On The Graph Of F(x)=sinx And The Interval [2,4), For What Value Of X Does F(x) Achieve A Minimum?

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Understanding the behavior of the sine function on specific intervals is fundamental in calculus and mathematical analysis. When analyzing the function $f(x) = \sin x$ over the interval $[2, 4)$, a common question arises: At what value of x does $f(x)$ achieve its minimum? This inquiry not only deepens our grasp of the sine function's properties but also hones our skills in identifying critical points, understanding the nature of extrema, and applying calculus techniques to real-world problems.

In this comprehensive article, we will explore the sine function's behavior within the interval $[2, 4)$, determine the points where $f(x)$ attains its minimum, and discuss the underlying calculus principles involved. We will organize our discussion into clear sections, incorporating explanations, visual insights, and practical steps to identify the minimum value accurately.

Understanding the Sine Function and Its Key Properties

Before delving into the specific interval $[2, 4)$, it is essential to revisit the fundamental properties of the sine function.

The Basic Characteristics of $f(x) = \sin x$

- Periodicity: The sine function is periodic with a period of 2π. That means $\sin(x + 2\pi) = \sin x$ for all x.

- Range: The output of $\sin x$ always lies between -1 and 1. Therefore, $-1 \leq \sin x \leq 1$.

- Zeros: $\sin x = 0$ at integer multiples of π: $x = n\pi$, where n is an integer.

- Maximum and Minimum: The maximum value 1 is achieved at $x = \frac{\pi}{2} + 2k\pi$, and the minimum value -1 at $x = \frac{3\pi}{2} + 2k\pi$, where k is an integer.

- Shape and Symmetry: The graph of $\sin x$ oscillates smoothly between -1 and 1, with periodic peaks and troughs.

Analyzing $f(x) = \sin x$ on the Interval $[2, 4)$

Our goal is to find the value of x within $[2, 4)$ where $\sin x$ attains its minimum. Since $\sin x$ is continuous and differentiable everywhere, calculus tools like derivatives can help locate minima and maxima.

Step 1: Visualize the Graph of $\sin x$ in the Interval

Plotting or visualizing the graph between $x=2$ and $x=4$ provides immediate insight:

- At $x=2$, $\sin 2 \approx 0.9093$.

- At $x=3$, $\sin 3 \approx 0.1411$.

- At $x=3.14 \approx \pi$, $\sin \pi=0$.

- At $x=4$, $\sin 4 \approx -0.7568$.

From these approximate values, it appears the sine function decreases from roughly 0.9093 at $x=2$ down to about -0.7568 at $x=4$. Therefore, the minimum within this interval is likely near $x=4$, but we need to verify precisely.

Step 2: Find Critical Points by Derivative Analysis

The first derivative of $f(x)=\sin x$ is:

$$
f'(x) = \cos x
$$

Critical points occur where $f'(x) = 0$, i.e., where:

$$
\cos x = 0
$$

The solutions to $\cos x = 0$ are:

$$
x = \frac{\pi}{2} + k\pi, \quad k \in \mathbb{Z}
$$

Within the interval $[2, 4)$, identify which critical points fall into this range:

- For $k=0$:

$$
x = \frac{\pi}{2} \approx 1.5708
$$

which is less than 2, so outside our interval.

- For $k=1$:

$$
x = \frac{\pi}{2} + \pi \approx 1.5708 + 3.1416 \approx 4.7124
$$

which is greater than 4, outside the interval.

- For $k=-1$:

$$
x= \frac{\pi}{2} - \pi \approx 1.5708 - 3.1416 \approx -1.5708
$$

less than 2, outside the interval.

Thus, there are no critical points within $[2, 4)$. The minima or maxima in this interval are therefore likely to occur at the endpoints or where the derivative does not vanish but the function reaches a minimum.

Step 3: Evaluate $f(x)$ at Critical and Boundary Points

Since there are no critical points inside $[2, 4)$, examine the endpoints:

- At $x=2$: $\sin 2 \approx 0.9093$

- At $x=4$: $\sin 4 \approx -0.7568$

Note that the interval is open at 4, but since $\sin 4$ is continuous, the infimum (greatest lower bound) as x approaches 4 from the left is approximately -0.7568.

Additionally, check at some points within the interval to confirm:

- At $x=3$: $\sin 3 \approx 0.1411$

- At $x=3.5$: $\sin 3.5 \approx -0.3508$

- At $x=3.9$: $\sin 3.9 \approx -0.6770$

These values show the sine function decreases toward -0.7568 as x approaches 4.

Identifying the Minimum Value of $f(x) = \sin x$ on $[2,4)$

Based on the analysis:

- The maximum value within $[2,4)$ is approximately 0.9093 at $x=2$.

- The minimum value appears to be near $x=4$, with $\sin 4 \approx -0.7568$.

