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Understanding the result of multiplying fractions is fundamental in mathematics, especially when analyzing whether the product exceeds or falls short of the original factors. In this article, we will explore whether multiplying two fractions—specifically $\frac{7}{8}$ and $\frac{4}{5}$ —results in a product that is greater than either of the factors. We will delve into fraction multiplication rules, compare the product with the individual factors, and clarify the underlying concepts with detailed explanations and examples.

Understanding Fraction Multiplication

Before analyzing the specific fractions $\frac{7}{8}$ and $\frac{4}{5}$, it is essential to understand how multiplication involving fractions works.

Rules of Fraction Multiplication

- Multiplying Numerators and Denominators: To multiply fractions, multiply the numerators together and the denominators together.

```
\[
\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}
\]
```

- Simplification: After multiplying, simplify the resulting fraction to its lowest terms if possible.

- Negative Fractions: The sign of the product depends on the signs of the factors.

Calculating the Product of $\frac{7}{8}$ and $\frac{4}{5}$

Applying the rule:

```
\[
\frac{7}{8} \times \frac{4}{5} = \frac{7 \times 4}{8 \times 5} =
\frac{28}{40}
\]
```

Simplify the fraction:

```
\[
\frac{28}{40} = \frac{7}{10}
\]
```

Thus, the product of $7/8$ and $4/5$ is $7/10$.

Comparing the Product to the Original Factors

Now, the core question: Is $7/10$ greater than either $7/8$ or $4/5$? To answer this, we need to compare $7/10$ with each factor individually.

Comparison with $7/8$

- Convert both fractions to decimal form:

```
\[
7/8 = 0.875
\]
```

```
\[
7/10 = 0.7
\]
```

- Clearly, $0.7 < 0.875$, so:

```
\[
\frac{7}{10} < \frac{7}{8}
\]
```

- Conclusion: The product ($7/10$) is less than $7/8$.

Comparison with $4/5$

- Convert to decimal:

```
\[
4/5 = 0.8
\]
```

```
\[
7/10 = 0.7
\]
```

- Since $0.7 < 0.8$, it follows that:

```
\[
\frac{7}{10} < \frac{4}{5}
\]
```

- Conclusion: The product ($7/10$) is less than $4/5$.

Implications of the Comparison

From the above comparisons, it is evident that:

- The product of $\frac{7}{8}$ and $\frac{4}{5}$ (which is $\frac{7}{10}$) is less than both factors.
- In other words, multiplying two fractions less than 1 results in a product that is smaller than either of the original fractions.

Why Is the Product Less Than the Factors?

This phenomenon is rooted in the properties of fractions less than 1. When multiplying two positive fractions less than 1:

- The product is always less than each factor.
- The factors represent parts of a whole; multiplying them effectively finds a fraction of a fraction, which reduces the overall size.

Mathematically:

If $(0 < a < 1)$ and $(0 < b < 1)$, then:

$$\begin{aligned} & \left[\right. \\ & a \times b < a \quad \text{and} \quad a \times b < b \\ & \left. \right] \end{aligned}$$

Applying this to our example:

$$\begin{aligned} & \left[\right. \\ & \frac{7}{8} \times \frac{4}{5} = \frac{7}{10} < \frac{7}{8} \quad \text{and} \\ & \quad \frac{7}{10} < \frac{4}{5} \\ & \left. \right] \end{aligned}$$

General Rules for Fraction Products and Their Magnitudes

Understanding when the product of two fractions is greater than, equal to, or less than the original factors involves analyzing their values:

When Is the Product Greater Than the Factors?

- If both fractions are greater than 1: Multiplying them results in a product larger than each factor.

- If one or both fractions are less than 1: The product is usually less than at least one of the factors.

When Is the Product Equal to a Factor?

- When one factor is 1 and the other is any fraction, the product equals that fraction.

When Is the Product Less Than Both Factors?

- When both fractions are less than 1, the product is less than both.

Additional Examples to Clarify the Concept

To further illustrate, here are some examples:

- Multiplying $\frac{3}{4}$ by $\frac{2}{3}$:

- Product: $\left(\frac{3}{4} \times \frac{2}{3}\right) = \frac{6}{12} = \frac{1}{2}$

- Comparisons:

- $\frac{1}{2} < \frac{3}{4}$ (0.75)

- $\frac{1}{2} < \frac{2}{3}$ (0.666...)

- Multiplying $\frac{5}{2}$ by $\frac{4}{3}$:

- Product: $\left(\frac{5}{2} \times \frac{4}{3}\right) = \frac{20}{6} = \frac{10}{3}$

- Comparisons:

- $\frac{10}{3} \approx 3.333...$ $> \frac{5}{2}$ (2.5)

- $\frac{10}{3} > \frac{4}{3}$ (1.333...)

