Let Be A Random Variable With Cdf Fx 0, X<0 1-e^-4x Find (round Off To Second Decimal Place).

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Understanding the behavior of random variables and their cumulative distribution functions (CDFs) is fundamental in probability theory and statistical analysis. In this article, we will explore a specific problem involving a random variable X with a given CDF $F_X(x)$, and we will systematically determine the probability that X falls within a particular range, rounding our final answer to two decimal places. This process involves understanding the properties of the CDF, calculating probabilities, and applying appropriate mathematical techniques.

Introduction to Cumulative Distribution Functions (CDFs)

Before delving into the specific problem, it’s essential to understand what a CDF is and how it relates to a random variable.

What is a CDF?

A cumulative distribution function (CDF) of a random variable X, denoted by $F_X(x)$, is a function that maps each real number x to the probability that X takes a value less than or equal to x. Mathematically, it is defined as:

$$
F_X(x) = P(X \leq x)
$$

The properties of a CDF include:

- Non-decreasing: $F_X(x)$ never decreases as x increases.
- Right-continuous: The limit from the right at any point x equals $F_X(x)$.
- Limits: $\lim_{x \to -\infty} F_X(x) = 0$ and $\lim_{x \to \infty} F_X(x) = 1$.

Given CDF and Its Domain

In our problem, the CDF $F_X(x)$ is given as:

$$
F_X(x) =
\begin{cases}
0, & x < 0 \\
1 - e^{-4x}, & x \geq 0
\end{cases}
$$

This indicates that the random variable X is supported on $[0, \infty)$, with zero probability for negative values.

Understanding the Problem Statement

The task involves:

- Recognizing the form of the CDF.
- Calculating the probability that X lies between specific bounds.
- Rounding the answer to two decimal places.

Let's formalize the problem:

Find $P(a \leq X \leq b)$, where a and b are specific bounds, using the provided CDF $F_X(x)$.

While the original statement mentions "X<0" and $F_X(x) = 0$ for $x<0$, and $F_X(x) = 1 - e^{-4x}$ for $x \geq 0$, it does not specify the particular interval for which the probability is sought. For illustrative purposes, assume the problem asks for:

Find $P(0.5 \leq X \leq 1.2)$, rounded to two decimal places.

(If the actual bounds are different, the same process applies.)

Note: The calculation method remains the same regardless of the bounds.

Calculating Probabilities Using the CDF

The probability that a random variable X lies between two values a and b (with $a < b$) can be calculated as:

$$
P(a \leq X \leq b) = F_X(b) - F_X(a)
$$

This follows directly from the properties of the CDF.

Step-by-Step Calculation

Given the specific bounds $a = 0.5$ and $b = 1.2$:

1. Calculate $F_X(1.2)$:

Since $1.2 \geq 0$,

$$
F_X(1.2) = 1 - e^{-4 \times 1.2} = 1 - e^{-4.8}
$$

2. Calculate $F_X(0.5)$:

Since $0.5 \geq 0$,

$$
F_X(0.5) = 1 - e^{-4 \times 0.5} = 1 - e^{-2}
$$

3. Compute the numerical values:

- $e^{-4.8} \approx e^{-4.8} \approx 0.0082$
- $e^{-2} \approx 0.1353$

4. Calculate the probability:

$$
P(0.5 \leq X \leq 1.2) = (1 - 0.0082) - (1 - 0.1353) = (0.9918) - (0.8647) = 0.1271
$$

5. Round to two decimal places:

$$
\boxed{0.13}
$$

Thus, the probability that X falls between 0.5 and 1.2 is approximately 0.13.

Generalization for Different Intervals

The same approach can be applied for any interval $[a, b]$:

- If both a and b are ≥ 0:

$$
P(a \leq X \leq b) = F_X(b) - F_X(a) = (1 - e^{-4b}) - (1 - e^{-4a}) = e^{-4a} - e^{-4b}
$$

- If $a < 0$ and $b \geq 0$:

$$
P(a \leq X \leq b) = F_X(b) - F_X(a) = (1 - e^{-4b}) - 0 = 1 - e^{-4b}
$$

- If both a and b are < 0:

$$
P(a \leq X \leq b) = 0
$$

Understanding the Distribution: Properties and Implications

The given CDF $F_X(x) = 1 - e^{-4x}$ for $x \geq 0$ resembles the CDF of an exponential distribution with rate parameter $\lambda = 4$.

Key Properties

- Type of distribution: Exponential.
- Mean (Expected value): $\frac{1}{\lambda} = \frac{1}{4} = 0.25$.
- Variance: $\frac{1}{\lambda^2} = \frac{1}{16} = 0.0625$.

Implications for Probability Calculations

Knowing the distribution type allows for alternative calculation methods, such as using the probability density function (PDF), which for this distribution is:

$$
f_X(x) = \frac{d}{dx} F_X(x) = 4 e^{-4x}
$$

for $x \geq 0$.

Using the PDF, the probability between a and b can also be calculated as:

$$
P(a \leq X \leq b) = \int_a^b 4 e^{-4x} dx
$$

which should yield the same result as using the CDF difference.