Since $f(x)$ is decreasing in this interval (as indicated by the negative derivative when $\cos x<0$), and there are no critical points, the minimum value on $[2,4)$ is approached at the right endpoint, $x \to 4^-$.

Therefore, $f(x)$ achieves its minimum value at $x \to 4^-$, with the minimum value approximately -0.7568.

Conclusion: The Precise Value of x for the Minimum

While the function does not attain $\sin x = -0.7568$ exactly at any point within $[2,4)$ (since $x=4$ is not included in the interval), it approaches this value as x approaches 4 from the left.

In the context of the question:

- The x-value where $f(x)$ attains its minimum within the interval $[2,4)$ is approaching $x=4$.

- The minimum value of $f(x)$ is $\sin x \approx -0.7568$, approached as $x \to 4^-$.

If the question requires the exact point where the minimum occurs within the interval, the answer is that the minimum is achieved arbitrarily close to, but not including, $x=4$.

Additional Insights and Practical Implications

Understanding the behavior of $\sin x$ on a specific interval like $[2, 4)$ has practical applications in physics, engineering, and data analysis:

- Wave Analysis: The sine function models oscillations; knowing where it reaches minimums helps in analyzing signal phases.

- Optimization Problems: Determining extrema within intervals is vital for optimizing functions in engineering design.

- Signal Processing: Recognizing the points where signals reach their minima or maxima enhances filtering and noise reduction.

Summary of Key Points

- The derivative \(f'(x) = \cos x\) helps locate critical points, but none exist inside \([2, 4)\).

- The sine function decreases on \([2, 4)\), approaching \(-0.7568\) at \(x=4\).

- The minimum value of \(f(x)=\sin x\) on \([2

Frequently Asked Questions

What is the function being analyzed in the interval [2,4) in the graph?

The function is $f(x) = \sin x$.

Within the interval [2,4), at which value of x does f(x) reach its minimum?

$f(x)$ attains its minimum at $x = 3\pi/2$, which is approximately 4.712, but since the interval is [2,4), the minimum occurs at $x = 3$, where $\sin 3$ is approximately 0.1411, which is not the minimum. Actually, within [2,4), the minimum is at $x \approx 4$, where $\sin 4 \approx -0.7568$, so the minimum is at $x \approx 4$.

How do you find the minimum value of f(x) = sin x on the interval [2,4)?

Identify where $\sin x$ attains its lowest value within [2,4). Since sine reaches its minimum at $3\pi/2 \approx 4.712$, which is outside the interval, the minimum within [2,4) is at the endpoint closest to $3\pi/2$, which is $x = 4$, where $\sin 4 \approx -0.7568$.

Does the minimum of f(x) occur at a critical point or an endpoint within [2,4)?

The minimum occurs at an endpoint, specifically at $x = 4$, since $3\pi/2$ is outside the interval.

What is the approximate value of f(x) at the minimum point in [2,4)?

At $x \approx 4$, $f(x) = \sin 4 \approx -0.7568$, which is the minimum value in that interval.

Is the minimum of sin x in [2,4) a local or global minimum within the interval?

It is a local minimum within the interval [2,4), occurring at the endpoint x = 4.

How does the behavior of sin x influence the location of the minimum in the interval [2,4)?

Since sin x decreases from about 0.909 at x=2 to about -0.7568 at x=4, the minimum within [2,4) occurs at the endpoint x=4, where sin x is lowest.

Additional Resources

On The Graph Of F(x) = sin x And The Interval [2,4): For What Value Of x Does F(x) Achieve A Minimum?

Introduction: Exploring the Behavior of the Sine Function on a Restricted Interval

In the realm of mathematics, trigonometric functions occupy a foundational position, underpinning various fields from engineering to physics. Among these, the sine function is particularly notable for its periodic and oscillatory nature. When analyzing the behavior of $f(x) = \sin x$ over specific intervals, a compelling question often arises: Where does the sine function attain its minimum or maximum within that domain?

This inquiry is not merely academic; it has practical implications in signal processing, oscillation analysis, and optimization problems. In this article, we focus on the function $f(x) = \sin x$ restricted to the interval $[2, 4)$, aiming to determine the value of x within this domain where the function reaches its minimum.

Understanding the Sine Function: Key Properties and Behavior

Before delving into the specific interval, it's essential to revisit some fundamental properties of the sine function:

- Periodicity: $\sin x$ has a period of 2π, meaning $\sin(x + 2\pi) = \sin x$.

- Range: $\sin x \in [-1, 1]$ for all real x.

- Critical Points: The maximum values of $\sin x$ are 1 (occurring at $x = \frac{\pi}{2} + 2k\pi$), and the minima are -1 (at $x= \frac{3\pi}{2} + 2k\pi$).

- Monotonicity: On intervals where $\sin x$ is increasing or decreasing depends on the derivative $\cos x$.

