In A Sequence The First Term Is 8 And The Common Difference Is -5 The Fifth Term In The Sequence Is

Introduction

In A Sequence The First Term Is 8 And The Common Difference Is -5 The Fifth Term In The Sequence Is a question that invites us to explore the properties of an arithmetic sequence. Arithmetic sequences are fundamental concepts in mathematics, especially within the study of sequences and series. These sequences are characterized by a constant difference between consecutive terms, known as the common difference. Understanding how to determine specific terms within such sequences is vital for solving various mathematical problems, including those involving pattern recognition, algebra, and real-world applications. In this article, we will delve into the details of this particular sequence, analyze its structure, derive the formula for its terms, and specifically find the fifth term as requested.

Understanding Arithmetic Sequences

What Is an Arithmetic Sequence?

An arithmetic sequence is a list of numbers where each term after the first is obtained by adding a fixed number, called the common difference, to the previous term. The general form of an arithmetic sequence can be written as:

- Term 1: a_1
- Term 2: $a_2 = a_1 + d$
- Term 3: $a_3 = a_2 + d = a_1 + 2d$
- ...
- Term n: $a_n = a_1 + (n-1)d$

where:

- a_1 is the first term
- d is the common difference

- n is the term number

Key Properties of Arithmetic Sequences

1. Linear pattern: The sequence progresses in a straight line when plotted.
2. Constant difference: The difference between any two consecutive terms is always d.
3. Explicit formula: Allows direct calculation of any term without listing all previous terms.
4. Recursive formula: Defines each term based on the previous term, $a_n = a_{n-1} + d$.

Analyzing the Given Sequence

Identifying the Sequence Parameters

In our case, the sequence starts with a first term of 8, and the common difference is -5. This means the sequence decreases by 5 each time, starting from 8.

So:

- $a_1 = 8$
- $d = -5$

Writing the Explicit Formula

Using the general formula for the n-th term of an arithmetic sequence:

$a_n = a_1 + (n - 1)d$

Substituting the known values:

$a_n = 8 + (n - 1)(-5)$

This simplifies to:

- $a_n = 8 - 5(n - 1)$
- $a_n = 8 - 5n + 5$

- $a_n = (8 + 5) - 5n$
- $a_n = 13 - 5n$

Calculating the Fifth Term

Applying the Formula

To find the fifth term, substitute $n=5$ into the explicit formula:
$a_5 = 13 - 5 \times 5$
Calculating:

- $a_5 = 13 - 25$
- $a_5 = -12$

Thus, the fifth term in the sequence is -12.

Verification by Listing the Terms

To verify, let's list the first five terms based on the initial parameters:

1. $a_1 = 8$
2. $a_2 = 8 + (-5) = 3$
3. $a_3 = 3 + (-5) = -2$
4. $a_4 = -2 + (-5) = -7$
5. $a_5 = -7 + (-5) = -12$

As expected, the fifth term is -12, confirming our calculation.

Further Analysis of the Sequence

Pattern Recognition

The sequence starts at 8 and decreases by 5 each time, producing a pattern:

- 8, 3, -2, -7, -12, ...

This pattern continues indefinitely, with each subsequent term decreasing by 5. This kind of sequence is useful in modeling scenarios where quantities decrease at a constant rate over time.

Graphical Representation

If you plot the sequence's terms against their position (term number), the graph will be a straight line with a negative slope, specifically -5. The line passes through the point (1,8) and decreases linearly as the term number increases.

Graphing helps visualize the linear relationship and can be a useful tool for understanding the sequence's behavior over more extended ranges.

Applications of Arithmetic Sequences

Real-World Examples

- Decreasing stock prices over time with a consistent decline.
- Salary reductions that occur regularly, such as in certain contractual agreements.
- Modeling the depreciation of assets over time.
- Planning repayment schedules with fixed installment reductions.

Mathematical and Educational Significance

Understanding arithmetic sequences enhances problem-solving skills and provides a foundation for more complex topics like geometric sequences, series summation, and algebraic reasoning.

Calculating specific terms, as in this case, develops algebraic manipulation skills and introduces the concept of explicit versus recursive formulas.

Summary and Conclusion

In conclusion, given the first term of the sequence as 8 and the common difference as -5, we derived the explicit formula for the sequence as $a_n = 13 - 5n$. Applying this formula for $n=5$, we found that the fifth

term is -12. This example illustrates the straightforward process of analyzing arithmetic sequences, deriving general formulas, and applying them to find specific terms. Such sequences are fundamental in mathematics and have numerous practical applications across different fields.

Understanding the properties and calculations related to arithmetic sequences equips students and professionals with essential tools for problem-solving and analytical thinking, reinforcing the importance of foundational mathematical concepts.

Frequently Asked Questions

What is the first term of the sequence if it is 8 and the common difference is -5?

The first term is 8.

How do you find the fifth term in the sequence with first term 8 and common difference -5?

Use the formula for the nth term: $a_n = a_1 + (n - 1)d$. For the fifth term, $a_5 = 8 + (5 - 1)(-5) = 8 + 4(-5) = 8 - 20 = -12$.

What is the value of the fifth term in the sequence?

The fifth term is -12.

Is the sequence increasing or decreasing?

Since the common difference is -5, the sequence is decreasing.

