Rewrite The Expression 4+[tex]\sqrt{16-(4)(5)}[/tex]/2 As A Complex Number In Standard Form,a+bi .

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Understanding how to manipulate and rewrite mathematical expressions is essential for students and professionals working in fields such as mathematics, engineering, and computer science. One common task is transforming expressions involving radicals and real numbers into their complex number form, especially when dealing with square roots of negative numbers. This article provides a comprehensive, step-by-step guide to rewriting the given expression into the standard form $a + bi$, where a and b are real numbers and i is the imaginary unit.

Breaking Down the Given Expression

Before rewriting the expression into complex form, it's crucial to understand its components and evaluate it step by step.

Expression Overview

The given expression is:

$$ 4 + \frac{\sqrt{16 - (4)(5)}}{2} $$

Our goal is to simplify this expression and, if necessary, express it as a complex number $a + bi$.

Step 1: Simplify Inside the Square Root

The first step is to evaluate the expression under the radical:

$$ 16 - (4)(5) $$

Calculate the product:

$$ (4)(5) = 20 $$

Subtract:

$$ 16 - 20 = -4 $$

So, the radical simplifies to:

$$ \sqrt{-4} $$

Step 2: Understand the Square Root of a Negative Number

The square root of a negative number involves imaginary numbers. Recall:

$$ \sqrt{-x} = i \sqrt{x} \quad \text{for } x > 0 $$

Applying this to $\sqrt{-4}$:

$$ \sqrt{-4} = i \sqrt{4} = i \times 2 = 2i $$

Step 3: Rewrite the Expression with the Simplified Radical

Substitute $\sqrt{-4}$ with $2i$:

$$ 4 + \frac{2i}{2} $$

Simplify the fraction:

$$ \frac{2i}{2} = i $$

The entire expression now simplifies to:

$$ 4 + i $$

Step 4: Express the Result in Standard Form $a + bi$

The simplified expression:

$$4 + i$$

is already in the standard form, where:

- $a = 4$
- $b = 1$

Thus, the expression rewritten as a complex number is:

$$\boxed{4 + 1i} \quad \text{or simply} \quad 4 + i$$

Additional Considerations in Complex Number Representation

Understanding how to handle expressions involving radicals and negative numbers is fundamental in complex analysis. Here are some key points:

Handling Radicals of Negative Numbers

- The square root of a negative number always involves the imaginary unit i.
- $\sqrt{-x} = i \sqrt{x}$ for $x > 0$.
- This property allows for converting many radical expressions into complex form.

Expressing Results in Standard Form

- The standard form of a complex number is $a + bi$.
- The real part a is the coefficient of the real component.
- The imaginary part b is the coefficient of i.

Complex Number Operations

- Addition, subtraction, multiplication, and division follow specific rules, similar to algebraic operations.
- Remember that $i^2 = -1$.

Practical Applications of Converting Expressions to Complex Form

Translating expressions into complex form has many practical uses:

- Solving quadratic equations with negative discriminants.
- Analyzing electrical circuits involving alternating current (AC).
- Signal processing and control systems.
- Quantum mechanics and wave functions.

Summary of Steps to Rewrite the Expression as a Complex Number

To recap, here's a quick summary of the process:

1. Identify the radical: Recognize if the expression under the square root is negative.
2. Simplify inside the radical: Calculate the value under the radical.
3. Handle negative radicals: Convert $\sqrt{-x}$ to $i \sqrt{x}$.
4. Simplify the entire expression: Perform algebraic operations to reach the simplest form.
5. Express in standard form: Write the result as $a + bi$.

Additional Examples

Understanding the process is easier with more examples:

Example 1:

Rewrite $2 + \frac{\sqrt{9 - 16}}{3}$ as a complex number.

- Inside radical: $9 - 16 = -7$
- $\sqrt{-7} = i \sqrt{7}$
- Expression: $2 + \frac{i \sqrt{7}}{3}$
- Final form: $a + bi = 2 + \frac{\sqrt{7}}{3} i$

Example 2:

Express $\sqrt{-25}$ in standard form.

