L Is The Point (-2,4) And M Is The Point(3,-1).A Variable PointQ(x,y) In Such That /QL/-/QM/=10

L Is The Point (-2,4) And M Is The Point(3,-1). A Variable Point Q(x,y) In Such That |QL – QM| = 10

Introduction

Understanding the geometric relationships between points in a coordinate plane is fundamental in coordinate geometry. The problem at hand involves two fixed points, L and M, with coordinates L(-2, 4) and M(3, -1), respectively. We are asked to analyze the locus of a variable point Q(x, y) such that the absolute difference of the distances from Q to these fixed points is equal to 10, i.e.,

|QL – QM| = 10.

This problem is a classic example of exploring loci defined by difference of distances, which leads us into the realm of hyperbolas. In this article, we will delve deep into the geometric and algebraic aspects of this problem, providing a comprehensive understanding suitable for students and enthusiasts aiming to master the concepts of conic sections and coordinate geometry.

Understanding the Components of the Problem

Coordinates of Fixed Points L and M

- Point L: (-2, 4)
- Point M: (3, -1)

These serve as fixed points, also known as foci in the context of hyperbolas.

Variable Point Q(x, y)

- Point Q: (x, y)

Q is any point in the plane, and the locus of Q is determined by the condition on the distances from Q to L and M.

The Condition: |QL – QM| = 10

- Distance from Q to L: \(QL = \sqrt{(x + 2)^2 + (y - 4)^2} \)
- Distance from Q to M: \(QM = \sqrt{(x - 3)^2 + (y + 1)^2} \)

The absolute difference of these distances is a constant, 10.

Geometric Significance: The Hyperbola

What is a Hyperbola?

A hyperbola is a conic section characterized by the difference of distances from any point on the curve to two fixed points (foci) being constant. Specifically:

- For a hyperbola with foci F_1 and F_2, any point Q on the hyperbola satisfies:

$$
|QF_1 - QF_2| = 2a
$$

where $2a$ is the constant difference (or sum, depending on the hyperbola type).

Relation to Our Problem

In our case:

- The fixed points L and M can be considered as the foci of the hyperbola.
- The constant difference is 10.

Therefore, the locus of all points $Q(x, y)$ such that $|QL - QM| = 10$ is a hyperbola with foci at L(-2, 4) and M(3, -1).

Algebraic Derivation of the Hyperbola Equation

Step 1: Write the Distance Expressions

$$
QL = \sqrt{(x + 2)^2 + (y - 4)^2}
$$
$$
QM = \sqrt{(x - 3)^2 + (y + 1)^2}
$$

Step 2: Set Up the Hyperbola Condition

$$
|QL - QM| = 10
$$

which leads to two cases:

1. $QL - QM = 10$
2. $QM - QL = 10$

Because the absolute value is involved, the equation splits into two branches of the hyperbola.

Step 3: Square Both Sides to Eliminate Square Roots

Let's analyze the case $QL - QM = 10$:

$$
QL - QM = 10 \Rightarrow QL = QM + 10
$$

Squaring both sides:

$$
(QL)^2 = (QM + 10)^2
$$
$$
(QL)^2 = (QM)^2 + 20 \cdot QM + 100
$$

Expressed explicitly:

$$
(x + 2)^2 + (y - 4)^2 = (x - 3)^2 + (y + 1)^2 + 20 \cdot \sqrt{(x - 3)^2 + (y + 1)^2} + 100
$$

This equation involves the square root term, complicating algebraic manipulation.

Step 4: Simplify the Equation

To handle the radical, isolate it:

$$
(QL)^2 - (QM)^2 = 20 \cdot QM + 100
$$

Calculate the difference of squares:

$$
[(x + 2)^2 + (y - 4)^2] - [(x - 3)^2 + (y + 1)^2] = 20 \cdot QM + 100
$$

Expanding:

$$
(x^2 + 4x + 4) + (y^2 - 8y + 16) - (x^2 - 6x + 9) - (y^2 + 2y + 1) = 20 \cdot QM + 100
$$

Simplify:

$$
x^2 + 4x + 4 + y^2 - 8y + 16 - x^2 + 6x - 9 - y^2 - 2y - 1 = 20 \cdot QM + 100
$$

\]

Combine like terms:

\[
(4x + 6x) + (4 + 16 - 9 - 1) + (-8y - 2y) = 20 \cdot QM + 100
\]
\[
10x + 10 + (-10y) = 20 \cdot QM + 100
\]

Expressed simply:

\[
10x - 10y + 10 = 20 \cdot QM + 100
\]

Isolate \(QM\):

\[
20 \cdot QM = 10x - 10y + 10 - 100
\]
\[
20 \cdot QM = 10x - 10y - 90
\]
\[
QM = \frac{10x - 10y - 90}{20} = \frac{x - y - 9}{2}
\]

Recall that:

\[
QM = \sqrt{(x - 3)^2 + (y + 1)^2}
\]

Set the two equal:

\[
\sqrt{(x - 3)^2 + (y + 1)^2} = \frac{x - y - 9}{2}
\]

