Solve The Linear Sysiem By The Gauss - Jordan Elimination Method X+y-z=5 - x+y+5z=30 5x+2y+z=11

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Introduction to Gauss-Jordan Elimination Method

The Gauss-Jordan elimination method is a systematic procedure used to solve systems of linear equations. It is an extension of Gaussian elimination, progressing further to transform the augmented matrix into reduced row-echelon form (RREF). This method simplifies the process of finding solutions—whether they are unique, infinite, or nonexistent.

In this article, we will demonstrate how to solve the following system of equations using the Gauss-Jordan elimination method:

- $x + y - z = 5$
- $-x + y + 5z = 30$
- $5x + 2y + z = 11$

By the end of this guide, you'll understand each step involved in converting the system into a form where solutions can be directly read, and you'll gain insight into the power and efficiency of the Gauss-Jordan method.

Understanding the System of Equations

Before diving into the solution process, let's analyze the system:

$$
\begin{cases}
x + y - z = 5 \quad \text{(Equation 1)} \\
- x + y + 5z = 30 \quad \text{(Equation 2)} \\
5x + 2y + z = 11 \quad \text{(Equation 3)} \\
\end{cases}
$$

This is a system of three equations with three variables, which typically has a unique solution, infinitely many solutions, or no solution.

Converting the System into an Augmented Matrix

The initial step in Gauss-Jordan elimination is to write the augmented matrix representing the system:

$$
\begin{bmatrix}
1 & 1 & -1 & | & 5 \\
-1 & 1 & 5 & | & 30 \\
5 & 2 & 1 & | & 11 \\
\end{bmatrix}
$$

Each row corresponds to an equation, and each column to a variable x, y, z, with the last column representing constants.

Step-by-Step Gauss-Jordan Elimination Process

Step 1: Make the leading coefficient of the first row a 1 (Pivot position)

The first row already has a leading 1 in the first column. No action needed here.

Step 2: Zero out the entries below the pivot in column 1

- To eliminate -1 in row 2, add row 1 to row 2:

$$
R_2 = R_2 + R_1
$$
$$
(-1 + 1) = 0, \quad (1 + 1) = 2, \quad (5 + -1) = 4, \quad (30 + 5) = 35
$$

- To eliminate 5 in row 3, subtract 5 times row 1 from row 3:

$$
R_3 = R_3 - 5 \times R_1
$$
$$
(5 - 5 \times 1) = 0, \quad (2 - 5 \times 1) = -3, \quad (1 - 5 \times -1) = 6, \quad (11 - 5 \times 5) = -14
$$

Updated matrix:

$$
\begin{bmatrix}
1 & 1 & -1 & | & 5 \\
0 & 2 & 4 & | & 35 \\
0 & -3 & 6 & | & -14 \\
\end{bmatrix}
$$

Step 3: Make the leading coefficient of row 2 a 1

Divide row 2 by 2:

$$
R_2 = \frac{1}{2} R_2
$$
$$
(0, 2, 4, |, 35) \rightarrow (0, 1, 2, |, 17.5)
$$

Updated matrix:

$$
\begin{bmatrix}
1 & 1 & -1 & | & 5 \\
0 & 1 & 2 & | & 17.5 \\
0 & -3 & 6 & | & -14 \\
\end{bmatrix}
$$

Step 4: Zero out entries above and below the pivot in column 2

- To eliminate the 1 in row 1, subtract row 2 from row 1:

$$
R_1 = R_1 - R_2
$$
$$
(1 - 0) = 1, \quad (1 - 1) = 0, \quad (-1 - 2) = -3, \quad (5 - 17.5) = -12.5
$$

- To eliminate -3 in row 3, add 3 times row 2:

$$
R_3 = R_3 + 3 \times R_2
$$
$$
(0 + 0) = 0, \quad (-3 + 3) = 0, \quad (6 + 3 \times 2) = 6 + 6 = 12, \quad (-14 + 3 \times 17.5) = -14 + 52.5 = 38.5
$$

Updated matrix:

$$
\begin{bmatrix}
1 & 0 & -3 & | & -12.5 \\
0 & 1 & 2 & | & 17.5 \\
0 & 0 & 12 & | & 38.5 \\
\end{bmatrix}
$$

Step 5: Make the pivot in row 3 a 1

Divide row 3 by 12:

\[
R_3 = \frac{1}{12} R_3
\]
\[
(0, 0, 1, |, 3.2083...)
\]

Updated matrix:

\[
\begin{bmatrix}
1 & 0 & -3 & | & -12.5 \\
0 & 1 & 2 & | & 17.5 \\
0 & 0 & 1 & | & 3.2083 \\
\end{bmatrix}
\]

Step 6: Zero out entries above the pivot in row 3

- For row 1, add 3 times row 3:

\[
R_1 = R_1 + 3 \times R_3
\]
\[
(1, 0, -3 + 3 \times 1, |, -12.5 + 3 \times 3.2083) = (1, 0, 0, |, -12.5 + 9.625) = (1, 0, 0, |, -2.875)
\]

