The Area Bounded By The Curve Y = 1/2x^2, The X-axis Is, And The Lines X = K And X = 2, Where 0

The Area Bounded By The Curve Y = 1/2x^2, The X-axis Is, And The Lines X = K And X = 2, Where 0

Understanding the area enclosed between a curve and specific boundary lines is a fundamental topic in integral calculus. In this article, we explore in detail how to calculate the area bounded by the parabola $y = \frac{1}{2}x^2$, the x-axis, and the vertical lines $x = K$ and $x = 2$, where $0 \leq K \leq 2$. We will examine the mathematical principles involved, provide step-by-step calculations, and discuss the significance of this problem in the context of integral calculus and applications.

Introduction to the Bounded Area Problem

Calculating bounded areas under curves is a common problem that involves integrating a function over a specified interval. For the given problem:

- The curve: $y = \frac{1}{2}x^2$
- The boundary lines: $x = K$ and $x = 2$
- The x-axis: $y = 0$

The goal is to find the total area enclosed within these boundaries, which depends on the value of K.

Significance of the Problem

This type of problem appears frequently in physics, engineering, and economics, where understanding the area under a curve helps in calculating quantities like work done, probabilities, and accumulated quantities.

Mathematical Foundations

Before delving into the calculation, it's important to understand the key concepts involved:

Definite Integrals

A definite integral computes the signed area under a curve between two points:

$$
\int_{a}^{b} f(x) \, dx
$$

If $f(x) \geq 0$ on [a, b], then this integral represents the actual area between the curve and the x-axis.

Properties of the Function $y = \frac{1}{2}x^2$

- The function is a parabola opening upwards.
- Symmetric with respect to the y-axis.
- Non-negative for all x, since $y \geq 0$.

Step-by-Step Calculation of the Area

The total area A bounded by the parabola, the x-axis, and the lines $x = K$ and $x = 2$ is given by the integral of $y = \frac{1}{2}x^2$ over the interval $[K, 2]$:

$$
A = \int_{K}^{2} \frac{1}{2}x^2 \, dx
$$

Step 1: Set Up the Integral

$$
A = \frac{1}{2} \int_{K}^{2} x^2 \, dx
$$

Step 2: Compute the Integral

The integral of x^2 is:

$$
\int x^2 \, dx = \frac{x^3}{3}
$$

Applying the limits:

$$
A = \frac{1}{2} \left[\frac{x^3}{3} \right]_K^{2} = \frac{1}{2} \left(\frac{2^3}{3} - \frac{K^3}{3} \right)
$$

Step 3: Simplify the Expression

$$
A = \frac{1}{2} \times \frac{1}{3} \left(8 - K^3 \right) = \frac{1}{6} (8 - K^3)
$$

Thus, the area in terms of K is:

$$
\boxed{
A = \frac{8 - K^3}{6}
}
$$

Analyzing Special Cases and Limits

Understanding special values of K helps interpret the behavior of the area function.

When $K = 0$

$$
A = \frac{8 - 0}{6} = \frac{8}{6} = \frac{4}{3}
$$

This corresponds to the area between the parabola and the x-axis from 0 to 2.

When $K = 2$

$$
A = \frac{8 - 2^3}{6} = \frac{8 - 8}{6} = 0
$$

This makes sense because the interval collapses to a point at $x = 2$, so the area reduces to zero.

For $0 < K < 2$

The area decreases as K increases, since K^3 grows, reducing the numerator.

Graphical Representation and Interpretation

Visualizing the problem enhances understanding:

- The parabola $y = \frac{1}{2}x^2$ is symmetric about the y-axis.
- The region of interest extends from $x = K$ to $x = 2$.
- The area is the region between the parabola and the x-axis over this interval.

Plotting Tips:

- Draw the parabola from $x = 0$ to $x = 2$.
- Mark the vertical lines at $x = K$ and $x = 2$.
- Shade the region between these lines under the parabola.

