Pls HelpThe Equation Of A Line Parallel To 6x 2y + 10 = 0 And Passes Through The Point(-2,7).

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Introduction

Understanding the equations of lines and their properties is fundamental in coordinate geometry, a key branch of mathematics that deals with points, lines, and shapes in a coordinate plane. When working with lines, one of the most common tasks is to find the equation of a line that is parallel or perpendicular to a given line and passes through a specific point. In this guide, we will explore how to determine the equation of a line parallel to $6x + 2y + 10 = 0$ that passes through the point (-2, 7).

This problem involves concepts such as the slope of a line, the point-slope form of a line's equation, and the relationships between parallel lines. By understanding these concepts step-by-step, you will be able to solve similar problems with confidence.

Understanding the Given Line Equation

Before we find the line parallel to the given one, let's analyze the original line:

Standard form of the line: $6x + 2y + 10 = 0$

To work effectively, it's often easier to express the equation in slope-intercept form, which is:

$$ y = mx + c $$

where:

- m is the slope,
- c is the y-intercept.

Converting to slope-intercept form

Starting with:

$$ 6x + 2y + 10 = 0 $$

Subtract 6x and 10 from both sides:

$$ 2y = -6x - 10 $$

Divide both sides by 2:

$$ y = -3x - 5 $$

Slope of the given line

From the slope-intercept form, the slope m is:

$$ m = -3 $$

Concept of Parallel Lines

In coordinate geometry, parallel lines have the following key property:

- They share the same slope but have different y-intercepts.

Thus, for a line parallel to the given line:

$$ y = -3x + c $$

the slope will remain -3, but the y-intercept c will change depending on the point through which the new line passes.

Finding the Equation of the Parallel Line

Step 1: Identify the slope

From the previous section, the slope of the original line is:

$$ m = -3 $$

Since the new line must be parallel, it will have the same slope:

$$ m_{new} = -3 $$

Step 2: Use the point-slope form

The point-slope form of the line's equation is:

$$ y - y_1 = m (x - x_1) $$

where:

- (x_1, y_1) is the point the line passes through,

- m is the slope.

Given point:

$$(-2, 7)$$

and slope:

$$-3$$

Substitute into the point-slope form:

$$y - 7 = -3 (x + 2)$$

Step 3: Simplify the equation

Distribute:

$$y - 7 = -3x - 6$$

Add 7 to both sides:

$$y = -3x - 6 + 7$$

$$y = -3x + 1$$

Final equation of the parallel line:

$$\boxed{ y = -3x + 1 }$$

Verifying the Solution

Let's verify the solution by checking whether the point (-2, 7) lies on the derived line:

Substitute $x = -2$:

$$y = -3 \times (-2) + 1 = 6 + 1 = 7$$

which matches the y-coordinate of the point, confirming the correctness of the equation.

Additional Insights and Related Concepts

1. General Form of the Line

The equation:

$$y = -3x + 1$$

can be rewritten into the standard form:

$$3x + y = 1$$

which might be useful in certain contexts, such as when dealing with multiple lines or inequalities.

2. Distance Between Parallel Lines

If you are interested in the distance between two parallel lines with equations:

$$y = -3x + c_1$$

$$ y = -3x + c_2 $$

the distance d between them is given by:

$$ d = \frac{|c_2 - c_1|}{\sqrt{1 + m^2}} $$

where $m = -3$ in our case.

For example, if you need to find how far the original line is from the new parallel line, you can use this formula with the respective intercepts.

3. Applications of Parallel Lines

Understanding how to find and work with parallel lines is essential in various fields:
- Architecture and engineering for designing structures.
- Computer graphics for rendering and modeling.
- Navigation and mapping.
- Robotics for path planning.

Common Mistakes and Tips

- Confusing slope with intercept: Always convert lines to slope-intercept form to identify the slope easily.
- Forgetting the point substitution: When passing through a specific point, ensure to substitute both x and y correctly.
- Mixing up equations: Ensure to keep the signs consistent, especially when dealing with negative slopes or points with negative coordinates.
- Verifying the solution: Always verify your derived line by substituting the given point.

