

In XYZ, Z Is A Right Angle And Tan X=8/15. What Is Sin Y ? (F) 8/17 (G) 15/17 (H) 17/15 (I) 15/8

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Understanding the relationships within triangles, especially right triangles, is fundamental in trigonometry. This problem involves a triangle XYZ where angle Z is a right angle, and the tangent of angle X is given. The goal is to find the sine of angle Y, given multiple-choice options. To solve this, we'll explore the properties of right triangles, trigonometric ratios, and how to connect the given information to the unknown angle Y.

Basics of Right Triangles and Trigonometry

Before diving into the specifics of the problem, let's review essential concepts:

Right Triangle Fundamentals

- In a right triangle, one angle is 90 degrees.
- The sides are typically labeled relative to a specific angle:
- The side opposite the angle is called the opposite side.
- The side adjacent to the angle is called the adjacent side.
- The side opposite the right angle is called the hypotenuse, which is the longest side.

Trigonometric Ratios

For a given angle in a right triangle:

- Sine (sin) = Opposite side / Hypotenuse
- Cosine (cos) = Adjacent side / Hypotenuse
- Tangent (tan) = Opposite side / Adjacent side

These ratios help relate angles to side lengths, and are fundamental in solving for unknown angles or side lengths.

Understanding the Problem Setup

Given:

- Triangle XYZ with angle Z being a right angle.
- $\tan X = \frac{8}{15}$.

Question:

- What is $\sin Y$?

Options:

- (F) $\frac{8}{17}$
- (G) $\frac{15}{17}$
- (H) $\frac{17}{15}$
- (I) $\frac{15}{8}$

Let's analyze this setup step-by-step.

Key Observations

- Since Z is a right angle, the triangle XYZ is a right triangle with angles X, Y, and Z.
- The sum of angles in a triangle is 180° , so:

$$X + Y + Z = 180^\circ$$

- Because Z is 90° , then:

$$X + Y = 90^\circ$$

implying that angles X and Y are complementary.

This is a critical insight because it allows us to relate the trigonometric functions of X and Y.

Determining Side Lengths Based on $\tan X = \frac{8}{15}$

Given $\tan X = \frac{8}{15}$, we can interpret this as a ratio of sides in a right triangle containing angle X:

- Opposite side to X: 8 units
- Adjacent side to X: 15 units

To find the hypotenuse of this triangle, we use the Pythagorean theorem:

$$\begin{aligned} \text{Hypotenuse} &= \sqrt{(8)^2 + (15)^2} = \sqrt{64 + 225} = \sqrt{289} = 17 \end{aligned}$$

Thus, the sides are:

- Opposite to X: 8
- Adjacent to X: 15
- Hypotenuse: 17

This forms a classic 8-15-17 right triangle.

Side Lengths in Triangle XYZ

- The side opposite angle X is 8.
- The side adjacent to angle X is 15.
- The hypotenuse is 17.

Since the angles X and Y are complementary:

$$Y = 90^\circ - X$$

And because trigonometric functions of complementary angles are related:

$$\begin{aligned} \sin Y &= \cos X \\ \cos Y &= \sin X \end{aligned}$$

So, by finding $(\cos X)$, we can determine $(\sin Y)$.

Calculating $(\cos X)$ and $(\sin X)$

From the sides identified:

$$\begin{aligned} \end{aligned}$$

$$\sin X = \frac{\text{Opposite to } X}{\text{Hypotenuse}} = \frac{8}{17}$$

$$\cos X = \frac{\text{Adjacent to } X}{\text{Hypotenuse}} = \frac{15}{17}$$

Now, since $\sin Y = \cos X$, it follows that:

$$\boxed{\sin Y = \frac{15}{17}}$$

Matching this to the options given:

- (F) 8/17
- (G) 15/17
- (H) 17/15
- (I) 15/8

The correct answer is (G) 15/17.

Additional Insights and Applications

Understanding the relationships between angles and side ratios is invaluable in various fields, including engineering, physics, architecture, and navigation. This problem showcases how to leverage the properties of right triangles and trigonometric functions to find unknown angles or side lengths.

Practical Applications of Trigonometry

- Navigation and Geolocation: Calculating distances and angles between landmarks.
- Architecture and Construction: Ensuring structures are built with precise angles.
- Physics: Analyzing forces and vectors involving angles.
- Astronomy: Determining the position of celestial bodies.

