Please Help With This Question On Factorisation. Please Show Steps Because I Want To Understand.

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Factorisation is a fundamental concept in algebra that involves expressing a given algebraic expression as a product of its factors. Understanding how to factorise expressions is crucial because it simplifies complex algebraic problems, makes solving equations easier, and forms the basis for higher-level mathematics. If you're struggling with factorisation and want clear, step-by-step guidance, you've come to the right place. This article will walk you through various methods of factorisation, illustrate each with detailed examples, and provide tips to improve your understanding.

What Is Factorisation?

Factorisation, also known as factoring, is the process of rewriting a polynomial or algebraic expression as a product of simpler expressions, known as factors. These factors, when multiplied together, give back the original expression.

Example:
Express the quadratic expression $x^2 + 5x + 6$ as a product of factors.

Answer:
$(x + 2)(x + 3)$

This simple example illustrates what factorisation aims to achieve: breaking down an expression into components that are easier to work with.

Why Is Factorisation Important?

- Simplifies solving equations
- Helps find roots or solutions of algebraic expressions
- Aids in simplifying rational expressions
- Essential in calculus, polynomial division, and other advanced topics

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Common Types of Factorisation

Understanding the various types of factorisation is key to mastering the topic. Each type applies to different forms of algebraic expressions.

1. Factorising Monomials

- Involves extracting the highest common factors (HCF) from terms.

2. Factorising Quadratic Expressions

- Focuses on quadratic expressions (second-degree polynomials).

3. Factorising by Grouping

- Useful for four-term polynomials.

4. Special Products

- Recognises patterns like difference of squares, perfect square trinomials, and sum/difference of cubes.

Step-by-Step Guide to Factorisation

Let's explore each method with detailed steps and examples.

1. Factorising Monomials and Polynomials Using the Highest Common Factor (HCF)

Steps:

1. Identify the HCF of all terms in the expression.
2. Divide each term by the HCF.
3. Write the expression as the product of the HCF and the simplified expression.

Example:
Factorise $12x^3 + 18x^2$.

Solution:

- Find HCF of 12 and 18: HCF = 6.
- Find HCF of x^3 and x^2: x^2.
- HCF of entire terms: $6x^2$.

Now, factor out $6x^2$:

$$
12x^3 + 18x^2 = 6x^2 (2x + 3)
$$

Result:

$\boxed{6x^2(2x + 3)}$

---

2. Factorising Quadratic Expressions

Quadratics are expressions of the form $ax^2 + bx + c$.

Steps:

1. Identify coefficients a, b, and c.
2. Find two numbers that multiply to $a \times c$ and add to b.
3. Rewrite the middle term using these numbers.
4. Factor by grouping.

Example:

Factorise $x^2 + 7x + 12$.

Solution:

- $a=1$, $b=7$, $c=12$.
- Find two numbers that multiply to $1 \times 12 = 12$ and add to 7:

Numbers are 3 and 4.

- Rewrite:

$$
x^2 + 3x + 4x + 12
$$

- Group:

$$
(x^2 + 3x) + (4x + 12)
$$

- Factor each group:

$$
x(x + 3) + 4(x + 3)
$$

- Factor out common binomial:

$$
(x + 3)(x + 4)
$$

Answer:

$\boxed{(x + 3)(x + 4)}$

---

3. Factorising by Grouping

Useful for four-term polynomials.

Steps:

1. Split the polynomial into two groups with two terms each.
2. Factor out the HCF from each group.
3. If the binomials are the same, factor them out.

Example:

Factorise $ax + ay + bx + by$.

Solution:

- Group:

$$
(ax + ay) + (bx + by)
$$

- Factor each group:

$$
a x (1 + y) + b x (1 + y)
$$

- Factor out common binomial:

$$
(1 + y)(a x + b x) = (1 + y) x (a + b)

\]

Answer:

$\boxed{(1 + y) x (a + b)}$

---

4. Recognising and Using Special Products

Certain algebraic forms always factor in specific ways.

a. Difference of Squares

Pattern: $a^2 - b^2 = (a - b)(a + b)$

Example:

Factorise $x^2 - 9$.

Solution:

$$
x^2 - 3^2 = (x - 3)(x + 3)
$$

b. Perfect Square Trinomials

Pattern: $a^2 \pm 2ab + b^2 = (a \pm b)^2$

Example:

Factorise $x^2 + 6x + 9$:

$$
x^2 + 2 \times x \times 3 + 3^2 = (x + 3)^2
$$

c. Sum and Difference of Cubes

Sum: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Difference: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Example:
Factorise $x^3 + 8$:

$$
x^3 + 2^3 = (x + 2)(x^2 - 2x + 4)
$$

Practice Problems with Step-by-Step Solutions

Let's apply what we've learned with some practice examples.

Problem 1:

Factorise $3x^2 + 6x$.

Solution:

- HCF of coefficients: 3.
- HCF of variables: x.

Factor out $3x$:

$$
3x^2 + 6x = 3x (x + 2)
$$

Problem 2:

Factorise $x^2 - 16$.

Solution:

- Recognise as difference of squares:

$$
x^2 - 4^2 = (x - 4)(x + 4)
$$

Problem 3:

Factorise $2x^2 + 5x - 3$.