Step 5: Remove the Square Root

Square both sides:

\[
(x - 3)^2 + (y + 1)^2 = \frac{(x - y - 9)^2}{4}
\]

Multiply both sides by 4:

$$
4[(x - 3)^2 + (y + 1)^2] = (x - y - 9)^2
$$

Expand:

$$
4(x^2 - 6x + 9 + y^2 + 2y + 1) = (x - y - 9)^2
$$

Simplify the left side:

$$
4x^2 - 24x + 36 + 4y^2 + 8y + 4 = (x - y - 9)^2
$$

The right side expands as:

$$
(x - y - 9)^2 = (x - y)^2 - 18(x - y) + 81
$$

which simplifies to:

$$
x^2 - 2xy + y^2 - 18x + 18y + 81
$$

Now, equate both sides:

$$
4x^2 - 24x + 36 + 4y^2 + 8y + 4 = x^2 - 2xy + y^2 - 18x + 18y + 81
$$

Bring all to one side:

$$
4x^2 - 24x + 36 + 4y^2 + 8y + 4 - x^2 + 2xy - y^2 + 18x - 18y - 81 = 0
$$

Combine like terms:

$$
(4x^2 - x^2) + (-24x + 18x) + (36 + 4) + (4y^2 - y^2) + (8y - 18y) + 2xy - 81 = 0
$$

Simplify:

$$
3x^2 - 6x + 40 + 3y^2 - 10y + 2xy - 81 = 0
$$

Further simplification:

$$
3x^2 + 3y^2 + 2xy - 6x - 10y - 41 = 0
$$

This is the explicit algebraic equation of the hyperbola corresponding to the case $QL - QM = 10$.

Complete Equation of the Hyperbola

Similarly, for the case $QM - QL = 10$, the derivation proceeds analogously, leading to a similar equation with a sign change in the relevant terms. The overall locus is the union of these two branches.

Final Equation

Frequently Asked Questions

What are the coordinates of points L and M in the problem?

Point L is at (-2, 4) and point M is at (3, -1).

What is the condition given for the variable point Q(x, y)?

The condition is that the absolute difference of the distances from Q to L and Q to M equals 10, i.e., $|QL - QM| = 10$.

How do you express the distances QL and QM mathematically?

Distance $QL = \sqrt{(x + 2)^2 + (y - 4)^2}$, and $QM = \sqrt{(x - 3)^2 + (y + 1)^2}$.

What equation must Q(x, y) satisfy based on the given condition?

It must satisfy $|\sqrt{(x + 2)^2 + (y - 4)^2} - \sqrt{(x - 3)^2 + (y + 1)^2}| = 10$.

How can the given condition be simplified or approached algebraically?

Square both sides of the equation to eliminate the absolute value and then expand and simplify to find possible equations for Q.

Are there specific geometric figures associated with this problem?

Yes, this problem involves hyperbolas, as the set of points where the difference of distances to two fixed points is constant forms a hyperbola.

What is the significance of the constant 10 in the problem?

The constant 10 represents the fixed difference in distances from Q to L and M that defines the hyperbola's constant difference parameter.

How would you find the locus of all points Q satisfying the condition?

By deriving and simplifying the equation |QL - QM| = 10, you obtain the equation of a hyperbola with foci at L and M, describing the locus of Q.

Additional Resources

Geometric Analysis of Point Q Relative to Fixed Points L and M: An In-Depth Exploration

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Introduction
Geometry, at its core, is a fascinating exploration of the relationships between points, lines, and shapes within a coordinate plane. One particularly intriguing problem involves understanding the position of a variable point $Q (x, y)$ in relation to two fixed points L and M, and how the distances from Q to these points influence its location.

In our case, given fixed points:
- $L(-2, 4)$
- $M(3, -1)$

and a condition involving the distances:
$$ |QL| - |QM| = 10 $$

our goal is to analyze the locus of the point Q satisfying this condition. This problem combines elements of distance geometry, algebraic manipulation, and the understanding of conic sections.

This article provides an in-depth, step-by-step exploration of the problem, examining the geometric significance, algebraic formulation, and the nature of the locus, culminating in a comprehensive understanding of the geometric configuration.

Setting the Stage: Coordinates and Basic Concepts

Fixed Points L and M

Let's first establish the fixed points:
- $L(-2, 4)$
- $M(3, -1)$

These points serve as references within the coordinate plane. The distances from a variable point $Q(x, y)$ to these fixed points are:

$$
|QL| = \sqrt{(x + 2)^2 + (y - 4)^2}
$$
$$
|QM| = \sqrt{(x - 3)^2 + (y + 1)^2}
$$

The problem involves the difference of these distances:

$$
|QL| - |QM| = 10
$$

This absolute difference signifies the set of all points Q for which the difference of distances to L and M is exactly 10.

Geometric Interpretation of the Condition $|QL| - |QM| = 10$

Recognizing the Locus: A Hyperbola

The condition:

$$
|QL| - |QM| = \pm c
$$

for a constant c, is a classic definition of a hyperbola in the Euclidean plane.