Conclusion and Practical Applications

Understanding how to work with the CDF of a random variable is crucial for probability calculations. In this article, we demonstrated the process of calculating the probability that a random variable with an exponential distribution—characterized by the CDF $F_X(x) = 1 - e^{-4x}$—falls within a specific interval. We showcased the step-by-step method, including evaluating the CDF at bounds, performing subtraction, and rounding the final probability to two decimal places.

Practical applications of such calculations include:

- Reliability analysis (e.g., time until failure).
- Queuing systems (e.g., service times).
- Survival analysis in medical research.
- Any field where modeling of waiting times or lifespans using exponential distributions is relevant.

By mastering these techniques, statisticians and data analysts can accurately determine probabilities associated with various ranges of random variables, facilitating better decision-making and insights.

Summary of Key Steps for Probability Calculation

1. Identify the distribution type from the CDF.
2. Determine the bounds for the probability.

3. Calculate the CDF at the bounds.
4. Subtract the lower bound CDF from the upper bound CDF.
5. Round the result to the required decimal places.

Applying these steps ensures accurate and efficient probability calculations for random variables with known CDFs.

Remember: Always verify the support of the distribution when calculating probabilities to avoid errors, especially with distributions that are zero outside certain intervals.

Final note: If the original problem specifies different bounds or conditions, adapt the above methodology accordingly to obtain the precise probability value.

Frequently Asked Questions

What is the cumulative distribution function (CDF) of the random variable X?

The CDF of X is $F_X(x) = 0$ for $x < 0$, and $F_X(x) = 1 - e^{-4x}$ for $x \geq 0$.

Is the given function a valid CDF for a random variable?

Yes, because it is non-decreasing, right-continuous, with limits 0 as $x \to -\infty$ and 1 as $x \to \infty$, satisfying the properties of a CDF.

What is the probability density function (PDF) corresponding to the given CDF?

The PDF is $f_X(x) = d/dx$ of $F_X(x)$. For $x \geq 0$, $f_X(x) = 4e^{-4x}$; for $x < 0$, $f_X(x) = 0$.

What is the expected value E[X] of the random variable?

$E[X] = \int_0^\infty x \, 4e^{-4x} \, dx$. Computing this integral yields $E[X] = 1/4 = 0.25$.

What is the variance Var(X) of the random variable?

$Var(X) = \int_0^\infty (x - E[X])^2 \, 4e^{-4x} \, dx$. Calculating gives $Var(X) = 1/16 =$

0.0625.

How do you find P(a < X < b) for given a and b?

Calculate P(a < X < b) = F_x(b) - F_x(a). For example, for a=0.5 and b=1, substitute into the CDF to find the probability.

What is the median of the distribution?

The median m is found by solving F_x(m) = 0.5. So, 1 - e^(-4m) = 0.5 ⇒ e^(-4m) = 0.5 ⇒ m = - (1/4) ln(0.5) ≈ 0.1733.

Calculate P(X > 1) for the given distribution, rounded to two decimal places.

P(X > 1) = 1 - F_x(1) = e^(-41) = e^(-4) ≈ 0.0183, rounded to 0.02.

How would you simulate random samples from this distribution?

Use inverse transform sampling: generate a uniform random variable U in [0,1], then find X = - (1/4) ln(1 - U).

Additional Resources

Understanding the Random Variable and its CDF: A Comprehensive Review

In the realm of probability theory and statistics, understanding the behavior of random variables is fundamental. The cumulative distribution function (CDF) offers a complete description of the probability distribution of a random variable, enabling us to derive probabilities, expectations, variances, and other statistical measures. The problem at hand involves a particular random variable X, characterized by its CDF $F_X(x)$, with a specific form given as:

$$
F_X(x) =
\begin{cases}
0, & x < 0 \\
1 - e^{-4x}, & x \geq 0
\end{cases}
$$

Our goal is to analyze this random variable thoroughly, compute specific probabilities or expectations as needed, and provide a rounded numerical answer to two decimal places. This detailed review will explore the properties of X, interpret its distribution, and perform relevant calculations step by step.

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1. Introduction to the CDF $F_X(x)$

The cumulative distribution function (CDF) of a random variable X, denoted as $F_X(x)$, is defined as:

$$
F_X(x) = P(X \leq x)
$$

for all real numbers x. It is a non-decreasing, right-continuous function that satisfies:

$$
\lim_{x \to -\infty} F_X(x) = 0, \quad \text{and} \quad \lim_{x \to \infty} F_X(x) = 1
$$

In our case, the CDF is given explicitly as:

$$
F_X(x) =
\begin{cases}
0, & x < 0 \\
1 - e^{-4x}, & x \geq 0
\end{cases}
$$

This form indicates that X is supported on the non-negative real line ($x \geq 0$), with a specific exponential-like behavior.

Key observations:
- The CDF is zero for negative x, confirming the support starts at zero.
- For $x \geq 0$, the CDF resembles an exponential distribution's CDF with parameter $\lambda = 4$.

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2. Recognizing the Distribution Type

Given the form $F_X(x) = 1 - e^{-4x}$ for $x \geq 0$, this aligns with the cumulative distribution function of an exponential distribution with rate parameter $\lambda = 4$.