Summary

To find the equation of a line parallel to $6x + 2y + 10 = 0$ passing through $(-2,7)$:

1. Convert the given line to slope-intercept form:

$$ y = -3x - 5 $$

2. Note the slope:

$$ m = -3 $$

3. Use the point-slope form with the point $(-2,7)$:

$$ y - 7 = -3 (x + 2) $$

4. Simplify to get the slope-intercept form:

$$ y = -3x + 1 $$

5. Verify by substituting the point:

$$ y = -3(-2) + 1 = 7 $$

which confirms the correctness.

This process demonstrates how to determine the equation of a parallel line passing through a specific point, a vital skill in coordinate geometry.

Additional Resources

- Coordinate Geometry Tutorials: For beginners to advanced topics.
- Graphing Tools: Use graph paper or online graphing calculators to visualize lines.
- Practice Problems:
- Find the equation of a line parallel to \(y = 2x - 4 \) passing through \((3, 5) \).
- Determine the distance between two parallel lines.

Conclusion

Mastering the concept of parallel lines and their equations is essential for solving a wide array of problems in mathematics and applied sciences. By understanding how to convert lines into slope-intercept form, using point-slope form, and verifying solutions, you can confidently handle similar questions involving lines in the coordinate plane. Remember, the key to success lies in careful calculation, verification, and practice.

If you ever encounter a problem asking for the equation of a line parallel to a given line passing through a specific point, follow the outlined steps, and you'll find the solution efficiently and accurately.

Frequently Asked Questions

How do I find the equation of a line parallel to $6x - 2y + 10 = 0$?

First, rewrite the given equation in slope-intercept form to identify its slope. Then, use that slope for your new line and the given point to find its equation.

What is the slope of the line $6x - 2y + 10 = 0$?

Rearranging the equation to $y = mx + b$ form gives $y = 3x + 5$, so the slope is 3.

How do I write the equation of a line parallel to the given line passing through (-2, 7)?

Use the point-slope form $y - y_1 = m(x - x_1)$ with $m = 3$ and point (-2, 7).

What is the point-slope form of the line passing through (-2, 7) with slope 3?

The equation is $y - 7 = 3(x + 2)$.

How do I convert $y - 7 = 3(x + 2)$ to slope-intercept form?

Distribute and simplify: $y - 7 = 3x + 6$, then add 7 to both sides: $y = 3x + 13$.

What is the final equation of the line parallel to $6x - 2y + 10 = 0$ passing through (-2, 7)?

The equation in slope-intercept form is $y = 3x + 13$.

Can I verify if the point (-2, 7) lies on the new line $y = 3x + 13$?

Yes, substituting $x = -2$: $y = 3(-2) + 13 = -6 + 13 = 7$, which matches the y-coordinate, confirming the point lies on the line.

Is the new line parallel to the original line?

Yes, because both lines have the same slope of 3.

What are the key steps to find a parallel line passing through a specific point?

Identify the slope of the given line, use the point-slope form with the slope and the point, then convert to your preferred form (slope-intercept or standard).

Additional Resources

Pls Help The Equation Of A Line Parallel To 6x - 2y + 10 = 0 And Passes Through The Point (-2,7)

Introduction

Mathematics, particularly coordinate geometry, often presents learners with complex problems that challenge their understanding of fundamental concepts such as lines, slopes, and equations. Among these, problems involving the determination of lines parallel to a given line and passing through a specific point are common, yet they require a clear grasp of the underlying principles. The problem statement: "Pls Help The Equation Of A Line Parallel To 6x - 2y + 10 = 0 And Passes Through The Point (-2,7)", exemplifies such a scenario.

This article aims to dissect this problem thoroughly, exploring the mathematical principles involved, elucidating the step-by-step solution process, and providing insights into related concepts. Our goal is to offer a comprehensive review suitable for students, educators, and enthusiasts seeking to deepen their understanding of line equations in coordinate geometry.

Understanding the Given Line Equation

Standard Form of a Line

The given line equation is:

$$6x - 2y + 10 = 0$$

This is a linear equation in the standard form $Ax + By + C = 0$, where:

- $A = 6$
- $B = -2$
- $C = 10$

Converting to Slope-Intercept Form

To understand the slope of this line, it’s often helpful to convert it into the slope-intercept form:

$$y = mx + c$$

Starting with:

$$6x - 2y + 10 = 0$$

Subtract $6x$ and 10 from both sides:

$$-2y = -6x - 10$$

Divide through by -2 to solve for y:

$$y = 3x + 5$$

Key insight: The slope m of the given line is 3.

