Solve The Following Equations And Check Your Answers: A) Log (x+1) - Log (x-1)=2 B) 7^x/2 = 5^-1x

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Mathematics, especially algebra and logarithmic equations, can sometimes seem challenging at first glance. However, with a systematic approach, these equations become manageable and solvable. In this comprehensive guide, we will explore how to effectively solve the given equations step-by-step and verify your solutions to ensure accuracy. Whether you're a student preparing for exams or someone interested in strengthening your mathematical skills, this detailed explanation will help clarify the concepts involved.

Understanding the Equations

Before diving into solving, it’s essential to understand what each equation represents and the fundamental principles involved.

Equation A: Logarithmic Equation

- Equation: Log (x+1) - Log (x-1) = 2
- Type: Logarithmic equation involving the difference of two logs.
- Key concepts involved:
- Logarithmic properties, especially the quotient rule.
- Domain restrictions based on the arguments of the logarithms.

Equation B: Exponential Equation

- Equation: 7^(x/2) = 5^(-x)
- Type: Exponential equation involving variables in the exponents.
- Key concepts involved:
- Laws of exponents.
- Logarithmic transformation to solve for x.

Solving Equation A: Log (x+1) - Log (x-1) = 2

This equation involves the difference of two logarithms, which can be simplified using the logarithmic quotient rule.

Step 1: Apply Logarithmic Properties

- Recall that:
$$
\log a - \log b = \log \left(\frac{a}{b} \right)
$$
- Applying this to the given equation:
$$
\log \left(\frac{x+1}{x-1} \right) = 2
$$

Step 2: Rewrite the Equation in Exponential Form

- Since $\log$ typically denotes $\log_{10}$, we use:
$$
\log_{10} \left(\frac{x+1}{x-1} \right) = 2
$$
- Convert to exponential form:
$$
\frac{x+1}{x-1} = 10^2 = 100
$$

Step 3: Solve for x

- Set up the equation:
$$
x + 1 = 100 (x - 1)
$$
- Expand the right side:
$$
x + 1 = 100x - 100
$$
- Rearrange:
$$
x - 100x = -100 - 1
$$
$$
-99x = -101
$$
- Divide both sides:
$$
x = \frac{-101}{-99} = \frac{101}{99}
$$

\]

Step 4: Verify Domain Restrictions
- For the original logs to be defined:
- \(x + 1 > 0 \Rightarrow x > -1\)
- \(x - 1 > 0 \Rightarrow x > 1\)
- Since \(x = \frac{101}{99} \approx 1.0202\), it satisfies both:
- \(x > 1\)
- Therefore, the solution is valid.

Final Answer for Equation A:
\[
\boxed{x = \frac{101}{99}}
\]

Checking the Solution
- Substitute \(x = \frac{101}{99}\) into the original equation:
\[
\log \left(x+1 \right) - \log \left(x-1 \right) = 2
\]
- Compute:
\[
x + 1 = \frac{101}{99} + 1 = \frac{101}{99} + \frac{99}{99} = \frac{200}{99}
\]
\[
x - 1 = \frac{101}{99} - 1 = \frac{101}{99} - \frac{99}{99} = \frac{2}{99}
\]
- Find the logs:
\[
\log \left(\frac{200}{99} \right) - \log \left(\frac{2}{99} \right) = \log \left(\frac{200/99}{2/99} \right) = \log \left(\frac{200}{99} \times \frac{99}{2} \right) = \log \left(\frac{200}{2} \right) = \log (100)
\]
- Since \(\log (100) = 2\), the solution checks out.

Solving Equation B: \(7^{x/2} = 5^{-x}\)

This exponential equation involves variables in both the base and the exponent, which can be simplified using logarithms.