Applications of Finding Bounded Areas

The calculation of areas bounded by curves has numerous applications:

- Physics: Calculating work done by a variable force.
- Economics: Determining consumer surplus or producer surplus.
- Probability: Calculating probabilities when dealing with continuous distributions.
- Engineering: Analyzing stress and strain in materials.

In particular, problems involving quadratic functions like $y = \frac{1}{2}x^2$ often appear in projectile motion, optimization problems, and in the analysis of quadratic cost functions.

Extensions and Generalizations

The principles used here can be extended to more complex regions and functions:

- Multiple curves: Finding the area between two curves, e.g., $y = \frac{1}{2}x^2$ and $y = x$.
- Different boundary lines: Changing K to other values or extending the interval.
- Higher dimensions: Calculating volumes using similar integral methods with rotation or revolution.

Conclusion

Calculating the bounded area under a parabola is a fundamental application of integral calculus. For the specific case of $y = \frac{1}{2}x^2$ between $x = K$ and $x = 2$, the area is succinctly given by:

$$
A = \frac{8 - K^3}{6}
$$

This expression allows for quick computation for different values of K, providing insight into how the enclosed area varies with the boundary line $x = K$. Understanding these calculations enhances one's ability to analyze similar problems in various scientific and engineering contexts.

References and Further Reading

- Stewart, James. Calculus: Early Transcendentals. Cengage Learning.
- Thomas, George B., and Franklin D. Finney. Calculus and Analytic Geometry. Pearson.
- Khan Academy. Integral Calculus Overview. https://www.khanacademy.org/math/integral-calculus
- Paul's Online Math Notes. Calculus II - Definite Integrals. https://tutorial.math.lamar.edu/Classes/CalcII/DefiniteIntegrals.aspx

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Note: This article aims to provide a comprehensive understanding of how to approach and calculate the area bounded by a parabola, the x-axis, and vertical boundary lines. Mastery of these concepts forms a strong foundation for more advanced topics in calculus and its applications.

Frequently Asked Questions

What is the geometric interpretation of the area bounded by the curve $y = (1/2)x^2$, the x-axis, and the lines $x = K$ and $x = 2$?

It represents the region on the xy-plane enclosed between the parabola $y = (1/2)x^2$, the x-axis, and the vertical lines at $x = K$ and $x = 2$, forming a finite area between these boundaries.

How do you set up the integral to find the area of the region bounded by $y = (1/2)x^2$, $x = K$, and $x = 2$?

You evaluate the definite integral of $(1/2)x^2$ with respect to x from $x = K$ to $x = 2$, i.e., area = $\int_K^2 (1/2)x^2 \, dx$.

What is the value of the definite integral $\int (1/2)x^2 \, dx$ from $x = K$ to $x = 2$?

The integral evaluates to $(1/6)x^3$, so the area is $(1/6)(2)^3 - (1/6)(K)^3 = (1/6)(8 - K^3)$.

How does the value of K affect the area bounded by the curve, x-axis, and lines $x = K$ and $x = 2$?

The area decreases as K increases, since the lower limit of integration shifts rightward, reducing the interval over which the parabola is integrated, ultimately affecting the total enclosed area.

What is the significance of choosing the limits $x = K$ and $x = 2$ in calculating the area?

These limits define the specific segment of the parabola being analyzed, allowing for the calculation of the area between the parabola, x-axis, and the vertical lines at these points.

Can the area be negative in this context, and why or why not?

No, the area cannot be negative because it is a measure of a region's size, which is always non-negative; the integral's value represents the magnitude of the enclosed area.

If K is set to 0, what is the area bounded by the parabola, x-axis, and lines x = 0 and x = 2?

The area is (1/6)(8 - 0^3) = (1/6) 8 = 4/3 units squared.

How would the area calculation change if the parabola was y = x^2 instead of y = (1/2)x^2?

