Q33) Given The Function F (x, Y) = Y Sin X + E* Cos Y, Determine A) Fx B) Fy C) Fax D) Fug E) Fry

Q33) Given The Function F (x, Y) = Y Sin X + E Cos Y, Determine A) Fx B) Fy C) Fax D) Fug E) Fry

Introduction

Understanding the derivatives of multivariable functions is a fundamental aspect of calculus, especially in fields like physics, engineering, and economics. When working with functions of two variables, such as F(x, y) = y sin x + E cos y, it’s essential to compute various derivatives to analyze the behavior of the function, find tangent planes, optimize functions, or study rates of change. This article provides a comprehensive guide to calculating the partial derivatives Fx and Fy, as well as the mixed derivatives Fxy and F yx, for the given function.

Understanding the Function F(x, y)

The function is given as:

F(x, y) = y sin x + E cos y

Here:
- **y sin x** couples the variables y and x, involving a product of y and the sine of x.
- **E cos y** is a term involving a constant E multiplied by the cosine of y.

Before proceeding with derivatives, note that:
- E is treated as a constant (possibly Euler's number, but unless specified, consider it a constant parameter).
- sin x and cos y are standard trigonometric functions.
- The variables x and y are independent variables.

Calculating the First-Order Partial Derivatives

A) Derivative of F with respect to x (Fx)

To find Fx, treat y as a constant and differentiate F(x, y) with respect to x.

Given:
F(x, y) = y sin x + E cos y

Since y is constant:
- The derivative of y sin x with respect to x is y cos x.
- The derivative of E cos y with respect to x is zero because it does not depend on x.

Therefore:
Fx = y cos x

B) Derivative of F with respect to y (Fy)

To find Fy, treat x as constant and differentiate F(x, y) with respect to y.

Given:
F(x, y) = y sin x + E cos y

- The derivative of y sin x with respect to y is sin x (since sin x acts as a constant multiplier).
- The derivative of E cos y with respect to y is -E sin y (by chain rule).

Thus:
Fy = sin x - E sin y

Calculating the Second-Order and Mixed Derivatives

C) The mixed partial derivative Fxy (or Fax)

This involves differentiating Fx with respect to y.

Recall:
Fx = y cos x

Treat x as constant and differentiate with respect to y:
- The derivative of y cos x with respect to y is cos x (since cos x is constant with respect to y).

Therefore:
Fxy = cos x

D) The mixed partial derivative Fy (or Fry)

This involves differentiating Fy with respect to x.

Recall:
Fy = sin x - E sin y

Treat y as constant:
- The derivative of sin x with respect to x is cos x.
- The derivative of -E sin y with respect to x is zero (since sin y is constant in terms of x).

Hence:

Fyx = cos x

Note: In most functions with continuous second derivatives, Fxy = Fyx (Clairaut’s theorem).

Summary of Derivatives

Derivative	Expression	Description
Fx	y cos x	Partial derivative with respect to x
Fy	sin x - E sin y	Partial derivative with respect to y
Fxy (or Fax)	cos x	Mixed second derivative (x then y)
Fyx (or Fry)	cos x	Mixed second derivative (y then x)

Applications and Significance of These Derivatives

1. Analyzing the Behavior of the Function

The derivatives help understand how the function F(x, y) changes as x or y varies. For example, Fx indicates how F changes when x increases slightly, holding y constant.

2. Finding Critical Points and Optimization

Setting Fx and Fy to zero can locate stationary points:
- Fx = 0 → y cos x = 0
- Fy = 0 → sin x = E sin y

Solving these equations can help find potential maxima, minima, or saddle points.

3. Tangent Planes and Linear Approximations

Using the partial derivatives, one can construct tangent planes at points, which are useful for approximations and understanding local behavior:

$$ z \approx F(x_0, y_0) + Fx(x_0, y_0)(x - x_0) + Fy(x_0, y_0)(y - y_0) $$

4. Differential Equations and Further Analysis

Second derivatives like Fxy and Fyx are essential in the study of the curvature and in applying second derivative tests for functions of two variables.

Special Considerations

Constant E

- If E is Euler’s number (~2.718), then it remains constant throughout differentiation.
- If E is a parameter, the derivatives involving E treat it as a constant.

Differentiability and Continuity

- Since sine and cosine are continuous and differentiable everywhere, and E is a constant, the function F(x, y) is smooth, ensuring the validity of mixed derivatives and Clairaut's theorem.

Conclusion

The derivatives of the function F(x, y) = y sin x + E cos y provide insight into its behavior in the xy-plane. The first-order partial derivatives, Fx and Fy, describe the slope of the function in the x and y directions, respectively. The mixed derivatives, Fxy and Fyx, reveal the interaction between the variables and are instrumental in analyzing the function's curvature and in optimization tasks. Mastery of these derivatives is crucial for advanced calculus applications, including multivariable optimization, differential equations, and modeling real-world phenomena.