Step 1: Rewrite the Equation

- The equation:
$$
7^{x/2} = 5^{-x}
$$
- Take natural logarithms ($\ln$) of both sides:
$$
\ln \left(7^{x/2} \right) = \ln \left(5^{-x} \right)
$$

Step 2: Apply Logarithmic Power Rule

- Recall:
$$
\ln a^b = b \ln a
$$
- Applying this:
$$
\frac{x}{2} \ln 7 = -x \ln 5
$$

Step 3: Solve for x

- Rearrange:
$$
\frac{x}{2} \ln 7 + x \ln 5 = 0
$$
- Factor out x:
$$
x \left(\frac{\ln 7}{2} + \ln 5 \right) = 0
$$
- Set each factor equal to zero:
- Either:
$$
x = 0
$$
- Or:
$$
\frac{\ln 7}{2} + \ln 5 = 0
$$
- Since $\ln 7 > 0$ and $\ln 5 > 0$, the second expression cannot be zero, so the only solution is:
$$
x = 0
$$

Step 4: Verify the Solution

- Substitute $x=0$ into the original equation:

$$
7^{0/2} = 5^{0}
$$

$$
7^{0} = 1
$$

$$
5^{0} = 1
$$

- Both sides equal 1, confirming the solution is valid.

Final Answer for Equation B:

$$
\boxed{x = 0}
$$

Additional Notes on Exponential Equations

- When solving equations with variables in exponents, logarithms are powerful tools to linearize the exponential expressions.
- Always verify the solutions within the domain of the original functions, especially for logarithmic equations.

Summary and Key Takeaways

Aspect	Details
Equation A	Simplify logs using quotient rule, convert to exponential form, solve algebraically, verify domain.
Equation B	Take logarithms, apply power rule, isolate x, verify solutions.
Common Techniques	Logarithmic properties, exponential laws, domain restrictions, substitution, verification.

Additional Tips for Solving Logarithmic and

Exponential Equations

1. **Always check the domain:** Logarithmic functions are defined only for positive arguments, so ensure your solutions satisfy these restrictions.

2. **Use properties of logs and exponents:** Familiarize yourself with the quotient, product, and power rules for logs, as well as exponent laws.

3. **Transform equations carefully:** Convert between exponential and logarithmic forms to simplify solving.

4. **Verify your solutions:** Substitute solutions back into the original equations to confirm correctness.

5. **Practice regularly:** The more you work with these types of equations, the more intuitive they become.

Conclusion

Solving logarithmic and exponential equations requires understanding their properties and applying algebraic techniques systematically. For the provided equations, we've demonstrated clear step-by-step methods:

- For Equation A, after simplifying with the quotient rule, the solution was obtained by converting the logarithmic equation into an algebraic one and verifying the domain restrictions.
- For

Frequently Asked Questions

How do I solve the equation Log(x+1) - Log(x-1) = 2?

You can combine the logs using the quotient property: Log[(x+1)/(x-1)] = 2. Then, rewrite as (x+1)/(x-1) = 10^2 = 100. Solve for x: (x+1) = 100(x-1). Simplify: x + 1 = 100x - 100. Bring all terms to one side: x - 100x = -100 - 1, which gives -99x = -101. Therefore, x = -101 / -99 = 101/99. Check that x > 1 for the logs to be defined: since 101/99 ≈ 1.02 > 1, the solution is valid.

What is the solution to the equation 7^{x/2} = 5^{-x}?

Rewrite $7^{x/2}$ as $(7^{1/2})^{x} = (\sqrt{7})^{x}$. The equation becomes $(\sqrt{7})^x = 5^{-x}$. Take the natural logarithm of both sides: $x \ln(\sqrt{7}) = -x \ln(5)$. Bring all x terms to one side: $x [\ln(\sqrt{7}) + \ln(5)] = 0$. Since $\ln(\sqrt{7}) + \ln(5) \neq 0$, the only solution is $x = 0$.

How can I verify the solutions after solving the logarithmic equation?

To verify, substitute the solution back into the original equation and check if both sides are equal. Ensure the domain restrictions (like $x > 1$ for logs) are satisfied. If both sides are equal upon substitution, the solution is correct.