These examples show that when both fractions are greater than 1, the product can be greater than the factors, contrasting with our original case where

both fractions are less than 1.

Summary and Final Thoughts

- The product of $\frac{7}{8}$ and $\frac{4}{5}$ is $\frac{7}{10}$.
- When multiplying two fractions less than 1, the product is always less than each of the original fractions.
- Therefore, the product ($\frac{7}{10}$) is not greater than either $\frac{7}{8}$ or $\frac{4}{5}$.
- This aligns with the general principle that multiplying fractions less than 1 results in a smaller fraction, representing a part of a part.

In conclusion, multiplying $\frac{7}{8}$ by $\frac{4}{5}$ yields a product smaller than both factors because both are less than 1, and the multiplication of such fractions results in an even smaller value. This understanding helps in solving more complex fraction multiplication problems and in grasping the behavior of fractions in various mathematical contexts.

SEO Keywords: fraction multiplication, $\frac{7}{8}$ times $\frac{4}{5}$, is product greater than factors, fraction comparison, multiplying fractions less than 1, fraction multiplication rules, fraction comparison examples, mathematical concepts of fractions

Frequently Asked Questions

What is the product of $\frac{7}{8}$ and $\frac{4}{5}$?

The product is $(\frac{7}{8}) \times (\frac{4}{5}) = \frac{28}{40} = \frac{7}{10}$.

Is the product of $\frac{7}{8}$ and $\frac{4}{5}$ greater than $\frac{7}{8}$?

No, the product $\frac{7}{10}$ is less than $\frac{7}{8}$, so it is not greater.

Is the product of $\frac{7}{8}$ and $\frac{4}{5}$ greater than $\frac{4}{5}$?

No, since $\frac{7}{10}$ is less than $\frac{4}{5}$ (which is $\frac{8}{10}$), the product is not greater.

Why is the product of two fractions less than either of the factors?

Because multiplying fractions typically results in a smaller value unless one factor is greater than 1, which is not the case here.

Can the product of two fractions ever be greater than both factors?

Only if both factors are greater than 1, which isn't true in this case since both are less than 1.

What is the key reason the product of $\frac{7}{8}$ and $\frac{4}{5}$ is less than either factor?

Because multiplying fractions less than 1 reduces their value, resulting in a product smaller than either factor.

Does multiplying $\frac{7}{8}$ by $\frac{4}{5}$ change the order of which is larger?

No, since both are less than 1, their product remains less than each of the original factors.

In general, when multiplying two proper fractions, will the product be greater than either?

No, the product of two proper fractions (less than 1) will always be less than each of the factors.

Additional Resources

If $\frac{7}{8}$ Is Multiplied By $\frac{4}{5}$, Will The Product Be Greater Than Either Of The Two Factors? Explain

When working with fractions, a common question that arises is whether multiplying two fractions results in a product that is greater than either of the original factors. Specifically, in the case of $\frac{7}{8}$ multiplied by $\frac{4}{5}$, learners and math enthusiasts alike wonder: Will the product be greater than $\frac{7}{8}$ or $\frac{4}{5}$? Understanding this concept not only deepens comprehension of fraction multiplication but also enhances overall mathematical intuition. This article provides a comprehensive analysis of this problem, exploring the principles behind fraction multiplication, comparing the product to the original factors, and explaining the reasoning with clear examples.

Understanding Fraction Multiplication

Before diving into the specific problem, it's essential to understand the foundational concepts of multiplying fractions.

What Does Multiplying Fractions Mean?

Multiplying fractions involves taking parts of parts. If you have $\frac{a}{b}$ and $\frac{c}{d}$, their product is calculated as:

$$(\frac{a}{b}) \times (\frac{c}{d}) = (\frac{a \times c}{b \times d})$$

This operation essentially scales one fraction by the other. Because fractions represent parts of a whole, multiplying two fractions results in a smaller (or sometimes equal) quantity, depending on the values involved.

Visualizing Fraction Multiplication

Visual models help clarify how the product compares to the original factors:

- Area models: Imagine a rectangle representing 1 whole unit. Divide it into b equal parts for the first fraction, and shade a parts. Repeat for the second fraction to see how the overlapping shaded area corresponds to the product.

- Number line models: Mark the fractions on a number line to see how their product relates to the original points.

Key Concept: Multiplying Fractions Generally Produces a Smaller Number

Except when multiplying by fractions greater than 1, the product tends to be less than either of the factors. This is because fractions less than 1 represent parts of a whole, and multiplying two such parts typically results in an even smaller part.

Analyzing the Specific Case: $7/8 \times 4/5$

Let's now examine the specific problem:

Is the product of $7/8$ and $4/5$ greater than $7/8$ or $4/5$?

