Suppose That The Demand Of A Certain Item Is X=10+(1/p^2)Evaluate The Elasticity At 0.7E(0.7) =

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Introduction to Price Elasticity of Demand

Understanding the concept of price elasticity of demand is fundamental in economics, especially when analyzing how consumers respond to changes in the price of goods and services. It measures the responsiveness of the quantity demanded to a change in price, providing insights into consumer behavior and helping businesses and policymakers make informed decisions.

In this context, we are given a demand function:

$$X = 10 + \frac{1}{p^2}$$

where:

- X represents the quantity demanded of the item.
- p is the price of the item.

We are asked to evaluate the price elasticity of demand at the specific price point $p = 0.7$. The goal is to compute the elasticity $E(p)$ at $p = 0.7$, which involves understanding the mathematical formulation of elasticity, applying it to the given demand function, and interpreting the result.

Understanding the Demand Function

The Given Demand Equation

The demand function:

$$X(p) = 10 + \frac{1}{p^2}$$

has two components:

- A constant term (10), which indicates a base level of demand independent of price.
- A price-dependent term ($1/p^2$), which diminishes as the price increases.

This function suggests that as the price p increases, the quantity demanded X decreases, consistent with the law of demand.

Behavior of the Demand Function

- When $p \to 0$, $1/p^2 \to \infty$, so demand becomes very high.
- When $p \to \infty$, $1/p^2 \to 0$, approaching $X \to 10$.

This indicates that at very low prices, demand is extremely high, and as price increases, demand approaches a steady level of 10 units.

Mathematical Foundations of Price Elasticity

Definition of Price Elasticity of Demand

Price elasticity of demand $E(p)$ measures how much the quantity demanded X responds to a change in price p. Formally:

$$E(p) = \frac{dX}{dp} \times \frac{p}{X}$$

where:

- $\frac{dX}{dp}$ is the derivative of X with respect to p,
- p is the price at which elasticity is evaluated,
- X is the quantity demanded at that price.

This formula calculates the percentage change in demand divided by the percentage change in price.

Interpreting Elasticity Values

- $|E(p)| > 1$: demand is elastic (responsive to price changes).
- $|E(p)| < 1$: demand is inelastic (less responsive).
- $|E(p)| = 1$: demand is unit elastic.

Calculating the Derivative $\frac{dX}{dp}$

Given:

$$X(p) = 10 + \frac{1}{p^2}$$

we differentiate with respect to p:

$$\frac{dX}{dp} = 0 + \frac{d}{dp} \left(p^{-2} \right)$$

Using power rule:

$$\frac{d}{dp} \left(p^{-2} \right) = -2 p^{-3} = - \frac{2}{p^3}$$

Hence:

$$\frac{dX}{dp} = - \frac{2}{p^3}$$

Evaluating Elasticity at $p = 0.7$

Calculate $X(0.7)$

Plugging $p = 0.7$ into the demand function:

$$X(0.7) = 10 + \frac{1}{(0.7)^2}$$

Compute $(0.7)^2$:

$$(0.7)^2 = 0.49$$

Then:

$$X(0.7) = 10 + \frac{1}{0.49}$$

Calculate $1/0.49$:

$$1/0.49 \approx 2.0408$$

So:

$$X(0.7) \approx 10 + 2.0408 = 12.0408$$

Calculate $\frac{dX}{dp}$ at $p = 0.7$

$$\frac{dX}{dp} = - \frac{2}{(0.7)^3}$$

Calculate $(0.7)^3$:

$$(0.7)^3 = 0.343$$

Now:

$$\frac{dX}{dp} = - \frac{2}{0.343} \approx -5.83$$

Compute Elasticity $E(0.7)$

Using the elasticity formula:

$$E(0.7) = \frac{dX}{dp} \times \frac{p}{X}$$

Plugging in the values:

$$E(0.7) = -5.83 \times \frac{0.7}{12.0408}$$

Calculate numerator:

$$-5.83 \times 0.7 \approx -4.081$$

Now divide by 12.0408:

$$E(0.7) \approx \frac{-4.081}{12.0408} \approx -0.339$$

Interpreting the Results

The calculated elasticity:

$$E(0.7) \approx -0.339$$

The negative sign indicates that the demand is inversely related to price, consistent with the law of demand. The magnitude:

$$|\!E(0.7)| \approx 0.339$$

which is less than 1, signifies that the demand at $p=0.7$ is inelastic. In practical terms:

- Inelastic demand implies that a percentage change in price causes a smaller percentage

change in quantity demanded.
- Implication for pricing strategies: Raising prices slightly might not significantly decrease demand, potentially increasing revenue.