Summary of Key Steps to Solve Similar Problems

1. Identify the right triangle and its known angles.
2. Use given ratios (like $\tan X$) to determine side lengths via the Pythagorean theorem.

3. Express the desired trigonometric function (like $\sin Y$) in terms of known quantities.
4. Apply complementary angle relationships to simplify calculations.
5. Match the calculated ratio to the options provided.

Conclusion

By analyzing the given data and applying fundamental trigonometric principles, we determined that $\sin Y = \frac{15}{17}$. This aligns with option (G). The problem demonstrates the elegance of right triangle properties and the importance of understanding trigonometric ratios, which serve as essential tools for solving a wide range of mathematical and real-world problems.

Understanding these concepts not only helps in academic contexts but also enhances problem-solving skills applicable across numerous technical fields. Whether dealing with simple geometry or complex engineering designs, mastery of right triangle trigonometry is a vital skill.

References:

- Trigonometry Textbooks
- Khan Academy Trigonometry Tutorials
- Geometry Tools and Resources
- Standard Right Triangle Ratios and Pythagorean Theorem

Keywords: Right Triangle, Trigonometry, Sine, Cosine, Tangent, Pythagorean Theorem, Complementary Angles, Triangle XYZ, Angle Y, Triangle Side Ratios, Trigonometric Ratios in Geometry

Frequently Asked Questions

In triangle XYZ, Z is a right angle and $\tan X = 8/15$. What is $\sin Y$?

15/17

Given a right triangle with Z as the right angle and $\tan X = 8/15$, what is the value of $\sin Y$?

15/17

In triangle XYZ, Z is a right angle, $\tan X = 8/15$, and Y is the other non-right angle. What is $\sin Y$?

15/17

If in triangle XYZ, $Z = 90^\circ$, $\tan X = 8/15$, what is $\sin Y$?

15/17

In a triangle with a right angle at Z and $\tan X = 8/15$, find $\sin Y$.

15/17

Given triangle XYZ with a right angle at Z and $\tan X = 8/15$, what is $\sin Y$?

15/17

In triangle XYZ, Z is a right angle and $\tan X = 8/15$. Which of the following is $\sin Y$?

15/17

For triangle XYZ with a right angle at Z and $\tan X = 8/15$, what is the value of $\sin Y$?

15/17

In triangle XYZ where Z is a right angle and $\tan X = 8/15$, determine $\sin Y$.

15/17

In triangle XYZ, Z is a right angle and $\tan X = 8/15$. What is the value of $\sin Y$?

15/17

Additional Resources

In XYZ, Z is a right angle and $\tan X = 8/15$. What is $\sin Y$?

This classic trigonometric problem invites us into the intriguing world of triangles, angles, and ratios. It combines fundamental concepts of right-angled triangles, the tangent function, and the relationships between angles to ultimately determine the sine of angle Y. Whether you're a student brushing up for exams or a math enthusiast interested in problem-solving techniques, understanding how to approach such questions deepens your grasp of trigonometry's core principles.

Understanding the Problem

Let's first break down what the problem states:

- In triangle XYZ, the notation suggests that points X, Y, and Z are vertices of a triangle.
- Z is a right angle, meaning that triangle XYZ is a right triangle with $\angle Z = 90^\circ$.
- $\tan X = 8/15$, which gives us a ratio involving the sides opposite and adjacent to $\angle X$.
- The goal is to find $\sin Y$.

Setting Up the Triangle: Coordinates and Notation

To analyze this problem systematically, visualize or construct the triangle XYZ with the following considerations:

- Since Z is the right angle, the triangle XYZ is right-angled at Z.
- The angles are $\angle X$, $\angle Y$, and $\angle Z$, with $\angle Z = 90^\circ$.
- The sum of angles in any triangle is 180° , so:

$$\begin{aligned} & \angle X + \angle Y + 90^\circ = 180^\circ \\ & \Rightarrow \angle X + \angle Y = 90^\circ \end{aligned}$$

Therefore, $\angle Y = 90^\circ - \angle X$.

This relationship suggests that angles X and Y are complementary, which will be crucial when expressing their sine and cosine values.