Additional Resources

- "Calculus: Early Transcendentals" by James Stewart
- Khan Academy’s Multivariable Calculus Course
- Online derivative calculators for practice and verification

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This comprehensive guide should serve as a valuable resource for students and professionals seeking to understand the derivatives of functions involving multiple variables, particularly for the given function involving sine and cosine components.

Frequently Asked Questions

What is the function F(x, y) given in the problem?

The function is $F(x, y) = y \sin x + e^x \cos y$.

How do you find the partial derivative of F with respect to x, denoted as Fx?

To find Fx, differentiate $F(x, y)$ with respect to x, treating y as a constant: $F_x = y \cos x + e^x \cos y$.

What is the partial derivative of F with respect to y, denoted as Fy?

To find Fy, differentiate $F(x, y)$ with respect to y, treating x as a constant: $F_y = \sin x - e^x \sin y$.

How is the partial derivative of Fx with respect to x, denoted as Fxx, calculated?

Fxx is obtained by differentiating $F_x = y \cos x + e^x \cos y$ with respect to x: $F_{xx} = - y \sin x + e^x \cos y$.

What is the partial derivative of Fy with respect to y, denoted as Fyy?

Fyy is found by differentiating $F_y = \sin x - e^x \sin y$ with respect to y: $F_{yy} = - e^x \cos y$.

How do you compute the mixed partial derivative Fxy?

Fxy is the derivative of $F_x = y \cos x + e^x \cos y$ with respect to y: $F_{xy} = \cos x - e^x \sin y$.

How do you compute the mixed partial derivative Fyx?

Fyx is the derivative of $F_y = \sin x - e^x \sin y$ with respect to x: $F_{yx} = \cos x - e^x \sin y$.

Are the mixed partial derivatives Fxy and Fyx equal for this function?

Yes, Fxy and Fyx are equal: both are $\cos x - e^x \sin y$, consistent with Clairaut's theorem on mixed partial derivatives.

What is the value of Fx at the point (x, y)?

Fx at (x, y) is $y \cos x + e^x \cos y$.

What is the value of Fy at the point (x, y)?

Fy at (x, y) is sin x - e^x sin y.

Additional Resources

Given the Function F(x, y) = y sin x + e cos y: A Comprehensive Investigation of Partial Derivatives and Their Applications

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Introduction

In the realm of multivariable calculus, understanding the behavior of functions with respect to their variables is foundational. Among these functions, the ones involving trigonometric and exponential components are particularly rich in structure and applications. In this detailed exploration, we examine the function:

$$F(x, y) = y \sin x + e \cos y$$

and delve into the computation and implications of its partial derivatives:

- F_x (partial derivative with respect to x)
- F_y (partial derivative with respect to y)
- F_{xx} (second partial derivative with respect to x)
- F_{xy} (mixed second partial derivative)
- F_{yy} (second partial derivative with respect to y)

This analysis aims to clarify the mathematical structure, interpret the derivatives' significance, and explore potential applications. The discussion is structured into in-depth sections, ensuring a comprehensive understanding suitable for researchers, students, and practitioners.

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1. The Fundamental Structure of the Function $F(x, y)$

The function combines linear, trigonometric, and exponential terms:

$$F(x, y) = y \sin x + e \cos y$$

where:
- $y \sin x$: introduces a coupling between y and x with a linear dependence on y and oscillatory behavior via $\sin x$.
- $e \cos y$: involves the exponential constant e multiplied by a cosine function, emphasizing the oscillatory nature in y.

This form is prevalent in various physical models, such as wave phenomena, oscillatory systems, and certain economic models where variables interact through sinusoidal and exponential relationships.

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2. Computing the First-Order Partial Derivatives

Understanding the behavior of F requires first derivatives with respect to each variable, which reveal the slopes, rates of change, and directional sensitivities.

2.1. Partial Derivative with Respect to x: F_x

Applying the rules of differentiation:

- $y \sin x$: differentiates with respect to x, treating y as constant, yielding $y \cos x$.
- $e \cos y$: independent of x, derivative is zero.

Result:

$$
F_x(x, y) = y \cos x
$$

Interpretation:
- The rate of change of F with respect to x depends linearly on y and oscillates with $\cos x$.
- When $y = 0$, the function is insensitive to x; when y is large, the effect is amplified.

2.2. Partial Derivative with Respect to y: F_y

Differentiating:

- $y \sin x$: treats x as constant, derivative is $\sin x$.
- $e \cos y$: derivative with respect to y is $-e \sin y$.