Broader Context and Implications

Price Sensitivity and Business Decisions

Understanding elasticity helps businesses determine optimal pricing strategies:

- If demand is elastic ($|E(p)| > 1$), lowering prices could increase total revenue.
- If demand is inelastic ($|E(p)| < 1$), increasing prices might boost revenue without a substantial decrease in demand.

Given the inelastic demand at $p=0.7$, the firm might consider a price increase to maximize revenue, assuming other factors remain constant.

Economic Significance of the Demand Function

The demand function's structure reveals:

- The impact of price on demand diminishes as p increases, since $1/p^2$ decreases.
- The demand's sensitivity to price changes is higher at lower prices, as reflected in the magnitude of elasticity.

Summary and Key Takeaways

- The demand function $X = 10 + 1/p^2$ exhibits a decreasing trend in demand as price increases.
- The derivative $\frac{dX}{dp} = -2/p^3$ quantifies the rate of change of demand concerning price.
- Evaluating at $p=0.7$:

- Demand $X \approx 12.0408$,
- Derivative ≈ -5.83,
- Elasticity ≈ -0.339.

- The demand is inelastic at this price point, indicating limited responsiveness to price changes.

- Practitioners can use this information to inform pricing policies, revenue optimization, and market analysis.

Final Thoughts

Calculating and interpreting price elasticity is vital in economic analysis and business strategy. By applying calculus to demand functions, analysts can quantify consumer responsiveness precisely. The example discussed demonstrates how to perform such calculations step-by-step, emphasizing the importance of understanding the underlying mathematics and economic implications.

In conclusion, at $p=0.7$, the demand for the item exhibits inelastic behavior with an elasticity of approximately -0.339. This insight can guide pricing decisions, helping businesses maximize revenue while maintaining customer satisfaction.

Note: To deepen your understanding, consider exploring how elasticity varies at different price points and how other factors like income, substitutes, and market conditions influence demand elasticity.

Frequently Asked Questions

What is the demand function given in the problem?

The demand function provided is $X = 10 + (1/p^2)$.

How do you interpret the elasticity of demand in this context?

Elasticity measures how much the quantity demanded responds to a change in price, calculated as $(dX/dp)\,(p/X)$.

How do you compute the derivative of X with respect to p?

Given $X = 10 + (1/p^2)$, the derivative dX/dp is $-2/p^3$.

What is the formula to calculate the point elasticity at p = 0.7?

Elasticity $E(p) = (dX/dp)\,(p / X)$.

How do you evaluate the demand at p = 0.7?

Substitute p = 0.7 into X = 10 + (1/p^2): X = 10 + 1/(0.7)^2 = 10 + 1/0.49 ≈ 10 + 2.0408 ≈ 12.0408.

What is the value of the elasticity at p = 0.7?

Using dX/dp = -2/p^3 = -2 / (0.7)^3 ≈ -2 / 0.343 = -5.83, then E(0.7) = (-5.83) (0.7 / 12.0408) ≈ -5.83 0.0581 ≈ -0.339.

What does an elasticity of approximately -0.339 imply about the demand at p = 0.7?

It indicates that demand is inelastic at this price point, meaning a 1% change in price results in about a 0.339% change in quantity demanded.

Additional Resources

Suppose That The Demand Of A Certain Item Is X=10+(1/p^2) Evaluate The Elasticity At 0.7E(0.7) =

Understanding demand elasticity is fundamental in economics as it provides insights into how the quantity demanded of a good responds to changes in its price. In this context, we are given a specific demand function, $X = 10 + \frac{1}{p^2}$, and asked to evaluate the price elasticity of demand at $p = 0.7$. This process involves differentiating the demand function with respect to price and applying the elasticity formula. This article aims to thoroughly dissect the problem, explain the calculations involved, interpret the results, and explore the broader implications of demand elasticity in economic analysis.

Introduction to Demand Function and Elasticity

Before delving into the specific problem, it is essential to understand the foundational concepts:

Demand Function

A demand function relates the quantity demanded of a good to its price, often expressed as $X = f(p)$. In our case, the demand function is:

$$
X = 10 + \frac{1}{p^2}
$$

This function implies that at different price levels, the demand varies according to this specified relationship.

