# If Three Numbers 36,54 And Another Number Have A G.C.D Of 6 And L.C.M Of 216, Find The Other Number

**If Three Numbers 36, 54 And Another Number Have A G.C.D Of 6 And L.C.M Of 216, Find The Other Number**

Understanding how to work with the Greatest Common Divisor (G.C.D) and Least Common Multiple (L.C.M) is fundamental in solving various problems in number theory. When given certain conditions involving multiple numbers, such as their G.C.D and L.C.M, we can often determine unknown numbers through systematic analysis. In this article, we will explore the problem: "If three numbers 36, 54, and another number have a G.C.D of 6 and an L.C.M of 216, find the other number." We will walk through the concepts, step-by-step calculations, and the logical reasoning needed to find the unknown number.

## Understanding G.C.D and L.C.M

Before diving into the problem, it’s essential to understand what G.C.D and L.C.M mean and how they relate to numbers.

### What is G.C.D?

- The Greatest Common Divisor (G.C.D) of two or more numbers is the largest positive integer that divides each of the numbers without leaving a remainder.
- Example: G.C.D of 36 and 54 is 6 because 6 is the largest number dividing both 36 and 54 evenly.

### What is L.C.M?

- The Least Common Multiple (L.C.M) of two or more numbers is the smallest positive integer that is divisible by each of the numbers.
- Example: L.C.M of 36 and 54 is 216 because 216 is the smallest number divisible by both 36 and 54.

## Given Data and Our Goal

Let’s restate the problem with the given data:

- Known numbers: 36 and 54
- Unknown number: x
- G.C.D of 36, 54, and x: 6

- L.C.M of 36, 54, and x: 216

Our goal: Find the value of x.

# Step 1: Prime Factorization of Known Numbers

Prime factorization helps us understand the divisibility and common factors of the numbers involved.

## Prime factorization of 36

- $36 = 2^2 \times 3^2$

## Prime factorization of 54

- $54 = 2 \times 3^3$

# Step 2: Analyzing the G.C.D of 36 and 54

- G.C.D of 36 and 54:
- Take the minimum powers of common prime factors.
- Both have primes 2 and 3.
- For 2: min exponent is 1 (since 36 has $2^2$, 54 has $2^1$)
- For 3: min exponent is 2 (36 has $3^2$, 54 has $3^3$)
- $G.C.D(36,54) = 2^1 \times 3^2 = 2 \times 9 = 18$

# Step 3: Prime Factorization of the Unknown Number x

Since the G.C.D of all three numbers is 6, and the G.C.D of 36 and 54 is 18, the G.C.D of all three numbers being 6 implies:

- $G.C.D(36, 54, x) = 6$

Note:
- $6 = 2 \times 3$

Given that G.C.D of the three numbers is 6, the common prime factors across all three must include 2 and 3, but only to the first power (since $6 = 2^1 \times 3^1$).

This means:
- x must have at least one 2 and one 3 as factors to share 6 as G.C.D.

- x cannot have higher powers of 2 or 3 beyond what maintains G.C.D as 6.

## Step 4: Prime Factorization of x Based on G.C.D

Assuming x has the prime factorization:

$x = 2^a \times 3^b \times$ other primes (possibly)

Since G.C.D(36, 54, x) = 6 = $2^1 \times 3^1$, the following must be true:

- $a \geq 1$ (x has at least one 2)
- $b \geq 1$ (x has at least one 3)
- For the G.C.D to be exactly 6, the minimum exponents of 2 and 3 among the three numbers must be 1.

Also, because the G.C.D is 6, the exponents of 2 and 3 in x cannot be less than 1 (or else G.C.D would be less than 6), and cannot be more than 1 (or else G.C.D would be greater than 6).

Thus:
- $a = 1$
- $b = 1$

So, x includes $2^1$ and $3^1$ as factors.

## Step 5: Analyzing the L.C.M of the Three Numbers

Next, consider the L.C.M of the three numbers:

L.C.M(36, 54, x) = 216

Prime factorization of 216:

- $216 = 2^3 \times 3^3$

Recall:
- $36 = 2^2 \times 3^2$
- $54 = 2^1 \times 3^3$

The L.C.M is calculated by taking the maximum exponents of each prime factor among the numbers.

Given that, for the L.C.M to be $2^3 \times 3^3$:

- For prime 2:
- Max exponent among 36 ($2^2$), 54 ($2^1$), and x ($2^a$) must be 3.

- Since 36 has 2^2, to reach 2^3, x must have at least 2^3.
- But this conflicts with earlier conclusion that a=1 for G.C.D of 6, which requires x to have 2^1.

This suggests we need to reevaluate the assumption about the exponents.

Important note: For G.C.D to be 6, the minimum exponents for 2 and 3 across all numbers are 1.

But for L.C.M to be 216 (2^3 × 3^3), the maximum exponents among all three numbers must be 3 for 2 and 3.

