Let F Be The Function Defined By F(x)=xlnx For X>0. On What Open Interval Is F Decreasing?

Introduction

Let F Be The Function Defined By F(x)=xlnx For X>0. On What Open Interval Is F Decreasing?

Understanding the behavior of functions is a fundamental aspect of calculus, especially when analyzing how functions increase or decrease over specific intervals. The function $F(x) = x \ln x$, defined for all positive real numbers, is a classic example involving a natural logarithm and a linear term. Determining where this function is decreasing requires examining its derivatives and understanding the critical points that influence its monotonicity. This article provides an in-depth exploration of the function $F(x) = x \ln x$, focusing on identifying the open intervals where it is decreasing.

Preliminary Concepts

Function Behavior and Monotonicity

A function is said to be decreasing on an interval if, for any two points x and y within that interval where $x < y$, the function values satisfy $F(x) > F(y)$. Conversely, the function is increasing if $F(x) < F(y)$ for $x < y$. To determine where a function is decreasing or increasing, calculus techniques such as derivatives are employed.

Critical Points and Derivative Sign

The derivative of a function provides information about its slope at any point. Critical points occur where the derivative is zero or undefined and are potential locations where the function switches from increasing to decreasing or vice versa. By analyzing the sign of the derivative on different intervals, one can classify these intervals as increasing or decreasing.

Analyzing the Function $F(x) = x \ln x$

Domain of the Function

Since $\ln x$ is defined only for $x > 0$, the domain of $F(x) = x \ln x$ is $(0, \infty)$.

Calculating the Derivative

To analyze the monotonicity of F(x), we compute its first derivative F'(x):

```
F'(x) = d/dx [x ln x]
= (1)  ln x + x  (1/x)
= ln x + 1
```

This derivative simplifies to F'(x) = ln x + 1.

Understanding F'(x) = ln x + 1

- The derivative depends solely on ln x, shifted by 1.
- Since ln x is defined for x > 0, F'(x) is defined for all x > 0.
- The critical points occur where F'(x) = 0, i.e., where ln x + 1 = 0.

Identifying Critical Points

Solving for Critical Points

Set F'(x) = 0:

```
ln x + 1 = 0
ln x = -1
x = e^{-1} = 1/e
```

Thus, the only critical point in the domain is at x = 1/e.

Behavior of the Derivative Around the Critical Point

- For x < 1/e:
- Since ln x < -1 for x < 1/e, then F'(x) = ln x + 1 < 0.
- This indicates F(x) is decreasing on intervals where x < 1/e.
- For x > 1/e:
- Since ln x > -1 for x > 1/e, then F'(x) = ln x + 1 > 0.
- This indicates F(x) is increasing on intervals where x > 1/e.

Determining the Intervals of Decrease and Increase

Interval Where F(x) Is Decreasing

From the derivative analysis, F(x) decreases where F'(x) < 0, i.e., for:

```
x < 1/e
```

Given the domain is (0, ∞), the interval where F(x) decreases is:

```
(0, 1/e)
```

Interval Where F(x) Is Increasing

Similarly, F(x) increases where F'(x) > 0, i.e., for:

```
x > 1/e
```

Thus, the interval of increasing behavior is:

```
(1/e, ∞)
```

Graphical Interpretation

Shape of the Graph of F(x) = x ln x

The graph of F(x) exhibits a characteristic shape with a minimum point at x = 1/e. Since F(x) is decreasing on (0, 1/e) and increasing on (1/e, ∞), the graph drops to its lowest point at x = 1/e and then rises again. The point (1/e, F(1/e)) is a local minimum, providing visual confirmation of the analytical findings.

Implications of the Behavior

- Understanding where the function decreases is useful for optimization problems, where one might want to find the minimum of F(x).
- The decreasing interval (0, 1/e) indicates that as x approaches zero from the right, F(x) decreases towards negative infinity, since ln x tends to -∞.

- At x = 1/e, the function attains its minimum value, which can be calculated as follows:

Calculating the Minimum Value of F(x)

```
F(1/e) = (1/e)  ln(1/e) = (1/e)  (-1) = -1/e
```

Therefore, F(x) reaches its minimum value of -1/e at x = 1/e.