- When the difference of distances to two fixed points (foci) is constant, the locus of the point is a hyperbola.
- The sign of the difference determines the branch of the hyperbola (positive or negative).

In our case, with $|QL| - |QM| = 10$, the locus of Q is a branch of a hyperbola with foci at L and M.

Key points:
- The foci are at $L(-2, 4)$ and $M(3, -1)$.
- The constant difference $2a$ (where $a > 0$) is 10, so:

$$
2a = 10 \quad \Rightarrow \quad a = 5
$$

The hyperbola's definition is the set of points Q such that the difference of distances to the foci is $2a$.

Detailed Derivation of the Hyperbola Equation

To analyze this hyperbola explicitly, we'll derive its equation by substituting the distance formulas.

Step 1: Write the distance expressions

$$
|QL| = \sqrt{(x + 2)^2 + (y - 4)^2}
$$
$$
|QM| = \sqrt{(x - 3)^2 + (y + 1)^2}
$$

The defining equation:

$$
|QL| - |QM| = 10
$$

which can be rewritten as:

$$
|QL| = |QM| + 10
$$

or equivalently:

$$
\sqrt{(x + 2)^2 + (y - 4)^2} = \sqrt{(x - 3)^2 + (y + 1)^2} + 10
$$

Step 2: Isolate and square to eliminate the radicals

Let’s set:

$$
R_1 = \sqrt{(x + 2)^2 + (y - 4)^2}
$$
$$
R_2 = \sqrt{(x - 3)^2 + (y + 1)^2}
$$

Our equation:

$$
R_1 = R_2 + 10
$$

Squaring both sides gives:

$$
R_1^2 = (R_2 + 10)^2
$$

$$
(x + 2)^2 + (y - 4)^2 = R_2^2 + 20 R_2 + 100
$$

But $R_2^2 = (x - 3)^2 + (y + 1)^2$, so:

$$
(x + 2)^2 + (y - 4)^2 = (x - 3)^2 + (y + 1)^2 + 20 R_2 + 100
$$

Rearranging:

$$
[(x + 2)^2 - (x - 3)^2] + [(y - 4)^2 - (y + 1)^2] = 20 R_2 + 100
$$

Let’s compute the differences:

$$
(x + 2)^2 - (x - 3)^2 = [(x)^2 + 4x + 4] - [x^2 - 6x + 9] = (x^2 + 4x + 4) - (x^2 -6x + 9) = 10x - 5
$$

Similarly:

$$
(y - 4)^2 - (y + 1)^2 = (y^2 - 8y + 16) - (y^2 + 2y + 1) = -10 y + 15
$$

Adding these:

$$
(10x - 5) + (-10 y + 15) = 20 R_2 + 100
$$

$$
10x - 5 - 10 y + 15 = 20 R_2 + 100
$$

$$
10x - 10 y + 10 = 20 R_2 + 100
$$

Divide through by 10:

$$
x - y + 1 = 2 R_2 + 10
$$

Recall that:

$$
R_2 = \sqrt{(x - 3)^2 + (y + 1)^2}
$$

Thus:

$$
x - y + 1 = 2 \sqrt{(x - 3)^2 + (y + 1)^2} + 10
$$

Rearranged:

$$
x - y + 1 - 10 = 2 \sqrt{(x - 3)^2 + (y + 1)^2}
$$

$$
x - y - 9 = 2 \sqrt{(x - 3)^2 + (y + 1)^2}
$$

Squaring both sides again:

$$
(x - y - 9)^2 = 4 [(x - 3)^2 + (y + 1)^2]
$$

Final Equation of the Hyperbola

Expanding both sides:

Left:

$$
(x - y - 9)^2 = (x - y)^2 - 18 (x - y) + 81
$$

Right:

$$
4[(x - 3)^2 + (y + 1)^2] = 4(x^2 - 6x + 9 + y^2 + 2y + 1) = 4x^2 - 24x + 36 + 4 y^2 + 8 y + 4
$$

Now, expand the left:

$$
(x - y)^2 - 18 (x - y) + 81 = (x^2 - 2 x y + y^2) - 18x + 18 y + 81
$$

Putting all together:

$$
x^2 - 2 x y + y^2 - 18 x + 18 y + 81 = 4 x^2 - 24 x + 36 + 4 y^2 + 8 y + 4
$$

Bring all to one side:

$$
x^2 - 2 x y + y^2 - 18 x + 18 y + 81 - 4 x^2 + 24 x - 36 - 4 y^2 - 8 y - 4 = 0
$$

Combine like terms:

- $x^2 - 4 x^2 = -3 x^2$
- $-2 x y$ remains
- $y^2 - 4 y^2 = -3 y^2$
- $-18 x + 24 x = 6 x$
- $18 y - 8 y = 10 y$
- $81 - 36 - 4 = 41$

Thus, the equation simplifies to:

$$
-3 x^2 - 2 x y - 3 y^2 + 6 x + 10 y +
$$

L Is The Point 2 4 And M Is The Point 3 1 A Variable Pointq X Y In Such That Ql Qm 10

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