- Since 36 has 2^2, to reach 2^3 in the L.C.M, x must have at least 2^3.
- Since 54 has 2^1, and the maximum is 2^3, x must have 2^3.
- Similarly, 36 has 3^2, and to reach 3^3 in L.C.M, x must have 3^3.
- 54 has 3^3, so that’s sufficient.

Now, considering the G.C.D = 6 = 2^1 × 3^1, the minimal exponents for the G.C.D are 1 for 2 and 3.

- For the G.C.D to be 6, the exponents for 2 and 3 in all three numbers must be at least 1, and the G.C.D takes the minimum exponents.

- Since x must have at least 2^3 and 3^3 to achieve the L.C.M of 216, but the G.C.D is only 6, which has exponents 1 for both primes, this indicates that x's exponents cannot be higher than 1 for the G.C.D.

This apparent contradiction suggests a need to understand the relationship between G.C.D and L.C.M in the context of multiple numbers.

Key Property:

For any two numbers a and b:

a × b = G.C.D(a, b) × L.C.M(a, b)

Similarly, for three numbers a, b, c:

a × b × c = G.C.D(a, b, c) × L.C.M(a, b, c) × (some factors depending on the numbers)

But more straightforwardly, for three numbers:

G.C.D × L.C.M = Product of the numbers divided by their common factors.

Alternatively, the product of the numbers equals the product of their G.C.D and L.C.M, multiplied by the appropriate overlaps.

Step 6: Applying the Key Property

Let's verify the product of the known numbers:

- $36 \times 54 = 1944$

Suppose x is the unknown number.

The product of all three numbers:

- $36 \times 54 \times x$

From the property:

G.C.D(36,54,x) × L.C.M(36,54,x) = (product of the three numbers) / (common factors)

But, in the case of three numbers, the relationship is more complex, and it's often easier to work with prime factorizations directly.

Step 7: Prime Factorization Approach

From earlier steps:

- $36 = 2^2 \times 3^2$
- $54 = 2^1 \times 3^3$
- G.C.D = $6 = 2^1 \times 3^1$
- L.C.M = $216 = 2^3 \times 3^3$

# Frequently Asked Questions

## What is the given G.C.D of the three numbers 36, 54, and the unknown number?

The G.C.D is 6.

## What is the given L.C.M of the three numbers 36, 54, and the unknown number?

The L.C.M is 216.

## How can we verify the G.C.D of 36 and 54?

By finding the highest common factor of 36 and 54, which is 6.

## What is the relationship between the G.C.D and L.C.M of three numbers?

The product of the three numbers equals the product of their G.C.D and L.C.M: a b c = G.C.D L.C.M (some factor), but more practically, for two numbers, G.C.D L.C.M = product of the numbers.

## How can we find the unknown number given the G.C.D and L.C.M of the three numbers?

Use the relationships between the numbers, their G.C.D, and L.C.M, and factorization methods to determine the possible values of the unknown number.

## What is the first step to find the other number in this problem?

Express the known numbers in terms of their G.C.D and find their simplified forms.

## What is the final value of the unknown number in this problem?

The other number is 12.

# Additional Resources

If three numbers 36, 54, and another number have a G.C.D of 6 and an L.C.M of 216, find the other number — this problem combines fundamental concepts of number theory, specifically the Greatest Common Divisor (G.C.D.) and Least Common Multiple (L.C.M.), to determine an unknown value based on given constraints. Such problems are common in competitive exams, mathematical puzzles, and algebraic reasoning, and mastering them requires a clear understanding of the properties of divisibility and the relationships between numbers.

In this comprehensive guide, we will explore how to approach this problem systematically, breaking down the concepts involved, applying relevant formulas, and working through the steps to find the unknown number. Whether you're a student preparing for exams or a math enthusiast eager to deepen your understanding, this detailed analysis will clarify the process and provide useful tips to handle similar problems.

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Understanding the Problem

Let's first restate the problem in our own words:

Given three numbers: 36, 54, and an unknown number, say x. The G.C.D of these three numbers is 6, and their L.C.M is 216. Find the value of x.

This problem involves two key concepts:

- Greatest Common Divisor (G.C.D.): The largest number that divides all three numbers without leaving a remainder.
- Least Common Multiple (L.C.M.): The smallest number that is a multiple of all three numbers.

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Fundamental Concepts and Properties

Before diving into calculations, it's essential to understand the properties of G.C.D. and L.C.M.:

1. Relationship Between G.C.D. and L.C.M.

For any two positive integers a and b:

$$ a \times b = \text{G.C.D.}(a, b) \times \text{L.C.M.}(a, b) $$

For three numbers a, b, and c, the relationship extends as:

$$ a \times b \times c = \text{G.C.D.}(a, b, c) \times \text{L.C.M.}(a, b, c) \times \text{(some factor depending on overlaps)} $$

But more practically, when considering multiple numbers, the key property used is:

$$ \text{G.C.D.}(a, b, c) \times \text{L.C.M.}(a, b, c) = \text{Product of the three numbers divided by their common factors} $$

However, for simplicity, when dealing with three numbers, the relation involving pairwise G.C.D.s and the overall G.C.D. and L.C.M. can be used to deduce the unknown.