- $\sqrt{-25} = i \sqrt{25} = 5i$

Conclusion

Transforming radical expressions involving negative numbers into complex form is a fundamental skill in advanced mathematics. The process involves recognizing when the radical contains a negative number, applying the property $\sqrt{-x} = i \sqrt{x}$, and simplifying accordingly. In our initial problem, we determined that the radical simplifies to $2i$, leading us to the final expression in standard form: $4 + i$.

Mastering these techniques enhances your ability to analyze and solve complex equations, a skill invaluable in many scientific and engineering disciplines. Whether you're working with quadratic equations with negative discriminants or analyzing complex signals, understanding how to rewrite expressions in the form $a + bi$ is essential.

Meta Description: Learn how to rewrite the expression $4 + \frac{\sqrt{16 - (4)(5)}}{2}$ as a complex number in standard form $a + bi$. Step-by-step guide with detailed explanations.

Frequently Asked Questions

How do you start rewriting the expression $4 + [\sqrt{(16 - (4)(5))}]/2$ as a complex number in standard form?

Begin by simplifying inside the square root: $16 - (4)(5) = 16 - 20 = -4$. Then, take the square root of -4, which involves imaginary numbers, and proceed to simplify the entire expression step by step.

What is the value of the square root $\sqrt{(16 - (4)(5))}$ in the given expression?

Since $16 - 20 = -4$, $\sqrt{(-4)} = 2i$, because the square root of a negative number introduces the imaginary unit i.

How do you simplify the expression $4 + [\sqrt{(16 - (4)(5))}]/2$ after computing the square root?

Replace $\sqrt{(16 - (4)(5))}$ with $2i$: $4 + (2i)/2$. Then, simplify $(2i)/2$ to i, resulting in $4 + i$.

What is the standard form a + bi for the given expression?

The expression simplifies to 4 + i, where a = 4 and b = 1.

Why does the square root of a negative number involve the imaginary unit i?

Because the square root of a negative number cannot be expressed as a real number, it is represented using the imaginary unit i, where $i^2 = -1$.

Can you show the step-by-step process to convert 4 + [√(16 - 20)]/2 into standard complex form?

Yes. First, compute inside the square root: 16 - 20 = -4. Then, √(-4) = 2i. Next, divide by 2: (2i)/2 = i. Finally, add 4: 4 + i.

Is the expression 4 + i in the form a + bi? If so, what are a and b?

Yes, 4 + i is in the form a + bi, with a = 4 and b = 1.

What is the importance of expressing complex numbers in standard form?

Expressing complex numbers in standard form a + bi provides a clear, consistent way to understand and perform operations like addition, subtraction, multiplication, and division involving complex numbers.

Can the original expression be simplified further, or is 4 + i the final form?

The simplified form 4 + i is the final form in standard complex number notation, as the expression has been fully simplified.

How does understanding complex numbers help in solving real-world mathematical problems?

Complex numbers are essential in various fields such as engineering, physics, and signal processing, as they help analyze and solve problems involving oscillations, waves, and systems that cannot be described using only real numbers.

Additional Resources

Rewrite The Expression 4+\(\displaystyle \frac{\sqrt{16-(4)(5)}}{2}\) As A Complex Number In Standard Form, a+bi

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Introduction

Mathematics often presents us with expressions that, at first glance, seem straightforward but require careful analysis to understand their true nature. One such expression is:

$$ 4 + \frac{\sqrt{16 - (4)(5)}}{2} $$

While it appears simple, rewriting it into the standard complex form $a + bi$ involves understanding the nature of the square root—whether it yields a real or imaginary number—and performing precise algebraic manipulations. This process not only deepens our comprehension of complex numbers but also highlights the importance of systematic problem-solving in mathematics. In this article, we will explore step-by-step how to convert this expression into the standard form, delving into the concepts of radicals, real versus imaginary parts, and the fundamental principles underlying complex numbers.

Understanding the Expression: Breaking Down the Components

Before rewriting the expression into the complex form, it is essential to analyze its components thoroughly.