Step 1: Analyzing the Given Tangent Ratio

Given $\tan X = 8/15$, we interpret this as:

$$\tan X = \frac{\text{Opposite side to } X}{\text{Adjacent side to } X} = \frac{8}{15}$$

In the context of a right triangle, this ratio involves the sides relative to angle X:

- Opposite side to X: Let's denote it as $\text{opposite}X$.
- Adjacent side to X: Let's denote it as $\text{adjacent}X$.

Construct a right triangle where:

- Opposite side to X: 8 units
- Adjacent side to X: 15 units

To find the hypotenuse, apply the Pythagorean theorem:

$$\text{hypotenuse} = \sqrt{(8)^2 + (15)^2} = \sqrt{64 + 225} = \sqrt{289} = 17$$

Step 2: Determine the Sides of Triangle XYZ

Now, considering the triangle XYZ with the right angle at Z, and the sides constructed as above:

- Side opposite X: 8
- Side opposite Y: Since $\angle X$ and $\angle Y$ are complementary, the side opposite Y in triangle XYZ will be related to the same triangle's sides.

In a right triangle, the sides are typically labeled as follows:

- Hypotenuse (XY): 17
- Side adjacent to X: 15
- Side opposite X: 8

Because the angles are within the same triangle, the sides correspond directly to the angles:

Angle	Opposite Side	Adjacent Side	Hypotenuse
X	8	15	17

| Y | ? | ? | ? |

Given that $\angle Y = 90^\circ - \angle X$, the triangle's sides for Y are related to the same right triangle:

- The side opposite Y is the side adjacent to X in the original setup, which is 15.
- The side adjacent to Y is the side opposite X, which is 8.

Alternatively, since the triangle is fixed, the sides are consistent.

Step 3: Computing $\sin Y$

Knowing that $\angle Y = 90^\circ - \angle X$, and that the sine of an angle is the ratio of its opposite side to hypotenuse, we can use the complementary angle relationship:

$$\begin{aligned} & \backslash[\\ & \sin Y = \cos X \\ & \backslash] \end{aligned}$$

because:

$$\begin{aligned} & \backslash[\\ & \sin (90^\circ - \theta) = \cos \theta \\ & \backslash] \end{aligned}$$

From earlier, we found:

$$\begin{aligned} & \backslash[\\ & \cos X = \frac{\text{Adjacent to } X}{\text{Hypotenuse}} = \frac{15}{17} \\ & \backslash] \end{aligned}$$

Therefore:

$$\begin{aligned} & \backslash[\\ & \boxed{\sin Y = \cos X = \frac{15}{17}} \\ & \backslash] \end{aligned}$$

which matches one of the given options.

Final Answer and Explanation

The value of $\sin Y$ is $\frac{15}{17}$.

This matches option (G) 15/17.

Additional Insights: Why Does This Work?

The key to solving this problem lies in understanding the relationships between angles and their trigonometric functions in right triangles, especially the complementary nature of angles:

- Complementary angles: In a right triangle, two non-right angles sum to 90° , and their sine and cosine are interrelated:

$$\sin(90^\circ - \theta) = \cos \theta$$

- Using the given tangent: The tangent ratio provided allows us to determine the sides relative to angle X , which then helps find the cosine of X .

- Connecting to $\sin Y$: Since $\angle Y = 90^\circ - \angle X$, the sine of Y is simply the cosine of X , enabling a straightforward calculation.

Summary and Key Takeaways

- Identify the right angle: Recognize Z as the right angle, which constrains the triangle's shape.
- Use the tangent ratio: Convert $\tan X = 8/15$ into side lengths using Pythagoras.
- Relate angles: Recall that $\angle Y = 90^\circ - \angle X$, which leads to $\sin Y = \cos X$.
- Compute the cosine of X : Using the sides, find $\cos X = 15/17$.
- Conclude: $\boxed{\sin Y = 15/17}$.

This problem is a perfect illustration of how fundamental trigonometric identities and relationships between angles can simplify seemingly complex problems. Mastery of these concepts enables quick and accurate solutions to a variety of geometry and trigonometry challenges.

Final Note

Understanding the relationships between angles and sides in right triangles, along with trigonometric identities, is essential for solving advanced geometry problems. With practice, recognizing these patterns becomes intuitive, making problem-solving more efficient and enjoyable.

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