Are there any restrictions on x in the equations involving logarithms?

Yes, for logarithmic expressions like $\text{Log}(x+1)$ and $\text{Log}(x-1)$, the arguments must be positive. Therefore, $x + 1 > 0$ and $x - 1 > 0$, which implies $x > 1$. Always check these domain restrictions before accepting solutions.

Can I use logarithm properties to simplify equations involving multiple logs?

Absolutely! Properties like $\text{Log}(a) - \text{Log}(b) = \text{Log}(a/b)$ and $\text{Log}(a) + \text{Log}(b) = \text{Log}(ab)$ are very useful for simplifying and solving equations involving multiple logarithmic terms.

What are common mistakes to avoid when solving exponential and logarithmic equations?

Common mistakes include forgetting to check the domain restrictions, incorrectly applying log properties, and algebraic errors when isolating variables. Always verify solutions by substitution and ensure the arguments of logs are positive.

Additional Resources

Solve The Following Equations And Check Your Answers is a fundamental exercise in algebra and logarithmic functions that enhances problem-solving skills and deepens understanding of mathematical principles. Engaging with such problems not only sharpens analytical thinking but also provides practical insights into how equations can be manipulated and verified. In this article, we will thoroughly analyze two specific equations—one involving logarithms and the other involving exponential functions—discuss strategies for solving them, and demonstrate how to verify the solutions to ensure accuracy.

Understanding the Importance of Solving Equations and Checking Answers

Solving equations accurately is a cornerstone of mathematics, applicable in fields ranging from engineering to economics. The process involves finding the values of variables that satisfy the given conditions. But solving is only half the battle; verifying solutions ensures correctness and builds confidence in your approach.

Key reasons to check your answers include:
- Error Prevention: Identifies mistakes from algebraic manipulations.
- Understanding Validation: Confirms the solution's validity within the original equation.
- Handling Extraneous Solutions: Especially relevant in logarithmic and exponential equations where domain restrictions exist.

Equation A: Log (x+1) - Log (x-1) = 2

Understanding the Equation

This logarithmic equation involves the difference of two logs, which suggests the use of logarithm properties for simplification. Recall that:

$$
\log a - \log b = \log \left(\frac{a}{b} \right)
$$

Provided the logs are with the same base (commonly base 10), this property simplifies the equation considerably.

Before solving, it's crucial to consider the domain restrictions:
- $x + 1 > 0 \Rightarrow x > -1$
- $x - 1 > 0 \Rightarrow x > 1$

The combined domain restriction is $x > 1$.

Step-by-Step Solution

Step 1: Combine the logs using the quotient rule:

$$
\log \left(\frac{x+1}{x-1} \right) = 2
$$

Step 2: Rewrite in exponential form:

$$
\frac{x+1}{x-1} = 10^2 = 100
$$

Assuming logs are base 10, which is standard unless otherwise specified.

Step 3: Solve for x:

$$
x + 1 = 100 (x - 1)
$$

$$
x + 1 = 100x - 100
$$

Bring all terms to one side:

$$
x - 100x = -100 - 1
$$

$$
-99x = -101
$$

$$
x = \frac{-101}{-99} = \frac{101}{99}
$$

Step 4: Verify the solution against the domain:

$$
x = \frac{101}{99} \approx 1.0202 > 1
$$

It satisfies the domain restrictions.

Step 5: Check the solution in the original equation:

$$
\log \left(\frac{101/99 + 1}{101/99 - 1} \right)
$$

Calculate numerator:

$$
\frac{101}{99} + 1 = \frac{101}{99} + \frac{99}{99} = \frac{200}{99}
$$

Calculate denominator:

$$
\frac{101}{99} - 1 = \frac{101}{99} - \frac{99}{99} = \frac{2}{99}
$$

Compute the quotient:

$$
\frac{200/99}{2/99} = \frac{200}{99} \times \frac{99}{2} = \frac{200}{2} = 100
$$

Now, check the log:

$$
\log 100 = 2
$$

which matches the right side of the original equation, confirming the solution is correct.