Result:

$$
F_y(x, y) = \sin x - e \sin y
$$

Interpretation:
- The sensitivity of F to changes in y combines a constant $\sin x$ term with an oscillatory $-e \sin y$ term.
- The balance between these terms determines local maxima, minima, and saddle points.

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3. Computing the Second-Order Partial Derivatives

Second derivatives provide information about the curvature of the surface $z = F(x, y)$. They are essential in classifying critical points and understanding concavity.

3.1. F_{xx}: Second partial with respect to x

Differentiate $F_x = y \cos x$:

$$
F_{xx} = \frac{\partial}{\partial x} (y \cos x) = y (-\sin x) = - y \sin x
$$

Implication:
- The concavity in the x-direction varies with both y and x.
- When $y > 0$, the curvature is negative where $\sin x > 0$, indicating concave down regions, and vice versa.

3.2. F_{yy}: Second partial with respect to y

Differentiate $F_y = \sin x - e \sin y$:

$$
F_{yy} = \frac{\partial}{\partial y} (\sin x - e \sin y) = 0 - e \cos y = - e \cos y
$$

Implication:
- The curvature in the y-direction oscillates with y due to $\cos y$, scaled by $-e$.
- The sign of F_{yy} alternates, indicating regions of convexity and concavity.

3.3. F_{xy} and F_{yx}: Mixed second derivatives

The mixed derivatives are:

$$
F_{xy} = \frac{\partial}{\partial y} (F_x) = \frac{\partial}{\partial y} (y \cos x) = \cos x
$$

Since mixed partials are equal if F is sufficiently smooth:

$$
F_{yx} = \frac{\partial}{\partial x} (F_y) = \frac{\partial}{\partial x} (\sin x - e \sin y) = \cos x
$$

Implication:
- Both mixed derivatives are equal, confirming the symmetry of second derivatives.
- The mixed partial $F_{xy} = \cos x$ indicates how the slope in x changes as y varies.

---

4. Interpretation and Applications of Derivatives

Having computed the derivatives, their significance extends into various analyses:

4.1. Critical Points and Stationary Points

Critical points occur where the first derivatives vanish:

$$
F_x = y \cos x = 0
$$
$$
F_y = \sin x - e \sin y = 0
$$

These conditions lead to the system:

$$
\begin{cases}
y \cos x = 0 \\
\sin x = e \sin y
\end{cases}
$$

Analysis of solutions involves considering cases where $y = 0$ or $\cos x = 0$:

- When $y=0$, from $F_y = 0$:

$$
\sin x = e \sin 0 = 0
$$

which implies $\sin x = 0$, so $x = n\pi$:

- Critical points at $(x, y) = (n\pi, 0)$.

Alternatively, when $\cos x=0$:

$$
x = \frac{\pi}{2} + n\pi
$$
and from $F_y = 0$:

$$
\sin x = e \sin y
$$

At these points, $\sin x = \pm 1$, leading to:

$$
\pm 1 = e \sin y \Rightarrow \sin y = \pm \frac{1}{e}
$$

which is feasible since $|1/e| < 1$. Therefore, the critical points are:

$$
\left(\frac{\pi}{2} + n\pi, \arcsin \left(\pm \frac{1}{e}\right)\right)
$$

---

5. Hessian and Nature of Critical Points

The second derivative test involves the Hessian determinant:

$$
D = F_{xx} F_{yy} - (F_{xy})^2
$$

At the critical points:

$$
F_{xx} = - y \sin x
$$
$$
F_{yy} = - e \cos y
$$
$$
F_{xy} = \cos x
$$

Evaluation at specific critical points enables classification into maxima, minima, or saddle points.

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6. Practical Applications and Broader Implications

The detailed understanding of these derivatives reveals insights into the behavior of the function in physical systems:

- Wave mechanics: Oscillatory behavior modeled by sine and cosine terms.
- Optimization problems: Identifying maxima and minima.
- Surface analysis: Curvature assessments from second derivatives.
- Economic modeling: Interactions between variables with oscillatory or exponential influences.

The interplay between the linear, oscillatory, and exponential components makes $F(x, y)$ a versatile prototype for complex systems.

Conclusion

This comprehensive investigation of $F(x, y) = y \sin x + e \cos y$ underscores the importance of partial derivatives in understanding multivariable functions. From first derivatives revealing sensitivities to second derivatives indicating curvature, each component plays a crucial role. The critical points and their classification provide a pathway to analyzing stability, maxima, minima, and saddle points.

Furthermore, the function exemplifies the rich structure encountered in mathematical modeling, where combined effects of trigonometric and exponential functions generate intricate behaviors. Mastery of these derivatives not only enhances theoretical understanding but also equips practitioners with tools to

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