Solution:

- Multiply a and c: $2 \times -3 = -6$.
- Find two numbers that multiply to -6 and add to 5: 6 and -1.

Rewrite middle term:

$$
2x^2 + 6x - x - 3
$$

Group:

$$
(2x^2 + 6x) - (x + 3)
$$

Factor each group:

$$
2x(x + 3) -1(x + 3)
$$

Factor out common binomial:

$$
(2x - 1)(x + 3)
$$

Tips for Mastering Factorisation

- Always look for the greatest common factor first.
- Recognise special patterns early.
- Practice with a variety of problems.
- Use systematic approaches like the methods outlined above.
- Check your factors by expanding to verify correctness.

Conclusion

Factorisation may seem challenging at first, but with practice and understanding of the various methods, it becomes a straightforward skill. Remember to break down expressions step-by-step, look for patterns, and verify your answers. Whether you're working with simple monomials or complex quadratics, the techniques covered in this guide will help you approach factorisation confidently and effectively.

If you encounter a specific problem, follow the steps outlined, and don't hesitate to revisit basic principles like finding the HCF or recognizing special products. With persistence, you'll master factorisation and strengthen your overall algebra skills.

Happy factoring!

Frequently Asked Questions

How do I factorise a quadratic expression like $x^2 + 5x + 6$?

To factorise $x^2 + 5x + 6$, look for two numbers that multiply to 6 and add to 5. These are 2 and 3. Rewrite the expression as $(x + 2)(x + 3)$.

What are the steps to factorise a quadratic trinomial step-by-step?

First, identify the quadratic in $ax^2 + bx + c$ form. Then, find two numbers that multiply to ac and add to b. Use these to split the middle term, and factor by grouping. Finally, write the expression as a product of two binomials.

How do I factorise a difference of squares like $x^2 - 9$?

Recognise that $x^2 - 9$ is a difference of squares: $x^2 - 3^2$. Use the formula $a^2 - b^2 = (a - b)(a + b)$. So, $x^2 - 9 = (x - 3)(x + 3)$.

Can you show me how to factorise a cubic polynomial like $x^3 + 3x^2 + 3x + 1$?

Yes. Notice that $x^3 + 3x^2 + 3x + 1$ resembles a binomial expansion: $(x + 1)^3$. Expand $(x + 1)^3$ to verify: $x^3 + 3x^2 + 3x + 1$. So, the factorised form is $(x + 1)^3$.

What is the step-by-step method to factorise trinomials where a □ 1?

First, multiply a and c. Find two numbers that multiply to ac and add to b. Then, split the middle term using these numbers, and factor by grouping. Finally, factor out common terms to get the binomial factors.

How do I factorise a quadratic when the middle term is negative, like $x^2 - 7x + 12$?

Identify two numbers that multiply to 12 and add to -7. These are -3 and -4. Rewrite the quadratic as $(x - 3)(x - 4)$.

What are common mistakes to avoid when factorising expressions?

Common mistakes include missing signs, forgetting to factor out the greatest common factor first, and not checking if the factors multiply back to the original expression. Always verify your factors by expansion.

Can you explain how to factorise an expression like $2x^2 + 8x$?

Yes. First, find the GCF of the coefficients, which is 2. Factor out 2: $2(x^2 + 4x)$. Then, factor the quadratic inside: $x(x + 4)$. So, the fully factorised form is $2x(x + 4)$.

Additional Resources

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Introduction

Factorisation is a fundamental concept in algebra that often poses challenges for students and learners new to the subject. Whether you're tackling quadratic expressions, polynomials, or more complex algebraic structures, understanding the step-by-step process of factorising expressions is crucial for developing a solid mathematical foundation. This article aims to delve into the intricacies of factorisation, providing a comprehensive guide that explains the concepts clearly, demonstrates detailed steps, and offers practical strategies to enhance understanding.

Understanding the Importance of Factorisation

Before diving into the mechanics, it's essential to grasp why factorisation matters:

- Simplifying expressions: It allows for easier manipulation of algebraic expressions.
- Solving equations: Factoring transforms complex equations into solvable linear or quadratic forms.
- Finding roots: It helps identify the solutions of polynomial equations.
- Mathematical analysis: Factorisation is key in calculus, number theory, and other advanced areas.

Fundamental Concepts and Terminology

To effectively approach factorisation, familiarity with key concepts is essential:

- Polynomial: An algebraic expression involving variables and coefficients, e.g., $ax^n + bx^{n-1} + \dots + c$.
- Factor: A polynomial that divides another polynomial evenly.
- Factorisation: The process of rewriting an expression as a product of its factors.
- Common factors: Factors shared by all terms in an expression.
- Special products: Recognisable patterns like difference of squares, perfect square trinomials, etc.

Step-by-Step Approach to Factorisation

Approach to factorising an algebraic expression depends on its form, but several common methods can be systematically applied:

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1. Factoring Out the Greatest Common Factor (GCF)

Step 1: Identify the GCF of all terms.

Step 2: Factor out the GCF from each term.

Example:

Factor $6x^3 + 9x^2 - 15x$.