The Expression in Detail:

$$ 4 + \frac{\sqrt{16 - (4)(5)}}{2} $$

- The number 4 is a constant term, purely real.
- The radical expression $\sqrt{16 - (4)(5)}$ involves a subtraction inside the radical.
- The entire radical is divided by 2.

Step 1: Simplify the radicand

The radicand (the expression inside the square root):

$$ 16 - (4)(5) $$

Calculate the product:

$$ 4 \times 5 = 20 $$

Now, substitute back:

$$ 16 - 20 = -4 $$

Thus, the expression simplifies to:

$$ 4 + \frac{\sqrt{-4}}{2} $$

Handling the Square Root of a Negative Number

The key step in rewriting the expression involves understanding how to interpret $\sqrt{-4}$.

The Nature of $\sqrt{-4}$

- The square root of a negative number is not a real number; instead, it involves the imaginary unit i, where:

$$ i = \sqrt{-1} $$

- Therefore,

$$ \sqrt{-4} = \sqrt{4 \times -1} = \sqrt{4} \times \sqrt{-1} = 2i $$

Implications for the Expression

Replacing $\sqrt{-4}$ with $2i$, the expression becomes:

$$ 4 + \frac{2i}{2} $$

which simplifies further.

Simplification to Standard Form

Now, perform the division:

$$ \frac{2i}{2} = i $$

Thus, the entire expression reduces to:

$$ 4 + i $$

Final Form: $a + bi$

In the standard form of a complex number $a + bi$:

- The real part a is 4.
- The imaginary part b is 1 (since it's multiplied by i).

Therefore, the complex number form is:

$$ \boxed{4 + 1i} $$

or simply:

$$ \boxed{4 + i} $$

Additional Considerations: When Radicals are Non-Real

While in this specific problem, the radical turned out to be imaginary, it's instructive to consider scenarios where the radicand is positive.

When is the radical real?

- If $16 - (4)(5) \geq 0$, then the radical yields a real number.
- For example, if the expression inside the radical is positive or zero, the square root is real.

When is the radical imaginary?

- If $16 - (4)(5) < 0$, the radical yields an imaginary number.
- In such cases, rewriting into $a + bi$ explicitly shows the imaginary component.

Broader Applications: Recognizing Complex Numbers in Different Contexts

Understanding how to convert expressions into complex form is fundamental in various fields such as engineering, physics, and applied mathematics.

Examples of applications:

- Electrical engineering: Impedance calculations often involve complex numbers.
- Signal processing: Complex representations simplify Fourier transforms.
- Quantum physics: Wave functions are expressed with complex amplitudes.

Key skills include:

- Recognizing when radicals involve imaginary parts.
- Simplifying radicals involving negative numbers.
- Expressing results in standard form $a + bi$.

Summary and Conclusion

Rewriting the given expression:

$$ 4 + \frac{\sqrt{16 - (4)(5)}}{2} $$

into the standard complex form $a + bi$ involves a clear sequence of steps:

1. Simplify the radicand:

$$
16 - 20 = -4
$$

2. Handle the square root of a negative number:

\[
\sqrt{-4} = 2i
\]

3. Divide by 2:

\[
\frac{2i}{2} = i
\]

4. Combine with the real part:

\[
4 + i
\]

The final answer is:

\[\boxed{4 + i} \]

This process exemplifies the importance of understanding radicals, imaginary units, and the standard form of complex numbers. Whether dealing with purely real, purely imaginary, or complex expressions, proficiency in these steps is essential for accurate mathematical analysis and problem-solving.

Final Thoughts

Transforming expressions into the complex number form is a fundamental skill that enhances mathematical literacy and problem-solving capabilities. Recognizing the nature of radicals, especially those involving negatives, is crucial for accurate interpretation. As demonstrated, even simple expressions can reveal rich structures when viewed through the lens of complex numbers. Whether you're a student, educator, or professional, mastering these techniques opens up a deeper understanding of the mathematical universe.

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