualization

- Knowing the center, vertices, and foci allows for accurate sketching.
- Asymptotes help approximate the hyperbola's branches.

Real-World Applications

- Satellite dish design.
- Planetary orbits.
- Signal processing.

Summary and Final Remarks

Accurately determining the center, vertices, and foci of a hyperbola is essential for understanding its geometric properties and for practical applications. For the equation assumed to be $\frac{(x - 1)^2}{9} - \frac{(y - 3)^2}{4} = 1$, the key features are:

- Center: \((1, 3)\)
- Vertices: \((-2, 3)\) and \((4, 3)\)
- Foci: \(\left(1 - \sqrt{13}, 3\right)\) and \(\left(1 + \sqrt{13}, 3\right)\)

These components define the hyperbola's shape, orientation, and key characteristics. Understanding how to derive these features from the standard form enhances one's ability to analyze and graph conic sections accurately.

Pros of mastering these concepts:
- Facilitates precise graphing of hyperbolas.
- Aids in solving real-world problems involving conic sections.
- Provides a foundation for understanding more complex mathematical and physical phenomena.

Cons:
- Requires familiarity with algebraic manipulation and standard forms.
- Misinterpretation of equations can lead to errors in identifying features.

In conclusion, mastering the identification of the center, vertices, and foci equips students and professionals with essential tools to analyze and utilize hyperbolas effectively. Continual practice and familiarity with the standard forms will foster

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