

Rewrite The Radical As A Rational Exponent. (2 Points) The Fourth Root Of 7 To The Fifth Power

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Understanding how to convert radicals into rational exponents is a fundamental skill in algebra that simplifies the process of manipulating and evaluating roots and powers. This article explores the concept thoroughly, focusing on the specific example of rewriting the fourth root of 7 raised to the fifth power as a rational exponent. By mastering this conversion, students and mathematicians can streamline calculations and deepen their understanding of exponential and radical expressions.

Introduction to Radicals and Rational Exponents

Radicals and exponents are two sides of the same mathematical coin. Radicals, often written using the radical symbol ($\sqrt{}$), denote roots, while exponents indicate powers. Recognizing their equivalence allows for flexible manipulations in algebraic expressions.

What Is a Radical?

A radical expression involves roots, such as square roots ($\sqrt{}$), cube roots ($\sqrt[3]{}$), or higher roots. The general form of a radical is:

$$\sqrt[n]{a}$$

which represents the n -th root of a . For example, the square root of 16 is written as $\sqrt{16}$, which equals 4.

What Is a Rational Exponent?

A rational exponent expresses roots as fractional powers. The general form is:

$$a^{\frac{m}{n}}$$

where:

- a is a non-zero real number,
- m and n are integers,
- $n \neq 0$.

This notation encodes both the power and the root within a single exponent.

Converting Radicals to Rational Exponents

The key to rewriting radicals as rational exponents lies in understanding the relationship:

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

This means the n -th root of a is equivalent to a raised to the power of $1/n$. When dealing with expressions involving powers of roots, such as $(\sqrt[n]{a})^m$, the conversion involves multiplying exponents.

Step-by-Step Conversion Process

Suppose you have the radical:

$$\left(\sqrt[n]{a}\right)^m$$

This can be rewritten as:

$$a^{\left(\frac{1}{n} \times m\right)} = a^{\frac{m}{n}}$$

This rule allows for straightforward conversion of composite radical and power expressions into a single rational exponent.

Applying the Concept: The Fourth Root of 7 to the Fifth Power

Let's analyze the specific example:

$$\left(\sqrt[4]{7}\right)^5$$

Our goal is to express this as a rational exponent.

Step 1: Rewrite the Radical as a Rational Exponent

Using the fundamental conversion:

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

we rewrite the radical:

$$\sqrt[4]{7} = 7^{\frac{1}{4}}$$

Step 2: Apply the Power to Power Rule

Since $(\sqrt[4]{7})^5$ is equivalent to $(7^{\frac{1}{4}})^5$, we use the rule:

$$\left(a^p\right)^q = a^{p \times q}$$

which simplifies the expression to:

$$7^{\frac{1}{4} \times 5} = 7^{\frac{5}{4}}$$

Final Expression

Thus, the radical expression:

$$\left(\sqrt[4]{7}\right)^5$$

is equivalent to the rational exponent:

$$7^{\frac{5}{4}}$$

This is a concise and algebraically friendly form that facilitates further calculations.

Implications and Uses of Rational Exponent Forms

Expressing radicals as rational exponents offers numerous advantages in algebra and calculus:

- **Simplification:** Rational exponents often simplify complex expressions, making them easier to manipulate.
- **Combining Terms:** Like bases can be combined using exponent rules, streamlining calculations.
- **Differentiation and Integration:** Rational exponents are essential in calculus for applying power rules.
- **Solving Equations:** Rational exponents facilitate solving equations involving roots and powers.

Additional Examples and Practice

To deepen understanding, consider these examples:

Example 1: Rewrite $\sqrt[3]{x^2}$ as a rational exponent.

- Solution:

$$\sqrt[3]{x^2} = x^{2/3}$$

Example 2: Simplify $\left(\sqrt[5]{y^3}\right)^4$ as a rational exponent.

- Step 1: Rewrite the radical:

$$\sqrt[5]{y^3} = y^{3/5}$$

- Step 2: Apply the power:

$$\left(y^{\frac{3}{5}}\right)^4 = y^{\left(\frac{3}{5} \times 4\right)} = y^{\frac{12}{5}}$$

Key Rules for Manipulating Rational Exponents

Understanding and applying the following rules is essential:

- Product Rule:** $a^p \times a^q = a^{p+q}$
- Quotient Rule:** $\frac{a^p}{a^q} = a^{p-q}$
- Power of a Power:** $\left(a^p\right)^q = a^{p \times q}$
- Negative Exponents:** $a^{-p} = \frac{1}{a^p}$
- Fractional Exponents:** $a^{\frac{p}{q}} = \sqrt[q]{a^p}$

Applying these rules correctly ensures accurate simplification and manipulation of expressions involving rational exponents.