Step 1: Compute the Product

Using the multiplication rule for fractions:

$$(7/8) \times (4/5) = (7 \times 4) / (8 \times 5) = 28 / 40$$

Simplify the fraction:

$$28 / 40 = (28 \div 4) / (40 \div 4) = 7 / 10$$

Thus,

$$7/8 \times 4/5 = 7/10$$

Step 2: Compare the Product to the Original Factors

Now, compare $7/10$ to $7/8$ and $4/5$.

- Compare $7/10$ and $7/8$:

Convert to common denominators or decimals:

$$7/10 = 0.7$$

$$7/8 \approx 0.875$$

Since $0.7 < 0.875$, $7/10$ is less than $7/8$.

- Compare $7/10$ and $4/5$:

$$4/5 = 0.8$$

$$7/10 = 0.7$$

Since $0.7 < 0.8$, $7/10$ is less than $4/5$.

Result: The product $7/10$ is less than both $7/8$ and $4/5$.

General Principles and Reasoning

The specific calculation reveals a general trend: the product of two fractions less than 1 is always less than either of the original fractions.

Why Is This the Case?

- Fractions less than 1 represent parts of a whole.
- When multiplying two such fractions, the result is:
less than each factor individually, because:
 - Multiplying by a fraction less than 1 reduces the magnitude.
 - The product is effectively a scaled-down version of either factor.

Formal Explanation:

Suppose:

- Both fractions are between 0 and 1:

$$0 < a/b < 1$$

$$0 < c/d < 1$$

- Then, their product:

$$(a/b) \times (c/d) < a/b$$

and

$$(a/b) \times (c/d) < c/d$$

because multiplying by a fraction less than one reduces the value.

When Can the Product Be Greater Than Either Factor?

- If either of the fractions is greater than 1, then the product may be greater than one or even greater than one of the factors.
- For example, multiplying $2/1$ (which equals 2) by $3/2$:

$$(2/1) \times (3/2) = (2 \times 3) / (1 \times 2) = 6/2 = 3$$

Which is greater than both original factors.

- But in the case of $7/8$ and $4/5$, both are less than 1, so the product is necessarily less than both.

Visual and Intuitive Explanation

Visual Model of $\frac{7}{8} \times \frac{4}{5}$

Imagine a rectangle representing a whole:

1. Divide the rectangle into 8 equal parts horizontally: each part is $\frac{1}{8}$.
2. Shade 7 parts to represent $\frac{7}{8}$.
3. Divide the same rectangle into 5 equal parts vertically: each part is $\frac{1}{5}$.
4. Shade 4 parts vertically to represent $\frac{4}{5}$.

The overlap of the shaded regions corresponds to the product:

- The overlapping area is $(\frac{7}{8}) \times (\frac{4}{5}) = \frac{7}{10}$ of the whole.

This confirms the calculation and visually shows that the product is less than either of the original fractions (which are $\frac{7}{8} \approx 0.875$ and $\frac{4}{5} = 0.8$).

Practical Implications and Summary

Summary of Findings:

- The product of $\frac{7}{8}$ and $\frac{4}{5}$ is $\frac{7}{10}$, which is approximately 0.7.
- This product is less than both $\frac{7}{8}$ (≈ 0.875) and $\frac{4}{5}$ (0.8).
- In general, when multiplying two fractions less than 1, the result is less than either of the two factors.

Practical Takeaways:

- Be cautious when estimating products of fractions: unless one factor exceeds 1, expect the product to be smaller.
- For fractions greater than 1, multiplication can produce a larger number than either factor.
- Visual models and decimal conversions are powerful tools for understanding and verifying these relationships.

Final Thoughts: Clarifying Common Misconceptions

Many students intuitively believe that multiplying two fractions might increase the value, especially if they think of multiplication as "scaling up." However, in the context of fractions less than 1, multiplication scales down the value. Recognizing this helps avoid misconceptions and enhances understanding.

In conclusion, if $\frac{7}{8}$ is multiplied by $\frac{4}{5}$, the product will not be greater than either of the two factors. Instead, it results in a smaller value, specifically $\frac{7}{10}$, exemplifying the general principle that multiplying two fractions less than 1 produces a smaller number.

In essence:

- $\frac{7}{8} \times \frac{4}{5} = \frac{7}{10}$, which is less than both $\frac{7}{8}$ and $\frac{4}{5}$.

- Multiplying fractions less than 1 diminishes their size.
- The question highlights the importance of understanding the nature of fractions and how basic operations affect their magnitude.

By mastering these concepts, learners can develop a more intuitive and accurate grasp of fractions, paving the way for more advanced mathematical reasoning.

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