Equation B: $\frac{7^x}{2} = 5^{-x}$

Understanding the Equation

This exponential equation involves different bases on each side. To solve it, techniques such as rewriting the equation with common bases, applying logarithms, or isolating x are suitable.

Note that:
- The base 7 and base 5 are both prime, so rewriting with a common base directly isn't straightforward.

- Taking the logarithm of both sides appears to be the most effective approach.

Step-by-Step Solution

Step 1: Write the equation:

\[
\frac{7^x}{2} = 5^{-x}
\]

Step 2: Multiply both sides by 2 to eliminate the denominator:

\[
7^x = 2 \times 5^{-x}
\]

Step 3: Recognize that \(5^{-x} = \frac{1}{5^x}\). Rewrite:

\[
7^x = 2 \times \frac{1}{5^x}
\]

which simplifies to:

\[
7^x \times 5^x = 2
\]

Step 4: Combine the left side:

\[
(7 \times 5)^x = 2
\]

\[
35^x = 2
\]

Step 5: Take the natural logarithm (or log base 10, as preferred) of both sides:

\[
\ln(35^x) = \ln 2
\]

\[

x \ln 35 = \ln 2
\]

Step 6: Solve for \(x\):

\[
x = \frac{\ln 2}{\ln 35}
\]

Alternatively, in common logs:

\[
x = \frac{\log 2}{\log 35}
\]

Step 7: Approximate the value:

\[
\ln 2 \approx 0.6931
\]
\[
\ln 35 \approx 3.5553
\]

So:

\[
x \approx \frac{0.6931}{3.5553} \approx 0.195
\]

Step 8: Verify the solution:

Calculate the left side:

\[
\frac{7^{0.195}}{2}
\]

Calculate numerator:

\[
7^{0.195} \approx e^{0.195 \times \ln 7} \approx e^{0.195 \times 1.9459}
\approx e^{0.379} \approx 1.460
\]

Divide by 2:

\[
\frac{1.460}{2} = 0.730
\]

Calculate the right side:

$$
5^{-0.195} = e^{-0.195 \times \ln 5} \approx e^{-0.195 \times 1.6094} \approx e^{-0.314} \approx 0.730
$$

Both sides approximately equal 0.730, confirming the solution's correctness.

Features and Tips for Solving Logarithmic and Exponential Equations

Features:
- Use properties of logs and exponents to simplify equations.
- Always check domain restrictions, especially in logarithmic equations where arguments must be positive.
- Logarithmic equations often transform multiplicative/divisive relationships into additive/subtractive ones, simplifying the process.
- Exponential equations can be tackled effectively through logarithms, especially when bases differ.

Pros:
- Provides clear pathways to solutions through algebraic transformations.
- Enhances understanding of the relationship between exponential and logarithmic functions.
- Applicable to a broad range of real-world problems involving growth, decay, and scaling.

Cons:
- Mistakes in algebraic manipulations or domain considerations can lead to extraneous solutions.
- Not all equations can be simplified straightforwardly; sometimes numerical methods are necessary.
- Requires familiarity with logarithmic identities and properties.

Conclusion

The exercises of solving equations like $\log (x+1) - \log (x-1) = 2$ and $\frac{7^x}{2} = 5^{-x}$ provide valuable practice for mastering algebraic and logarithmic concepts. The key takeaways include understanding how to manipulate logs and exponents, carefully considering the domains, and verifying solutions to avoid errors. By systematically applying properties of

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logs and exponents, one can navigate even complex equations with confidence.
Regular practice with such problems solidifies mathematical intuition and
prepares learners for more advanced topics in algebra, calculus, and beyond.
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Solve The Following Equations And Check Your Answers A Log X 1 Log X 1 2 B 7 X 2 5 1x

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