Common Mistakes to Avoid

While working with radicals and rational exponents, be aware of common pitfalls:

- Misinterpreting the exponent:** Remember that the radical $\sqrt[n]{a}$ is always $a^{\frac{1}{n}}$, not a^n .
- Incorrect multiplication of exponents:** When raising a radical to a power, multiply the exponents carefully.
- Ignoring the domain:** For real numbers, certain radical expressions are only defined under specific conditions (e.g., even roots require non-negative radicands).

Summary and Conclusion

Converting radicals into rational exponents is a powerful technique that simplifies algebraic expressions and enhances mathematical understanding. Specifically, the radical expression $\left(\sqrt[4]{7}\right)^5$ can be rewritten as $7^{\frac{5}{4}}$, a form that is easier to

manipulate and evaluate.

Mastering this conversion involves understanding the fundamental relationship:

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

and applying the exponent rules correctly. Whether in algebra, calculus, or advanced mathematics, expressing roots as rational exponents enables more straightforward calculations, problem-solving, and theoretical insights.

By practicing these conversions and rules, students can develop a strong foundation for tackling complex mathematical expressions confidently.

Frequently Asked Questions

How do you rewrite the fourth root of 7 to the fifth power as a rational exponent?

The fourth root of 7 to the fifth power can be written as $(7^5)^{\frac{1}{4}}$, which simplifies to $7^{\frac{5}{4}}$.

What is the general formula for rewriting roots as rational exponents?

A root n of a number a can be written as $a^{\frac{1}{n}}$, so the m th power of that root becomes $a^{\frac{m}{n}}$.

How do you simplify $(7^5)^{\frac{1}{4}}$ to a single exponential expression?

Using the rule $(a^b)^c = a^{bc}$, it simplifies to $7^{\frac{5}{4}}$.

What does the rational exponent $\frac{5}{4}$ represent in the expression $7^{\frac{5}{4}}$?

It indicates taking the fourth root of 7 raised to the fifth power, or equivalently, raising 7 to the 5th power and then taking the 4th root.

Is rewriting roots as rational exponents useful for simplifying algebraic expressions?

Yes, expressing roots as rational exponents often simplifies calculations and makes it easier to apply exponent rules.

Can you convert the expression 'the fourth root of 7 to the fifth power' into a radical form?

Yes, it can be written as $(\sqrt[4]{7})^5$, which is equivalent to $7^{\{5/4\}}$ in exponential form.

Additional Resources

Rewrite The Radical As A Rational Exponent. (2 Points) The Fourth Root Of 7 To The Fifth Power

Understanding how to rewrite radicals as rational exponents is a fundamental skill in algebra and higher mathematics. It allows for more streamlined calculations, easier manipulation of expressions, and a deeper comprehension of the relationships between roots and exponents. In this article, we will explore the process of converting radicals into rational exponents, specifically focusing on the example of the fourth root of 7 raised to the fifth power. This detailed examination will include definitions, step-by-step conversions, and practical applications, providing you with a comprehensive understanding of this important mathematical concept.

Introduction to Rational Exponents and Radicals

Before diving into the specific example, it is essential to understand the foundational concepts of radicals and rational exponents. A radical expression involves roots, such as square roots, cube roots, or any n th root. For example, the square root of a number is written as $\sqrt{a}$, which is equivalent to $a^{(1/2)}$.

Rational exponents, on the other hand, express roots and powers in a single exponent notation. The general rule for converting radicals to rational exponents is:

- The n th root of a^m can be written as $a^{(m/n)}$.

This conversion simplifies complex radical expressions, especially when dealing with multiple roots or powers, and facilitates algebraic operations such as multiplication, division, and exponentiation.

Key features of rational exponents:

- They unify roots and powers into a single notation.
- They simplify the process of raising expressions to powers or extracting roots.
- They are compatible with the rules of exponents, making algebraic manipulations more straightforward.

Converting The Fourth Root Of 7 To The Fifth

Power

Let's analyze the specific example: the fourth root of 7 raised to the fifth power. In radical notation, this is written as:

$$\sqrt[4]{7^5}$$

Our goal is to rewrite this radical expression as a rational exponent.