- For row 2, subtract 2 times row 3:

\[
R_2 = R_2 - 2 \times R_3
\]
\[
(0, 1, 2 - 2 \times 1, |, 17.5 - 2 \times 3.2083) = (0, 1, 0, |, 17.5 - 6.4167) = (0, 1, 0, |, 11.0833)
\]

The final augmented matrix in reduced row-echelon form:

\[
\begin{bmatrix}
1 & 0 & 0 & | & -2.875 \\
0 & 1 & 0 & | & 11.0833 \\
0 & 0 & 1 & | & 3.2083 \\
\end{bmatrix}
\]

Deriving the Solution

From the RREF matrix, the solutions are directly read:

- $x = -2.875$
- $y = 11.0833$
- $z = 3.2083$

Expressed precisely, these are:

$$
x = -\frac{23}{8} \quad \text{or} \quad -2.875,
$$
$$
y = \frac{67}{6} \quad \text{or} \quad 11.0833,
$$
$$
z = \frac{38}{12} = \frac{19}{6} \quad \text{or} \quad 3.2083.
$$

Verifying the Solution

To ensure the accuracy of the solution, substitute the values back into the original equations:

1. $x + y - z = 5$

$$
- \frac{23}{8} + \frac{67}{6} - \frac{19}{6} = ? \quad \Rightarrow
$$

Calculate:

$$
- \frac{23}{8} + \frac{67 - 19}{6} = - \frac{23}{8} + \frac{48}{6} = - \frac{23}{8} + 8
$$

Express 8 as $\frac{64}{8}$:

$$
- \frac{23}{8} + \frac{64}{8} = \frac{41}{8} =
$$

Frequently Asked Questions

What is the first step in solving the system using Gauss-Jordan elimination?

The first step is to write the system as an augmented matrix and then perform row operations to convert it into reduced row echelon form.

How do you set up the augmented matrix for the given system?

The augmented matrix is:
[[1, 1, -1, 5],
[-1, 1, 5, 30],
[5, 2, 1, 11]]

What is the purpose of using Gauss-Jordan elimination on this system?

The purpose is to systematically eliminate variables and obtain solutions directly from the reduced matrix.

How do you perform row operations to eliminate variables in this system?

You use operations like row swapping, multiplying a row by a scalar, and adding or subtracting multiples of one row from another to create zeros below and above the pivot positions.

What are the solutions to the system after applying Gauss-Jordan elimination?

The solutions are $x = 2$, $y = 3$, and $z = 0$.

Can Gauss-Jordan elimination detect if the system has no solution or infinitely many solutions?

Yes, if during the elimination process you encounter a row with all zeros in the coefficients but a non-zero constant, the system has no solution. If you have free variables after elimination, it indicates infinitely many solutions.

Is the Gauss-Jordan method suitable for solving larger systems efficiently?

Yes, Gauss-Jordan elimination can be extended to larger systems, but for very large systems, more efficient algorithms like LU decomposition may be preferred.

What is the significance of converting the matrix to reduced row echelon form in Gauss-Jordan elimination?

Converting to reduced row echelon form simplifies reading off the solutions directly, as each leading 1 corresponds to a variable with a clear value or parametric form if free variables exist.

Additional Resources

Solve The Linear System By The Gauss-Jordan Elimination Method X + y - z = 5, -x + y + 5z = 30, 5x + 2y + z = 11

Introduction

In the realm of linear algebra, solving systems of equations is a foundational skill that unlocks a deeper understanding of mathematical relationships and real-world applications. Among the various techniques available, the Gauss-Jordan elimination method stands out for its systematic approach to transforming matrices into reduced row-echelon form, ultimately revealing the solutions to the system. This article explores how to apply the Gauss-Jordan elimination method to a specific system of three equations:

- $x + y - z = 5$
- $-x + y + 5z = 30$
- $5x + 2y + z = 11$

Through detailed steps and explanations, we will demonstrate how this method provides an effective pathway to find the values of x, y, and z, making the process transparent and accessible for learners and practitioners alike.

Understanding the System of Equations

Before diving into the elimination process, it's essential to understand the structure and components of the given system:

1. Equations:

1. $x + y - z = 5$
2. $-x + y + 5z = 30$
3. $5x + 2y + z = 11$

2. Variables:

The unknowns are $x, y,$ and z.

3. Objective:

Find the specific values of $x, y,$ and z that satisfy all three equations simultaneously.

Representing the System as an Augmented Matrix

The first step in applying Gauss-Jordan elimination is to convert the system into an augmented matrix, which consolidates coefficients and constants into a tabular form:

$$
\begin{bmatrix}
1 & 1 & -1 & | & 5 \\
-1 & 1 & 5 & | & 30 \\
5 & 2 & 1 & | & 11 \\
\end{bmatrix}
$$

This matrix represents the coefficients of variables in each equation and the constants on the right side.

Step-by-Step Gauss-Jordan Elimination Process

The goal is to manipulate this matrix to reach the reduced row-echelon form, where the leading coefficient (pivot) in each row is 1, and all other entries in the pivot's column are zero.

Step 1: Make the first pivot a 1 (Already achieved in row 1).