Summary and Conclusion

In this comprehensive analysis, we examined the function F(x) = x ln x over its domain (0, ∞). By calculating the derivative, F'(x) = ln x + 1, we identified the critical point at x = 1/e. The sign of the derivative revealed that F(x) is decreasing on the interval (0, 1/e) and increasing on (1/e, ∞). Consequently, the open interval where F(x) is decreasing is precisely (0, 1/e). This interval encompasses all positive x-values less than 1/e, where the function slopes downward, reaching its minimum at x = 1/e with a minimum value of -1/e.

Understanding the decreasing intervals of functions like F(x) = x ln x is essential in various fields, including optimization, economics, and natural sciences, where such functions often model real-world phenomena. The analytical approach using derivatives provides a clear and rigorous method to determine these intervals, reinforcing the importance of calculus in analyzing function behavior.

Frequently Asked Questions

How do I find the interval where the function F(x) = x ln x is decreasing for x > 0?

To find where F is decreasing, first compute its derivative: F'(x) = ln x + 1. Since F' is decreasing when it is negative, set F'(x) < 0, which gives ln x + 1 < 0 → ln x < -1 → x < e^{-1}. Therefore, F is decreasing on the open interval (0, e^{-1}).

What is the derivative of the function F(x) = x ln x, and how does it help identify decreasing intervals?

The derivative is F'(x) = ln x + 1. By analyzing where F'(x) < 0, we find the intervals where F is decreasing. Specifically, F'(x) < 0 when x < e^{-1}, indicating F decreases on (0, e^{-1}).

Why is the interval (0, e^{-1}) where F(x)=x ln x is

decreasing for x > 0?

Because on (0, e^{-1}), the derivative F'(x) = ln x + 1 < 0, which means the function's slope is negative there, so F decreases on that interval. This occurs because ln x < -1 when x < e^{-1}.

Can you explain the significance of the critical point at x = e^{-1} for the function F(x) = x ln x?

Yes. The critical point at x = e^{-1} occurs where F'(x) = 0, i.e., ln x + 1 = 0. This point marks the transition from decreasing to increasing behavior, making it a local minimum point for F.

Is the function F(x) = x ln x decreasing on the entire domain x > 0?

No, F(x) is only decreasing on the interval (0, e^{-1}). For x > e^{-1}, the derivative F'(x) = ln x + 1 > 0, so the function is increasing there.

How does understanding the decreasing interval of F(x) = x ln x help in graphing or analyzing the function?

Knowing where F is decreasing helps identify the shape of the graph, specifically the declining side leading up to the minimum at x= e^{-1}. It aids in sketching the graph accurately and understanding the function's behavior across its domain.

Additional Resources

Understanding the Function F(x) = x ln x and Its Monotonic Behavior

Analyzing the behavior of functions is a fundamental aspect of calculus, providing insights into their increasing and decreasing intervals, extrema, and overall shape. The function in question, F(x) = x ln x for x > 0, is a classic example that combines algebraic and logarithmic components, leading to interesting properties worth exploring in detail.

Introduction to the Function F(x) = x ln x

Before diving into where the function is decreasing, it's essential to understand its basic structure, domain, and initial characteristics.

Domain of F(x)

- The domain is given as x > 0, owing to the natural logarithm ln x, which is only defined for positive real numbers.
- The function is continuous and differentiable over this domain, which allows us to analyze its increasing and decreasing behavior through derivatives.

Basic features of F(x)

- For x > 0, the function behaves differently depending on whether x is less than, equal to, or greater than 1.
- At x = 1, F(1) = 1 ln 1 = 0.
- As x → 0+, ln x → -∞, so F(x) → 0 (-∞), which requires careful analysis to understand the limit.

Calculating the Derivative of F(x)

To determine where F(x) is decreasing, we need to analyze its first derivative F'(x).

Derivative Calculation

Using the product rule for differentiation:
- If F(x) = x ln x, then
- F'(x) = d/dx [x ln x]

Applying the product rule:
- F'(x) = d/dx [x] ln x + x d/dx [ln x]
- F'(x) = 1 ln x + x (1/x)
- Simplify:
- F'(x) = ln x + 1

Result:
$$
F'(x) = \ln x + 1
$$

This derivative provides the key to understanding the monotonicity of F(x).

Determining Increasing and Decreasing Intervals

The sign of F'(x) determines whether F(x) is increasing or decreasing:

- $F(x)$ is increasing where $F'(x) > 0$
- $F(x)$ is decreasing where $F'(x) < 0$

Let's analyze where $F'(x) = \ln x + 1$ changes sign.

