# If The Length Breadth And Height Of 3x+y Cm 2x+2y Cm And 3y Cm Respectively And Find Its Volume

**If The Length Breadth And Height Of 3x+y Cm 2x+2y Cm And 3y Cm Respectively And Find Its Volume**

Understanding the problem of calculating the volume of a rectangular prism with variable dimensions is an essential aspect of geometry and algebra. In this article, we will explore how to determine the volume when the length, breadth, and height are expressed as algebraic expressions involving variables x and y. We will analyze the given dimensions: length as 3x + y cm, breadth as 2x + 2y cm, and height as 3y cm. By systematically working through the algebraic steps, we aim to derive a comprehensive formula for the volume and discuss its applications.

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## Understanding the Dimensions of the Rectangular Prism

Before diving into calculations, it's crucial to understand the nature of the dimensions involved:

- Length (L): 3x + y centimeters
- Breadth (B): 2x + 2y centimeters
- Height (H): 3y centimeters

These dimensions are expressed as algebraic expressions, indicating that the volume depends on the variables x and y. Such problems are common in algebra and geometry, especially when dealing with variable-sized objects or optimizing dimensions.

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## Step-by-Step Calculation of the Volume

The volume (V) of a rectangular prism is given by the formula:

$$ V = \text{Length} \times \text{Breadth} \times \text{Height} $$

Given the dimensions, substitute the expressions:

$$ V = (3x + y) \times (2x + 2y) \times 3y $$

Our goal is to expand and simplify this expression to find a formula for volume in terms of x and y.

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## Step 1: Simplify the Expression for Breadth

Notice that the breadth expression contains a common factor:

$$ 2x + 2y = 2(x + y) $$

So, the volume formula becomes:

$$ V = (3x + y) \times 2(x + y) \times 3y $$

This simplifies the algebra and makes factorization easier.

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## Step 2: Multiply the First Two Factors

Focus on multiplying:

$$ (3x + y) \times 2(x + y) $$

Distribute 2:

$$ (3x + y) \times 2x + 2y $$

Now, expand:

$$ (3x + y) \times 2(x + y) = 2(3x + y)(x + y) $$

Further expand:

$$ 2 \times [(3x + y)(x + y)] $$

Let's focus on expanding $ (3x + y)(x + y) $:

$$ (3x + y)(x + y) = 3x \times x + 3x \times y + y \times x + y \times y $$

Simplify:

$$ 3x^2 + 3xy + xy + y^2 $$

Combine like terms:

$$ 3x^2 + (3xy + xy) + y^2 = 3x^2 + 4xy + y^2 $$

Now, multiply this result by 2:

$$2 \times (3x^2 + 4xy + y^2) = 6x^2 + 8xy + 2y^2$$

So, the product of the first two factors simplifies to:

$$6x^2 + 8xy + 2y^2$$

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## Step 3: Multiply the Result by $3y$

Recall the full volume expression:

$$V = (first \; two \; factors) \times 3y$$

Substitute the simplified form:

$$V = (6x^2 + 8xy + 2y^2) \times 3y$$

Distribute $3y$:

$$V = 6x^2 \times 3y + 8xy \times 3y + 2y^2 \times 3y$$

Calculate each term:

- $6x^2 \times 3y = 18x^2 y$
- $8xy \times 3y = 24 xy^2$
- $2y^2 \times 3y = 6 y^3$

Therefore, the total volume expression is:

$$V = 18x^2 y + 24 xy^2 + 6 y^3$$

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# Final Formula for the Volume

Putting it all together, the volume of the rectangular prism with variable dimensions is:

$$\boxed{V = 18x^2 y + 24 xy^2 + 6 y^3} \quad \text{cubic centimeters}$$

This algebraic expression describes how the volume varies with the parameters x and y.

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# Applications and Implications of the Volume Formula

Understanding the volume in terms of x and y has several practical applications:

- Design Optimization: Engineers and architects can use this formula to optimize dimensions for maximum volume within constraints.
- Manufacturing: Adjusting x and y allows for customizing products based on space or material limitations.
- Mathematical Modeling: The formula exemplifies how algebra can describe real-world objects with variable parameters.
- Educational Purposes: Helps students understand the process of expanding algebraic expressions and applying geometric formulas.

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# Special Cases and Further Analysis

Analyzing specific values of x and y can yield insights into the shape and size of the object:

- Case 1: When y = 0

$$ V = 18x^2 \times 0 + 24 x \times 0^2 + 6 \times 0^3 = 0 $$

This indicates the volume collapses when y is zero, implying the prism reduces to a degenerate shape or no volume.