Considering these features, locating the minimum on $[2, 4)$ involves identifying the points within this interval where $\sin x$ reaches its lowest value.

Analyzing the Interval [2, 4): Critical Points and Behavior

Identifying Critical Points of $\sin x$

The critical points of $\sin x$ occur where its derivative, $\cos x$, equals zero:

$$
\frac{d}{dx} \sin x = \cos x = 0
$$

The solutions to $\cos x = 0$ are:

$$
x = \frac{\pi}{2} + k\pi, \quad k \in \mathbb{Z}
$$

Within the interval $[2, 4)$, we need to determine which of these critical points fall inside this range.

- For $k=0$:

$$
x = \frac{\pi}{2} \approx 1.5708
$$

which is less than 2, thus outside the interval.

- For $k=1$:

$$
x = \frac{\pi}{2} + \pi \approx 1.5708 + 3.1416 = 4.7124
$$

which exceeds 4, so not within the interval.

Since neither critical point from the derivative's zeros occurs within $[2, 4)$, we conclude that $\sin x$ has no critical points inside this interval.

Implication: Monotonic Behavior on $[2, 4)$

Given the absence of critical points in $[2, 4)$, the function $\sin x$ is monotonic within this interval. To determine whether it's increasing or decreasing, analyze the sign of $\cos x$ over the interval.

- For $x \in [2, 4)$:

- At $x = 2$:

$$
\cos 2 \approx -0.4161
$$

- At $x = 4$:

$$
\cos 4 \approx -0.6536
$$

Since $\cos x$ is negative throughout $[2, 4)$ (as $\cos x$ is decreasing from $\cos 2 \approx -0.4161$ to $\cos 4 \approx -0.6536$), the derivative $\cos x$ is negative, implying that $\sin x$ is decreasing over this interval.

Conclusion: On $[2, 4)$, $f(x) = \sin x$ is monotonically decreasing.

Determining the Minimum of $\sin x$ on $[2, 4)$

Since $\sin x$ is decreasing on $[2, 4)$, its minimum value within this interval occurs at the right endpoint, which is approaching but not including $x=4$. As x approaches 4 from below, $\sin x$ approaches $\sin 4$.

- Calculate $\sin 4$:

$$
\sin 4 \approx -0.7568
$$

Because 4 is not included in the interval (it's a half-open interval), the actual minimum value within the interval $[2, 4)$ is:

$$

\boxed{\text{approaching } \sin 4 \approx -0.7568}
\]

and the corresponding \(x\) value is approaching 4 from below.

Therefore, the x-value where \(f(x)\) achieves its minimum over \([2, 4)\) is any \(x\) arbitrarily close to 4 from below, with the minimum function value approaching \(-0.7568\).

Summary of Key Findings

- The sine function \(\sin x\) is monotonically decreasing on \([2, 4)\).
- No critical points exist within this interval, so the extremum points are at the boundary.
- The minimum value of \(\sin x\) on \([2, 4)\) is approached at \(x \to 4^-\), with:

\[
\boxed{\sin 4 \approx -0.7568}
\]

- The x-value where the minimum is approached is just less than 4, but since 4 is not included, the minimum is not attained at a specific point but approached in the limit.

Broader Implications and Applications

Understanding where a function attains its extremum over a specific interval is vital across numerous disciplines:

- Physics: Analyzing oscillations or wave minima/maxima.
- Engineering: Signal processing where phase shifts and amplitude minima are critical.
- Mathematics: Optimization problems involving periodic functions.
- Economics & Data Science: Modeling cyclical trends and identifying extremal points within bounded domains.

In the context of the sine function on \([2,4)\), this analysis highlights the importance of understanding the monotonic segments of functions, especially when critical points are absent, and emphasizes the role of boundary behavior in extremal value analysis.

Conclusion: The Final Takeaway

In conclusion, the minimum value of $f(x) = \sin x$ on the interval $[2, 4)$ is approached at $x \to 4^-$, with the function value nearing $\sin 4 \approx -0.7568$. Since the function is decreasing throughout this interval, the minimal point is not at an interior point but at the boundary approaching 4 from below. This example underscores an essential principle in calculus and analysis: the extremal values of continuous functions on closed or half-open intervals often occur at critical points or boundaries, with the behavior at the boundaries dictating the minima and maxima when critical points are absent within the domain.

References & Further Reading:

- Stewart, J. (2015). Calculus: Early Transcendentals. Cengage Learning.
- Thomas, G. B., & Finney, R. L. (2002). Calculus and Analytic Geometry. Pearson.
- Khan Academy. (n.d.). Graphing sine and cosine functions. [Online resource]

This comprehensive analysis provides a detailed understanding of the behavior of $\sin x$ on the interval $[2, 4)$, elucidating where and how the minimum value is approached, enriching both theoretical knowledge and practical application skills.

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