2. G.C.D. Divides All Numbers

Since the G.C.D. of 36, 54, and x is 6, it means:

- 6 divides 36
- 6 divides 54
- 6 divides x

3. Expressing the Numbers in Terms of G.C.D.

Any number with G.C.D 6 can be expressed as:

- $36 = 6 \times 6$
- $54 = 6 \times 9$
- $x = 6 \times$ some integer k

Let’s denote:

- $36 = 6 \times 6$
- $54 = 6 \times 9$
- $x = 6 \times k$

Since the G.C.D. of the three numbers is 6, the G.C.D. of their quotients (after dividing each by 6) should be 1.

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Step-by-Step Solution Approach

Step 1: Simplify the Numbers

Express all three numbers in terms of their G.C.D.:

- $a = 36 = 6 \times 6$
- $b = 54 = 6 \times 9$
- $c = x = 6 \times k$

Given that the G.C.D. of the three numbers is 6, the G.C.D. of 6, 9, and k must be 1:

$$
\text{G.C.D.}(6, 9, k) = 1
$$

Step 2: Use the L.C.M. Relationship

The L.C.M. of the three numbers is 216. Since:

$$
\text{L.C.M.}(36, 54, x) = 216
$$

and 36 and 54 are known, we can express the L.C.M. in terms of their factors.

Step 3: Find the L.C.M. of 36 and 54

Prime factorization:

- $36 = 2^2 \times 3^2$
- $54 = 2^1 \times 3^3$

To find their L.C.M., take the highest powers of each prime:

$$
\text{L.C.M.}(36, 54) = 2^{\max(2,1)} \times 3^{\max(2,3)} = 2^2 \times 3^3 = 4 \times 27 = 108
$$

So, the L.C.M. of 36 and 54 is 108.

Step 4: Incorporate the Unknown Number

The overall L.C.M. of 36, 54, and x is 216. Since the L.C.M. of 36 and 54 alone is 108, and the overall L.C.M. (including x) is 216, x must contribute additional prime factors or higher powers to reach 216.

Expressed mathematically:

$$
\text{L.C.M.}(36, 54, x) = \text{L.C.M.}(\text{L.C.M.}(36, 54), x) = 216
$$

Because the L.C.M. of 36 and 54 is 108, and:

$$
\text{L.C.M.}(108, x) = 216
$$

Step 5: Find Possible Values of x

Express x in terms of its prime factors, considering $x = 6 \times k$ and that G.C.D. of 6, 9, and k is 1.

Recall that:

- $108 = 2^2 \times 3^3$

Since the L.C.M. of 108 and x is 216, and $216 = 2^3 \times 3^3$, the prime factorization of x must contribute to raising the 2's exponent from 2 to 3 (to match 216).

Thus, x must have at least $2^3$ in its prime factorization component when combined with 108.

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Deducing the Value of the Unknown Number

Step 6: Prime Factor Analysis of x

Given the above, x must be such that:

$$
\text{L.C.M.}(108, x) = 216
$$

and x can be written as:

$$
x = 6 \times k = 2 \times 3 \times k
$$

Since x must contribute at least $2^3$ in the combined L.C.M., and 108 has $2^2$, x must have at least $2^3$.

Let’s consider x in prime factor form:

$$
x = 2^a \times 3^b \times m
$$

where m contains any other primes, but for simplicity, since 108 contains only 2 and 3, and the overall L.C.M is 216 = $2^3 \times 3^3$, x should be of the form:

$$
x = 2^a \times 3^b
$$

with:

- $a \geq 3$ (to raise the maximum power of 2 to 3)
- $b \geq 3$ (to match the maximum power of 3, which is already 3 in 108)

But since 108 has $3^3$, and the overall L.C.M. is 216 = $2^3 \times 3^3$, the maximum power of 3 in x can be 3 (or higher but not necessary), and maximum power of 2 in x must be 3.

Step 7: Possible Values for x

To achieve L.C.M. of 216:

- $a = 3$
- $b = 3$

Thus,

$$
x = 2^3 \times 3^3 = 8 \times 27 = 216
$$

Recall that:

$$
x = 6 \times k
$$

and

$$
x = 216
$$

So,

$$
6 \times k = 216 \Rightarrow k = \frac{216}{6} = 36
$$

Now, check whether G.C.D. of 6, 9, and k is 1.

- 6 = $2 \times 3$
- 9 = $3^2$
- 36 = $2^2 \times 3^2$

Compute:

\[
\text{G.C.D.}(6, 9, 36)
\]

Prime factors:

- 6: 2^1 × 3

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