Determining the Parallel Line Equation

Concept of Parallel Lines

Two lines are parallel if and only if they have equal slopes. Therefore, any line parallel to the given line must also have a slope of 3.

Equation of the Parallel Line

Using the point-slope form:

$$ y - y_1 = m (x - x_1) $$

where:

- $(x_1, y_1) = (-2, 7)$
- $m = 3$

Substituting:

$$ y - 7 = 3(x + 2) $$

Expanding:

$$ y - 7 = 3x + 6 $$

Adding 7 to both sides:

$$ y = 3x + 13 $$

This is the equation of the line passing through the point (-2,7) and parallel to the original line.

Expressing the Final Equation

The final equation can be written in various forms:

- Slope-intercept form: $y = 3x + 13$
- Standard form: $3x - y + 13 = 0$

Both are correct representations. For clarity and standardization, the standard form is often preferred in algebraic contexts.

Verifying the Solution

To ensure accuracy, verify whether the point (-2, 7) satisfies the final equation:

$$3(-2) - 7 + 13 = -6 - 7 + 13 = 0$$

Since the left side equals zero, the point lies on the line, confirming the correctness.

Deep Dive: The Significance of Parallel Lines and Their Equations

Mathematical Importance

Parallel lines retain a constant distance from each other and share the same slope but differ in their y-intercept. This property is fundamental in various applications:

- Design and architecture: ensuring consistent spacing.
- Navigation and mapping: plotting routes that maintain a consistent heading.
- Physics: understanding trajectories and fields.

Real-World Applications

Understanding how to find equations of lines parallel to given references is crucial in engineering, urban planning, and even computer graphics — where precise alignment and spacing are vital.

Additional Considerations and Related Concepts

1. Slope-Intercept vs. Standard Form

While the slope-intercept form is often more intuitive, the standard form is preferred in certain contexts, such as solving systems of equations or applying the elimination method.

2. The Role of Point-Slope Form

The point-slope form offers a straightforward way to write the equation of a line when a point and slope are known, simplifying many problems.

3. Techniques for Multiple Parallel Lines

Given one line, constructing multiple parallel lines involves varying the y-intercept while maintaining the same slope. For example, lines passing through different points with slope 3.

Common Mistakes and Misconceptions

- Confusing slope with y-intercept: Remember, the slope determines the line's tilt; the y-intercept is where the line crosses the y-axis.
- Not verifying the point: Always substitute the given point into your equation to confirm correctness.
- Misinterpreting the standard form: Ensure proper rearrangement when converting between forms.

Summary of Steps to Find the Equation

1. Identify the slope of the given line: Convert to slope-intercept form.

- Original: $6x - 2y + 10 = 0$
- Converted: $y = 3x + 5$, so $m = 3$.

2. Use the point-slope form with the given point and slope:

- $(x_1, y_1) = (-2, 7)$, $m = 3$

3. Write the equation:

- $y - 7 = 3(x + 2)$

4. Simplify to desired form:

- $y = 3x + 13$ (slope-intercept)
- $3x - y + 13 = 0$ (standard form)

5. Verify the solution:

- Substitute the point into the final form to confirm.

Final Remarks

The process of determining a line parallel to a given line and passing through a specific point illustrates core principles of coordinate geometry, including understanding line slopes, equations in various forms, and the importance of verification. Mastery of these concepts enables solving a broad class of problems with confidence.

This detailed exploration not only provides the specific answer to the posed problem but also reinforces foundational skills essential for advanced mathematical reasoning and real-world applications.

References and Further Reading

- Coordinate Geometry: Understanding Lines and Slopes
- Algebra Textbooks: Equations of Lines and Their Applications
- Online Resources: Khan Academy's Tutorials on Line Equations
- Mathematical Software: Desmos and GeoGebra for Visualizing Lines

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In conclusion, the equation of the line parallel to $6x - 2y + 10 = 0$ passing through (-2,7) is $y = 3x + 13$ or in standard form, $3x - y + 13 = 0$. This solution exemplifies the elegance and logical structure of coordinate geometry, demonstrating how fundamental concepts translate into practical problem-solving strategies.

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