Step-by-Step Conversion

Step 1: Recognize the radical notation and its equivalent rational exponent form.

- The fourth root of a number is expressed as the 4th root, which can be written as a fractional exponent of $\frac{1}{4}$.
- The expression 7^5 is already in exponential form, so we are dealing with the fourth root of 7^5 .

Step 2: Write the radical as a rational exponent.

Using the rule: $\sqrt[n]{a^m} = (a^m)^{(1/n)}$, the radical becomes:

$$(7^5)^{(1/4)}$$

Step 3: Simplify the expression by multiplying exponents.

When raising a power to another power, multiply the exponents:

$$(7^5)^{(1/4)} = 7^{\{5 (1/4)\}} = 7^{\{5/4\}}$$

Result:

The radical expression $\sqrt[4]{7^5}$ is equivalent to the rational exponent:

$$7^{\{5/4\}}$$

This single exponential expression is easier to manipulate algebraically and provides a clearer understanding of the relationship between roots and powers.

Features and Benefits of Using Rational Exponents

Converting radicals to rational exponents offers several advantages:

- Simplification: Rational exponents often simplify expressions, making calculations more

straightforward.

- Compatibility: They fit seamlessly with algebraic rules of exponents, allowing for easier manipulation.
- Efficiency: Rational exponents reduce the need for awkward radical notation, especially in complex expressions.
- Generalization: They provide a unified approach that applies to all roots and powers, regardless of the degree.

Pros of Using Rational Exponents:

- Streamlines complex expressions.
- Facilitates differentiation and integration in calculus.
- Enables easier exponentiation and root extraction.

Cons of Using Rational Exponents:

- May initially be less intuitive than radical notation for beginners.
- Requires understanding of fractional exponents and their properties.
- Can sometimes lead to fractional exponents that are less straightforward to interpret.

Practical Applications and Examples

Let's consider some practical scenarios where rewriting radicals as rational exponents is beneficial.

Example 1: Simplify $\sqrt[4]{7^5}$ as $7^{\{5/4\}}$.

This form allows for easier calculations when multiplying, dividing, or raising to powers. For instance, if you need to multiply this expression by another exponential, you can directly add exponents:

$$7^{\{5/4\}} 7^{\{3/4\}} = 7^{\{(5/4 + 3/4)\}} = 7^{\{8/4\}} = 7^{\{2\}}$$

Example 2: Calculate $(7^{\{5/4\}})^2$.

Using exponent rules:

$$(7^{\{5/4\}})^2 = 7^{\{(5/4) 2\}} = 7^{\{10/4\}} = 7^{\{5/2\}}$$

This demonstrates how rational exponents simplify repeated powers.

Example 3: Express the original radical in radical notation from the rational exponent.

Given $7^{\{5/4\}}$, the radical form is:

$$(7^{\{5/4\}}) = (7^{\{1/4\}})^5 = (\sqrt[4]{7})^5$$

This shows the equivalence between the radical and exponential forms.

Advanced Considerations and Variations

While the primary focus is on converting radicals to rational exponents, it's important to recognize some advanced considerations:

- Negative Exponents: If the exponent is negative, it indicates reciprocals, e.g., $7^{-3/4} = 1 / 7^{3/4}$.
- Simplification of Fractional Exponents: When possible, simplify the fractional exponent by factoring numerator and denominator.
- Multiple Radicals: When expressions involve nested radicals, rational exponents can help combine or separate them more efficiently.

Example: Simplify $\sqrt[4]{7^5} \sqrt[4]{7^3}$

Convert both to rational exponents:

$$7^{5/4} 7^{3/4} = 7^{\{5/4 + 3/4\}} = 7^{8/4} = 7^2$$

This showcases the power of rational exponents in simplifying complex radical expressions.

Conclusion

Rewriting radicals as rational exponents is a vital skill that enhances mathematical flexibility and efficiency. The example of the fourth root of 7 raised to the fifth power illustrates this process clearly: transforming $\sqrt[4]{7^5}$ into $7^{5/4}$ simplifies algebraic manipulations and provides a unified framework for understanding roots and powers. Whether in algebra, calculus, or applied mathematics, mastering the conversion between radicals and rational exponents opens the door to more advanced problem-solving techniques. As you become more comfortable with these concepts, you'll find that dealing with exponents and roots becomes more intuitive and less cumbersome, ultimately enriching your mathematical toolkit.

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