The first row has a leading 1 in the first column, so we proceed to eliminate the variables in the other rows using row operations.

Step 2: Eliminate x from rows 2 and 3

1. Eliminate x from row 2:

Add row 1 multiplied by 1 to row 2:

$$
R_2 \leftarrow R_2 + R_1
$$

Calculations:

$$
(-1 + 1) = 0, \quad (1 + 1) = 2, \quad (5 + -1) = 4, \quad (30 + 5) = 35
$$

Resulting row 2:

$$
[0 \quad 2 \quad 4 \quad | \quad 35]
$$

2. Eliminate x from row 3:

Subtract 5 times row 1 from row 3:

$$
R_3 \leftarrow R_3 - 5 \times R_1
$$

Calculations:

$$
(5 - 5 \times 1) = 0, \quad (2 - 5 \times 1) = 2 - 5 = -3, \quad (1 - 5 \times -1) = 1 + 5 = 6, \quad (11 - 5 \times 5) = 11 - 25 = -14
$$

Resulting row 3:

$$
[0 \quad -3 \quad 6 \quad | \quad -14]
$$

Updated matrix:

$$
\begin{bmatrix}
1 & 1 & -1 & | & 5 \\
0 & 2 & 4 & | & 35 \\
0 & -3 & 6 & | & -14 \\
\end{bmatrix}
$$

Step 3: Make the second pivot a 1 (Row 2)

Divide row 2 by 2:

$$
R_2 \leftarrow \frac{1}{2} R_2
$$

Results:

$$
[0 \quad 1 \quad 2 \quad | \quad 17.5]
$$

Updated matrix:

$$
\begin{bmatrix}
1 & 1 & -1 & | & 5 \\
0 & 1 & 2 & | & 17.5 \\
0 & -3 & 6 & | & -14 \\
\end{bmatrix}
$$

Step 4: Eliminate y from rows 1 and 3

1. Eliminate y from row 1:

Subtract row 2 from row 1:

$$
R_1 \leftarrow R_1 - R_2
$$

Calculations:

$$
(1 - 0) = 1, \quad (1 - 1) = 0, \quad (-1 - 2) = -3, \quad (5 - 17.5) = -12.5
$$

Resulting row 1:

$$
[1 \quad 0 \quad -3 \quad | \quad -12.5]
$$

2. Eliminate y from row 3:

Add 3 times row 2 to row 3:

$$
R_3 \leftarrow R_3 + 3 \times R_2
$$

Calculations:

$$
(0 + 3 \times 0) = 0, \quad (-3 + 3 \times 1) = 0, \quad (6 + 3 \times 2) = 6 + 6 = 12, \quad (-14 + 3 \times 17.5) = -14 + 52.5 = 38.5
$$

Resulting row 3:

$$
[0 \quad 0 \quad 12 \quad | \quad 38.5]
$$

Updated matrix:

$$
\begin{bmatrix}
1 & 0 & -3 & | & -12.5 \\
0 & 1 & 2 & | & 17.5 \\
0 & 0 & 12 & | & 38.5 \\
\end{bmatrix}
$$

Step 5: Make the third pivot a 1

Divide row 3 by 12:

$$
R_3 \leftarrow \frac{1}{12} R_3
$$

Results:

$$
[0 \quad 0 \quad 1 \quad | \quad 3.2083]
$$

Updated matrix:

$$
\begin{bmatrix}
1 & 0 & -3 & | & -12.5 \\
0 & 1 & 2 & | & 17.5 \\
0 & 0 & 1 & | & 3.2083 \\
\end{bmatrix}
$$

Step 6: Back-substitution to find variable values

Now, we perform operations to eliminate the z terms from rows 1 and 2, to arrive at solutions.

1. Eliminate z from row 1:

Add 3 times row 3 to row 1:

$$
R_1 \leftarrow R_1 + 3 \times R_3
$$

Calculations:

$$
-12.5 + 3 \times 3.2083 \approx -12.5 + 9.625 = -2.875
$$

Updated row 1:

$$
[1 \quad 0 \quad 0 \quad | \quad -2.875]
$$

2. Eliminate z from row 2:

Subtract 2 times row 3 from row 2:

$$
R_2 \leftarrow R_2 - 2 \times R_3
$$

Calculations:

$$
17.5 - 2 \times 3.2083 \approx 17.5 - 6.4166 = 11.0834
$$

Updated row 2:

$$
[0 \quad 1 \quad 0 \quad | \quad 11.0834]
$$

Step 7: Final solutions

From the transformed matrix, the solutions are:

- $x = -2.875$
- $y = 11.0834$
- $z = 3.2083$

Expressed more precisely:

$$
x \approx -2.875, \quad y \approx 11.083, \quad z \approx 3.208
$$

Interpreting the Results and Verifying

[Solve The Linear Sysiem By The Gauss Jordan Elimination Method X Y Z 5 X Y 5z 30 5x 2y Z 11]

Solve The Linear Sysiem By The Gauss Jordan Elimination Method X Y Z 5 X Y 5z 30 5x 2y Z 11

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