2.1. The Exponential Distribution

The exponential distribution is a continuous probability distribution often used to model waiting times between independent events that occur at a constant average rate.

Probability Density Function (PDF):

$$
f_X(x) = \lambda e^{-\lambda x}, \quad x \geq 0
$$

Cumulative Distribution Function (CDF):

$$
F_X(x) = 1 - e^{-\lambda x}
$$

In our scenario:

$$
f_X(x) = 4 e^{-4x}, \quad x \geq 0
$$

Support: $x \in [0, \infty)$

2.2. Confirming the Distribution

The given CDF matches the standard form of an exponential distribution with $\lambda=4$, confirming:

- $X \sim \text{Exponential}(4)$

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3. Fundamental Properties of the Random Variable X

Understanding the properties of X is crucial for any further analysis.

3.1. Probability Density Function (PDF)

From the CDF, the PDF is the derivative:

$$
f_X(x) = \frac{d}{dx} F_X(x) = \frac{d}{dx}(1 - e^{-4x}) = 4 e^{-4x}
$$

for $x \geq 0$, and zero elsewhere.

3.2. Expected Value $E[X]$

The mean of an exponential distribution:

$$
E[X] = \frac{1}{\lambda} = \frac{1}{4} = 0.25
$$

3.3. Variance $\text{Var}(X)$

Variance of the exponential distribution:

$$
\text{Var}(X) = \frac{1}{\lambda^2} = \frac{1}{16} = 0.0625
$$

3.4. Median

The median m satisfies:

$$
F_X(m) = 0.5 \Rightarrow 1 - e^{-4m} = 0.5
$$
$$
e^{-4m} = 0.5
$$
$$
-4m = \ln(0.5) \Rightarrow m = -\frac{1}{4} \ln(0.5)
$$

Since $\ln(0.5) = -\ln 2$:

$$
m = -\frac{1}{4} \times (-\ln 2) = \frac{\ln 2}{4} \approx \frac{0.6931}{4} \approx 0.1733
$$

---

4. Calculations Based on the Distribution

The problem specifies to "Find (round off to second decimal place)." While the exact query isn't explicitly provided, typical questions involving such a distribution include:

- Computing $P(a < X < b)$
- Finding $E[X]$ or $E[X^n]$
- Calculating probabilities at specific points
- Determining quantiles (e.g., median)

4.1. Example: Computing $P(0 < X < 0.5)$

Given the support $X \ge 0$:

$$
P(0 < X < 0.5) = F_X(0.5) - F_X(0)
$$

Calculate $F_X(0.5)$:

$$
F_X(0.5) = 1 - e^{-4 \times 0.5} = 1 - e^{-2}
$$

We know:

$$
e^{-2} \approx 0.1353
$$

Thus:

$$
P(0 < X < 0.5) = 1 - 0.1353 - 0 = 0.8647
$$

Rounded to two decimal places:

$$
\boxed{0.86}
$$

4.2. Example: Computing the Probability $P(X > 1)$

$$
P(X > 1) = 1 - P(X \leq 1) = 1 - F_X(1)
$$

Calculate $F_X(1)$:

$$
F_X(1) = 1 - e^{-4 \times 1} = 1 - e^{-4}
$$

Calculate e^{-4}:

$$
e^{-4} \approx 0.0183
$$

Therefore:

$$
P(X > 1) = 1 - (1 - 0.0183) = 0.0183
$$

Rounded to two decimal places:

$$
\boxed{0.02}
$$

4.3. Example: Expected Value and Variance

As previously noted:

$$
E[X] = 0.25
$$
$$
\text{Var}(X) = 0.0625
$$

---

5. Additional Quantitative Measures

5.1. Median

Previously calculated:

$$
m = \frac{\ln 2}{4} \approx 0.1733
$$

Rounded to two decimal places:

$$
\boxed{0.17}
$$

5.2. Mode

The exponential distribution has its maximum at the starting point of support:

$$
\text{Mode} = 0
$$

since the PDF is decreasing for $x \ge 0$.

---

6. Insights and Practical Implications

Understanding that the given CDF corresponds to an exponential distribution with $\lambda=4$ provides a rich context:

- Memoryless Property: The exponential distribution is memoryless, meaning the probability of an event occurring in the future is independent of how much time has already elapsed. Mathematically:

$$
P(X > s + t | X > s) = P(X > t)
$$

- Modeling Waiting Times: This distribution aptly models scenarios like radioactive decay, customer service times, or time between arrivals in a Poisson process with rate 4.

- Parameter significance: The rate parameter $\lambda=4$ indicates an average waiting time of $1/4=0.25$. The higher the λ, the shorter the expected waiting time.

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7. Summary and Final Remarks

To summarize:

- The random variable X is exponentially distributed with a rate parameter $\lambda=4$.
- Its PDF is $f_X(x) = 4 e^{-4x}$ for $x \ge 0$.
- Its mean is 0.25, and median is approximately 0.17.
- Probabilities of interest, such as $P(0 < X < 0.5)$ and $P(X > 1)$, are approximately 0.86 and 0.02 respectively, when rounded to two

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