What is the general formula for the nth term of this sequence?

The nth term is $a_n = 8 + (n - 1)(-5)$.

Can you find the sixth term in the sequence?

Yes. $a_6 = 8 + (6 - 1)(-5) = 8 + 5(-5) = 8 - 25 = -17$.

What is the pattern of the sequence starting with 8 and decreasing by 5?

The sequence decreases by 5 each time: 8, 3, -2, -7, -12, -17, and so on.

How can you verify the fifth term is -12?

Calculate using the formula: $a_5 = 8 + (5 - 1)(-5) = 8 - 20 = -12$, confirming the fifth term is -12.

Additional Resources

In A Sequence The First Term Is 8 And The Common Difference Is -5 The Fifth Term In The Sequence Is a classic example of an arithmetic progression (AP). Understanding how to find specific terms within an AP is fundamental in algebra and helps build a solid foundation for more advanced mathematical concepts. Whether you're a student tackling a homework problem or a mathematics enthusiast exploring sequences, this guide will walk you through the process step-by-step, providing clarity and insight into the mechanics of arithmetic sequences.

Understanding Arithmetic Sequences

An arithmetic sequence is a sequence of numbers in which the difference between any two successive terms is constant. This constant difference is called the common difference (denoted as d).

Key Components of an Arithmetic Sequence:
- First Term (a_1): The initial number in the sequence.
- Common Difference (d): The fixed amount added (or subtracted) to get from one term to the next.
- n-th Term (a_n): The term at position n in the sequence.

General Formula for the n-th Term:
The n-th term of an arithmetic sequence can be calculated using the formula:

$$a_n = a_1 + (n - 1)\, d$$

where:
- a_n = the n-th term
- a_1 = the first term
- d = common difference
- n = position of the term in the sequence

Defining the Sequence: First Term and Common Difference

Given the problem:

> The first term is 8 and the common difference is -5.

This immediately sets the parameters:

- $a_1 = 8$
- $d = -5$

The negative common difference indicates that the sequence decreases by 5 at each step.

Calculating the Fifth Term

Step 1: Identify the formula

Using the general formula:

$a_n = a_1 + (n - 1) d$

Step 2: Plug in known values

For the fifth term (n=5):

$a_5 = 8 + (5 - 1) (-5)$

Step 3: Simplify the expression

$a_5 = 8 + 4 (-5)$

$a_5 = 8 - 20$

Step 4: Final calculation

$a_5 = -12$

Therefore, the fifth term in the sequence is -12.

Visualizing the Sequence

Let's write out the first few terms to better understand the sequence:

Term Number (n)	Term (a_n)	Calculation
1	8	given
2	8 + (-5)	$a_2 = 8 + (2-1)(-5) = 8 - 5$
3	8 + 2(-5)	$a_3 = 8 + (3-1)(-5) = 8 - 10$
4	8 + 3(-5)	$a_4 = 8 + (4-1)(-5) = 8 - 15$
5	-12	$a_5 = 8 + (5-1)(-5) = 8 - 20$

The sequence progresses as: 8, 3, -2, -7, -12, ...

Additional Examples and Practice

To deepen understanding, let’s explore how to find other terms or solve similar problems.

Example 1: Find the 10th term

Using the formula:

$a_1 = 8$, $d = -5$, $n = 10$

$a_{10} = 8 + (10 - 1)(-5) = 8 + 9(-5) = 8 - 45 = -37$

The 10th term is -37.

Example 2: Find the sum of the first 5 terms

The sum of the first n terms of an AP is given by:

$S_n = n/2 (a_1 + a_n)$

For the first 5 terms:

$S_5 = 5/2 (8 + (-12)) = 2.5 (-4) = -10$

The sum of the first 5 terms is -10.

Exploring the Sequence’s Behavior

Given the first term and the common difference, students can analyze the sequence’s behavior:

- Decreasing sequence: Since d is negative, the sequence decreases as n increases.
- Limit: As n approaches infinity, the terms tend toward negative infinity.
- Number of terms: The sequence can be extended infinitely, or until a specific term is reached.

Practical Applications of Arithmetic Sequences

Understanding and calculating terms in an arithmetic sequence has real-world applications, including:

- Finance: Calculating installments or depreciation over time.
- Physics: Movement with constant acceleration.
- Population studies: Modeling populations with consistent growth or decline.
- Scheduling: Planning events at regular intervals.

Summary: Key Takeaways

- The sequence starts at 8 with a common difference of -5.
- The nth term formula is: $a_n = 8 + (n - 1)(-5)$.
- The fifth term, calculated as a_5, is -12.
- Subsequent terms follow a pattern decreasing by 5 each time.

Final Thoughts

Mastering the calculation of specific terms within an arithmetic sequence, such as In A Sequence The First Term Is 8 And The Common Difference Is -5 The Fifth Term In The Sequence Is, empowers students and learners to approach a variety of real-world problems and mathematical challenges with confidence. By understanding the fundamental formula, practicing with different values, and visualizing the sequence's progression, one can develop a robust comprehension of arithmetic progressions that extends beyond basic exercises into practical applications.

Whether you're exploring academic coursework or simply honing your problem-solving skills, the concepts outlined here serve as a vital foundation in the fascinating world of sequences and series.

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