Solving for Critical Points

Set $F'(x) = 0$ to find critical points:

$$
\ln x + 1 = 0
$$

$$
\ln x = -1
$$

$$
x = e^{-1} = \frac{1}{e}
$$

This is the only critical point in the domain $(0, \infty)$.

Sign Analysis of the Derivative

To determine when $F(x)$ is decreasing, analyze $F'(x) = \ln x + 1$:

- For $x < 1/e$:
- $\ln x < -1$, so $\ln x + 1 < 0$
- Therefore, $F'(x) < 0$
- The function is decreasing in this interval.

- For $x > 1/e$:
- $\ln x > -1$, so $\ln x + 1 > 0$
- Therefore, $F'(x) > 0$
- The function is increasing in this interval.

Thus:
- $F(x)$ is decreasing on $(0, 1/e)$
- $F(x)$ is increasing on $(1/e, \infty)$

Behavior at the Critical Point and Endpoints

Understanding the nature of the critical point at $x = 1/e$ helps in grasping the overall shape of the graph.

At x = 1/e

- $F'(1/e) = 0$, indicating a potential extremum.
- To classify the extremum (minimum or maximum), examine the second derivative.

Second Derivative Analysis

Calculate $F''(x)$:
- Recall $F'(x) = \ln x + 1$
- Therefore:

$$
F''(x) = \frac{d}{dx} (\ln x + 1) = \frac{1}{x}
$$

- Since $x > 0$, $F''(x) > 0$ everywhere.

This indicates F(x) is concave upward on its entire domain, meaning the critical point at $x = 1/e$ is a local minimum.

Summary of Monotonicity and Critical Points

- Decreasing interval: $(0, 1/e)$
- Increasing interval: $(1/e, \infty)$
- Critical point at: $x = 1/e$, a local minimum

Graphical Interpretation

Visualizing the graph helps solidify understanding:

- As $x \to 0+$, $F(x) \to 0$ (approaching from below)
- At $x = 1/e$, F(x) attains its minimum value:

$$
F(1/e) = \frac{1}{e} \ln \frac{1}{e} = \frac{1}{e} (-1) = -\frac{1}{e}
$$

- For $x > 1/e$, F(x) increases without bound, tending to infinity as $x \to \infty$

Conclusion: Open Interval of Decrease for F(x)

Based on the derivative analysis:

- The function $F(x) = x \ln x$ is strictly decreasing on the open interval:

$$
\boxed{
(0, \frac{1}{e})
}
$$

- On $(0, 1/e)$, $F'(x) < 0$, so $F(x)$ decreases as x increases toward $1/e$.
- At $x = 1/e$, $F(x)$ reaches its minimum point.
- Beyond $x = 1/e$, the function transitions to increasing behavior.

Additional Insights and Practical Implications

Understanding where $F(x)$ decreases has several practical applications:

- Optimization problems: Recognizing the decreasing interval helps in identifying minimum points.
- Modeling phenomena: Functions like $x \ln x$ appear in information theory, economics, and entropy calculations.
- Mathematical analysis: The derivative test exemplifies fundamental calculus techniques for analyzing functions.

Summary of Key Points

- The derivative $F'(x) = \ln x + 1$ is negative for $x < 1/e$ and positive for $x > 1/e$.
- The function $F(x)$ is decreasing on the open interval $(0, 1/e)$.
- The critical point at $x = 1/e$ is a local minimum.
- The behavior of $F(x)$ is well-understood through derivative tests, providing a clear picture of its increasing and decreasing intervals.

Final Remarks

The analysis of $F(x) = x \ln x$ exemplifies how derivatives serve as powerful tools in understanding function behavior. The decreasing interval $(0, 1/e)$ is particularly significant because it marks the region where the function exhibits a downward trend before reaching its minimum. Recognizing such intervals is crucial in various mathematical and applied contexts, from calculus coursework to real-world modeling.

By mastering the process of derivative calculation, critical point analysis, and sign testing, students and practitioners can confidently analyze the monotonicity of a wide range of functions, enhancing their mathematical intuition and problem-solving skills.

In summary:

The function $F(x) = x \ln x$ is decreasing precisely on the open interval $(0, 1/e)$, where x ranges from just above zero up to $1/e$. This interval corresponds to the region where the derivative F'(x) = \ln x + 1 is negative, indicating a downward trend in the function's graph. Understanding this behavior provides foundational insights into the function's shape and critical points, essential for further mathematical analysis and applications.

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