- Case 2: When x = 0

$$ V = 18 \times 0^2 \times y + 24 \times 0 \times y^2 + 6 y^3 = 6 y^3 $$

In this case, the volume depends solely on y, which reflects a specific configuration where the length and breadth are minimized.

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# Conclusion

Calculating the volume of a rectangular prism with dimensions expressed algebraically as functions of variables x and y involves systematic expansion and simplification. Starting from the given expressions:

- Length: 3x + y cm
- Breadth: 2x + 2y cm
- Height: 3y cm

we derived the volume formula:

$$ V = 18x^2 y + 24 xy^2 + 6 y^3 $$

This formula encapsulates the relationship between the variables and the overall volume, allowing for various applications in mathematical modeling, engineering, and design. Understanding these steps enhances problem-solving skills in algebra and geometry, demonstrating the power of algebraic manipulation in real-world contexts.

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Keywords: volume calculation, rectangular prism, algebraic expressions, geometric dimensions, variable dimensions, volume formula, algebra in geometry, problem-solving, mathematical modeling

# Frequently Asked Questions

## What are the dimensions of the rectangular box given in the problem?

The length is $3x + y$ cm, the breadth is $2x + 2y$ cm, and the height is $3y$ cm.

## How do you find the volume of a rectangular box with given dimensions?

The volume is calculated by multiplying the length, breadth, and height: Volume = length × breadth × height.

## What is the formula for the volume of the given box in terms of x and y?

The volume $V = (3x + y) \times (2x + 2y) \times 3y$.

## How can the volume expression be simplified before calculation?

First, expand the product $(3x + y)(2x + 2y)$, then multiply the result by $3y$ to find the volume.

## What is the expanded form of (3x + y)(2x + 2y)?

Expanding gives: $(3x)(2x) + (3x)(2y) + y(2x) + y(2y) = 6x^2 + 6xy + 2xy + 2y^2 = 6x^2 + 8xy + 2y^2$.

## How do you calculate the final volume after expansion?

Multiply the expanded expression $(6x^2 + 8xy + 2y^2)$ by $3y$ to get the volume: $V = 3y(6x^2 + 8xy + 2y^2)$.

## What is the simplified form of the volume expression?

V = 18x^2 y + 24xy^2 + 6y^3.

## Can the volume be expressed in terms of specific values of x and y?

Yes, if specific values of x and y are given, substitute them into the simplified formula to find the volume.

## Why is understanding the algebraic expansion important in calculating volume?

It helps in accurately simplifying complex expressions, making it easier to evaluate the volume for different values of x and y.

## Is the problem asking for a numerical answer or an algebraic expression?

The problem is asking for the algebraic expression of the volume in terms of x and y.

# Additional Resources

If The Length Breadth And Height Of 3x + y cm, 2x + 2y cm, And 3y cm Respectively And Find Its Volume

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Introduction

Mathematics often presents us with problems that seem straightforward at first glance but reveal layers of complexity upon closer inspection. One such problem involves calculating the volume of a rectangular prism where its dimensions are expressed algebraically in terms of variables \( x \) and \( y \). The problem statement—If the length, breadth, and height are \( 3x + y \) cm, \( 2x + 2y \) cm, and \( 3y \) cm respectively, find its volume—serves as an intriguing case study for algebraic manipulation, geometric interpretation, and problem-solving strategies.

This article aims to dissect this problem comprehensively, exploring the underlying concepts, potential pitfalls, and broader applications. We will examine the problem's structure, analyze the algebraic expressions involved, and demonstrate step-by-step calculations to arrive at the volume. Additionally, we will investigate how different values of \( x \) and \( y \) influence the volume, discuss the significance of such problems in mathematical education, and explore their relevance in real-world contexts.

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Contextualizing the Problem: Geometry Meets Algebra

At its core, the problem involves a three-dimensional rectangular object—commonly known as a rectangular prism or cuboid—with dimensions defined in algebraic terms. Understanding how to compute volume in such scenarios requires a solid grasp of the basic geometric formula:

$$
\text{Volume} = \text{Length} \times \text{Breadth} \times \text{Height}
$$

Given that the dimensions are expressed as algebraic expressions involving $x$ and $y$, the calculation becomes an exercise in algebraic expansion and simplification.

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Deep Dive into Dimensions

Length: $3x + y$ cm

This dimension suggests a linear combination of the variables $x$ and $y$, scaled by coefficients 3 and 1 respectively. Its significance lies in how variations in $x$ and $y$ directly influence the length.

Breadth: $2x + 2y$ cm

Expressed as $2x + 2y$, this dimension can be factored to highlight common factors:

$$
2x + 2y = 2(x + y)
$$

which simplifies the algebraic manipulation and offers insights into how the sum $x + y$ impacts the breadth.