Price Elasticity of Demand

Price elasticity of demand (E) measures the responsiveness of quantity demanded to a change in price. It is mathematically given by:

$$
E(p) = \frac{dX}{dp} \times \frac{p}{X}
$$

where:
- dX/dp is the derivative of the demand function with respect to price,
- p is the current price,
- X is the quantity demanded at that price.

A high absolute value of elasticity indicates that demand is sensitive to price changes, whereas a low value suggests inelastic demand.

Calculating the Derivative of the Demand Function

To evaluate elasticity at a specific price, we first need to compute dX/dp.

Given:

$$
X = 10 + \frac{1}{p^2}
$$

Differentiating with respect to p:

$$
\frac{dX}{dp} = 0 + \frac{d}{dp} \left(p^{-2} \right) = -2p^{-3} = -\frac{2}{p^3}
$$

This derivative tells us how the quantity demanded changes as the price varies.

Evaluating the Demand Function and Its Derivative at $p=0.7$

Next, we evaluate both X and $\frac{dX}{dp}$ at $p=0.7$:

1. Calculate X:

$$
X(0.7) = 10 + \frac{1}{(0.7)^2} = 10 + \frac{1}{0.49} \approx 10 + 2.0408 \approx 12.0408
$$

2. Calculate $\frac{dX}{dp}$:

$$
\frac{dX}{dp} = -\frac{2}{(0.7)^3} = -\frac{2}{0.343} \approx -\frac{2}{0.343} \approx -5.826
$$

Calculating the Price Elasticity at $p=0.7$

Using the elasticity formula:

$$
E(p) = \frac{dX}{dp} \times \frac{p}{X}
$$

Plugging in the values:

$$
E(0.7) = -5.826 \times \frac{0.7}{12.0408} \approx -5.826 \times 0.0581 \approx -0.338
$$

The negative sign indicates the typical inverse relationship between price and demand: as price increases, demand decreases.

Interpretation:

- The absolute value of elasticity is approximately 0.338.
- Since $|E| < 1$, demand at $p=0.7$ is inelastic, meaning that a 1% change in price results in less than a 1% change in quantity demanded.

Implications of the Elasticity Result

Understanding the calculated elasticity allows businesses and policymakers to make informed decisions:

Features and Significance

- Inelastic Demand ($|E| < 1$): Price changes have a relatively small effect on quantity demanded.
- Revenue Considerations: For inelastic goods, increasing price can lead to higher total revenue, as the decrease in demand is proportionally smaller than the increase in price.
- Pricing Strategies: Firms might consider raising prices if their product exhibits inelastic demand at current price points.

Pros and Cons

- Pros:
- Predicts consumer response to price adjustments.
- Helps in setting optimal pricing to maximize revenue.
- Assists in assessing tax impacts or market interventions.
- Cons:
- Elasticity can vary across different price ranges and consumer segments.
- Static elasticity does not capture dynamic market responses or external influences.
- Assumes ceteris paribus (all else held constant), which may not hold in real markets.

Broader Context and Applications

Demand elasticity is a central concept in various economic analyses:

Market Strategy and Policy

- Firms use elasticity to decide whether to raise or lower prices.
- Governments consider elasticity when imposing taxes or subsidies to predict revenue impacts.

Limitations and Challenges

- Elasticity calculations are based on assumptions of smooth demand curves, which may not reflect real-world complexities.
- External factors like income changes, substitute availability, and consumer preferences influence demand elasticity over time.

Alternative Approaches and Further Analysis

While the current analysis provides a snapshot at $p=0.7$, further insights can be gained through:

- Elasticity over a range of prices: Plotting $E(p)$ across different p values to understand demand responsiveness.
- Cross-price elasticity: Examining how demand responds to prices of related goods.
- Income elasticity: Analyzing how demand changes with consumer income levels.

Conclusion

The evaluation of the demand function $X=10 + \frac{1}{p^2}$ at $p=0.7$ reveals that the demand is inelastic with an elasticity of approximately -0.338. This suggests that at this price point, consumers are relatively insensitive to price changes, and firms could consider price increases without significant drops in quantity demanded. However, it is crucial to recognize that elasticity can vary across different price levels and market conditions. A comprehensive understanding of demand elasticity aids businesses and policymakers in making strategic decisions that optimize revenue, market stability, and consumer welfare. Ultimately, analyzing demand functions and their elasticities fosters a more nuanced grasp of market dynamics and economic behavior.

Note: The interpretation and calculations provided here serve as a detailed example of demand elasticity evaluation, demonstrating the step-by-step process and emphasizing its importance in economic analysis.

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