- GCF of 6, 9, and 15 is 3.
- GCF of x^3, x^2, x is x.

Solution:

- GCF of all coefficients and variables: $3x$.

- Factored form:

$$
6x^3 + 9x^2 - 15x = 3x(2x^2 + 3x - 5)
$$

---

2. Factoring Quadratic Expressions

Quadratic expressions are of the form $ax^2 + bx + c$. Most factorisation involves finding two numbers that multiply to ac and add to b.

2.1 Basic Quadratic Trinomials

Step 1: Multiply a and c.

Step 2: Find two numbers that multiply to ac and sum to b.

Step 3: Rewrite the middle term using these two numbers and factor by grouping.

Example:

Factor $x^2 + 5x + 6$.

- $a=1, b=5, c=6$.
- $ac=6$.
- Factors of 6: 1 & 6, 2 & 3.
- Sum: 1+6=7, 2+3=5 □ 2 and 3 are the numbers.

Rewrite:

\[

$$x^2 + 2x + 3x + 6$$

Factor by grouping:

$$x(x + 2) + 3(x + 2) = (x + 2)(x + 3)$$

2.2 When $a \neq 1$: Factoring $ax^2 + bx + c$

Method:

- Multiply a and c.
- Find two numbers that multiply to ac and add to b.
- Rewrite the middle term accordingly.
- Factor by grouping.

Example:

Factor $2x^2 + 7x + 3$.

- $ac=6$.
- Factors of 6: 1 & 6, 2 & 3.
- Sum: 1+6=7, 2+3=5.
- The pair that sums to 7: 1 and 6.

Rewrite:

$$
2x^2 + x + 6x + 3
$$

Factor:

$$
x(2x + 1) + 3(2x + 1) = (2x + 1)(x + 3)
$$

---

3. Recognising Special Products

Certain algebraic expressions follow specific patterns that can be directly factored.

3.1 Difference of Squares

Expression:

$$
a^2 - b^2 = (a - b)(a + b)
$$

Example:

Factor $x^2 - 9$.

$$
x^2 - 3^2 = (x - 3)(x + 3)
$$

3.2 Perfect Square Trinomials

Expression of the form:

$$
a^2 \pm 2ab + b^2 = (a \pm b)^2
$$

Example:

Factor $x^2 + 6x + 9$:

$$
(x + 3)^2
$$

3.3 Sum of Cubes and Difference of Cubes

- Sum of cubes:

$$

$$
a^3 + b^3 = (a + b)(a^2 - ab + b^2)
$$

- Difference of cubes:

$$
a^3 - b^3 = (a - b)(a^2 + ab + b^2)
$$

Example:

Factor $x^3 - 8$:

$$
x^3 - 2^3 = (x - 2)(x^2 + 2x + 4)
$$

Practical Strategies for Effective Factorisation

- Always look for the GCF first.
- Check for special patterns before attempting more complex methods.
- Use substitution if the expression involves higher degree polynomials.
- Practice with a variety of examples to become familiar with different forms.

Working Through a Complex Example

Let's consider a comprehensive example to illustrate the entire process:

Problem:

Factor $4x^3 + 12x^2 - 8x$.

Step 1: Find the GCF:

- Coefficients: 4, 12, 8 → GCF is 4.
- Variables: x^3, x^2, x → GCF is x.

- GCF of entire expression: $4x$.

Step 2: Factor out GCF:

$$
4x(x^2 + 3x - 2)
$$

Step 3: Factor the quadratic $x^2 + 3x - 2$:

- $a=1, b=3, c=-2$.
- $ac = -2$.
- Factors of -2: (1, -2), (-1, 2).
- Sum: 1 + (-2) = -1, -1 + 2 = 1, so none sum to 3. Let's revisit.

Actually, the pair that multiplies to -2 and sums to 3:

- (1, -2): sum = -1
- (-1, 2): sum = 1

No pair directly sums to 3. Therefore, quadratic cannot be factored over integers.

Solution:

Use quadratic formula:

$$
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{9 - 4 \times 1 \times (-2)}}{2} = \frac{-3 \pm \sqrt{9 + 8}}{2} = \frac{-3 \pm \sqrt{17}}{2}
$$

Since roots are irrational, the quadratic is not factorable over integers, so the complete factorisation over rationals is:

$$
4x \left(x^2 + 3x - 2 \right)
$$

which cannot be simplified further over integers.

Common Mistakes and How to Avoid Them

- Neglecting to factor out the GCF: Always check for common factors first.
- Misidentifying patterns: Recognise special products to save time.
- Forgetting to check the quadratic discriminant when roots are irrational.
- Attempting to factor without rewriting: Proper rewriting makes factorisation clearer.

Final Tips for Mastery

- Practice consistently with different types of expressions.

- Memorise key identities and formulas for special products.

- Break complex expressions into simpler parts.

- Use graphical understanding to confirm roots and factors.

- Develop a systematic approach to avoid missing steps.

Conclusion

Factorisation is an essential skill in algebra that, once mastered, significantly simplifies the process of solving equations and analyzing algebraic expressions. By following systematic steps—starting with the greatest

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