Height: $3y$ cm

The simplest of the three, height depends solely on $y$, scaled by 3. Its simplicity allows for straightforward substitution once the values of $y$ are known.

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Calculating the Volume: Step-by-Step Approach

Step 1: Write the volume formula with given dimensions

$$
V = (3x + y) \times (2x + 2y) \times 3y
$$

Step 2: Simplify the breadth expression

$$
2x + 2y = 2(x + y)
$$

\]

Thus,

\[
V = (3x + y) \times 2(x + y) \times 3y
\]

Step 3: Re-arrange for clarity

\[
V = (3x + y) \times 2(x + y) \times 3y
\]

which simplifies to:

\[
V = 2 \times 3 \times y \times (3x + y) \times (x + y)
\]

or

\[
V = 6 y (3x + y)(x + y)
\]

Step 4: Expand the product \( (3x + y)(x + y) \)

Applying distributive property:

\[
(3x + y)(x + y) = 3x \times x + 3x \times y + y \times x + y \times y
\]

Simplify:

\[
= 3x^2 + 3xy + xy + y^2
\]

Combine like terms:

\[
= 3x^2 + 4xy + y^2
\]

Step 5: Final expression for volume

Substitute back into the volume formula:

\[

$$
V = 6 y (3x^2 + 4xy + y^2)
$$

Distribute $6 y$:

$$
V = 6 y \times 3x^2 + 6 y \times 4xy + 6 y \times y^2
$$

which simplifies to:

$$
V = 18 x^2 y + 24 x y^2 + 6 y^3
$$

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## Interpretation and Analysis

The volume $V$ is expressed as a cubic polynomial in terms of $x$ and $y$:

$$
V = 18 x^2 y + 24 x y^2 + 6 y^3
$$

This expression encapsulates how variations in $x$ and $y$ influence the volume:

- When $x$ increases (with $y$ fixed), the volume increases proportionally to $x^2 y$ and $x y^2$.
- When $y$ increases (with $x$ fixed), the volume increases proportionally to $x y^2$ and $y^3$.

This polynomial form also hints at potential factorizations and the geometric relationships between the dimensions.

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## Special Cases and Numerical Examples

Case 1: $x = 1, y = 1$

$$
V = 18 (1)^2 (1) + 24 (1)(1)^2 + 6 (1)^3 = 18 + 24 + 6 = 48 \text{ cm}^3
$$

Case 2: $x = 2, y = 3$

$$
V = 18 (2)^2 (3) + 24 (2)(3)^2 + 6 (3)^3 = 18 \times 4 \times 3 + 24 \times 2 \times 9 + 6 \times 27
$$

Calculate each term:

$$
= 18 \times 12 + 24 \times 18 + 162 = 216 + 432 + 162 = 810 \text{ cm}^3
$$

These examples demonstrate how the volume scales with the variables and serve as validation points for the algebraic expression.

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## Broader Implications and Applications

### Mathematical Education

Problems like this reinforce the importance of algebraic manipulation in understanding geometric concepts. They serve as valuable exercises for students to develop proficiency in expanding, factoring, and interpreting polynomial expressions.

### Engineering and Design

In practical applications such as architecture and manufacturing, dimensions are often expressed in terms of variables representing design parameters. Being able to derive volume expressions allows engineers to optimize material usage, structural integrity, and cost-efficiency.

### Optimization and Calculus

The polynomial form of the volume allows for exploring maximum or minimum values through calculus techniques—critical in fields requiring optimal design solutions.

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## Potential Extensions and Further Investigations

- Parameter Analysis: Investigate how the volume behaves as $x$ and $y$ vary within specific ranges.
- Constraints: Explore scenarios where $x$ and $y$ must satisfy certain conditions (e.g., positive values, relationships).
- Inverse Problems: Given a fixed volume, determine possible pairs $(x, y)$ satisfying the dimensions.
- Graphical Representation: Use 3D plots to visualize how the volume changes over the domain of $x$ and $y$.

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## Conclusion

The problem of finding the volume of a rectangular prism with dimensions expressed algebraically as $3x + y$, $2x + 2y$, and $3y$ exemplifies the interplay between algebra and geometry. Through systematic expansion and simplification, we derive a comprehensive expression:

$$
\boxed{V = 18 x^2 y + 24 x y^2 + 6 y^3}
$$

This formula not only facilitates straightforward computation for specific values of $x$ and $y$ but also opens avenues for deeper analysis, optimization, and real-world application.

Mathematics, in this context, manifests as a powerful tool for understanding and manipulating the dimensions of space, highlighting the elegance and utility of algebraic techniques in solving tangible problems. Whether in education, engineering, or research, mastering such problems enhances analytical skills and fosters a deeper appreciation of the interconnectedness of mathematical concepts.

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