The integral would be □□^{2} x^2 dx, which evaluates to (1/3)x^3, so the area becomes (1/3)(2)^3 - (1/3)(K)^3 = (1/3)(8 - K^3).

Additional Resources

The Area Bounded By The Curve Y = 1/2x^2, The X-axis Is, And The Lines X = K And X = 2, Where 0 < K < 2

Understanding the area bounded by a curve, the axes, and specific vertical lines is a fundamental concept in calculus, especially in the study of integration. In this detailed guide, we will explore how to find the area bounded by the parabola Y = 1/2 x^2, the x-axis, and the vertical lines X = K and X = 2, where 0 < K < 2. This problem exemplifies how to set up and evaluate definite integrals, interpret the geometric meaning of the integral, and understand how parameters influence the bounded area. Whether you're a student preparing for calculus exams or a professional applying these techniques, this comprehensive analysis aims to clarify each step of the process.

Understanding the Problem: Geometric Setup and Key Concepts

Before diving into calculations, it’s essential to establish a clear geometric picture of the problem:

- Curve: $y = \frac{1}{2} x^2$ – a parabola opening upward, symmetric about the y-axis.
- X-axis: $y = 0$, which acts as the lower boundary.
- Vertical lines: $x = K$ and $x = 2$, with $0 < K < 2$. These lines define the vertical boundaries of the region of interest.
- Region: The area enclosed between the parabola, the x-axis, and the vertical lines $x = K$ and $x = 2$.

The goal is to compute the total area of this region.

Visualizing the Region

Imagine plotting the parabola $y = \frac{1}{2} x^2$. As x increases from 0 to 2, the parabola rises quadratically. The region of interest is the "slice" of this parabola between $x = K$ and $x = 2$, lying above the x-axis.

Key observations:

- Since $y = \frac{1}{2} x^2 \ge 0$ for all real x, the parabola is always above the x-axis in this interval.
- The area between the parabola and the x-axis from $x = K$ to $x = 2$ can be computed using definite integrals.

Setting Up the Integral

The General Approach

The area A bounded by the curve $y = \frac{1}{2} x^2$, the x-axis, and the lines $x = K$ and $x = 2$ is given by:

$$
A = \int_{x=K}^{x=2} y \, dx
$$

Substituting $y = \frac{1}{2} x^2$:

$$
A = \int_{K}^{2} \frac{1}{2} x^2 \, dx
$$

This integral calculates the accumulated area under the parabola from $x = K$ to $x = 2$.

Why this integral?

- The integral of the function over an interval gives the exact area under the curve.
- Because the curve is above the x-axis in this interval, the integral directly represents the area.

Calculating the Integral

Step 1: Write the integral explicitly

$$
A = \frac{1}{2} \int_K^{2} x^2 \, dx
$$

Step 2: Find the antiderivative

Recall that:

$$
\int x^n \, dx = \frac{x^{n+1}}{n+1} + C
$$

Applying this to x^2:

$$
\int x^2 \, dx = \frac{x^3}{3}
$$

Step 3: Evaluate the definite integral

$$
A = \frac{1}{2} \left[\frac{x^3}{3} \right]_K^{2}
$$

$$
A = \frac{1}{2} \left(\frac{2^3}{3} - \frac{K^3}{3} \right)
$$

$$
A = \frac{1}{2} \times \frac{1}{3} (8 - K^3)
$$

$$
A = \frac{1}{6} (8 - K^3)
$$

This is the formula for the area bounded by the parabola $y = \frac{1}{2} x^2$, the x-axis, and the vertical lines $x = K$ and $x = 2$.

Interpreting the Result

The expression:

$$
A = \frac{8 - K^3}{6}
$$

has several important implications:

- Dependence on K: As K varies between 0 and 2, the area changes accordingly.
- Behavior at the endpoints:
- When $K \to 0^+$:

$$
A \to \frac{8 - 0}{6} = \frac{8}{6} = \frac{4}{3}
$$

The area from $x=0$ to $x=2$.

- When $K \to 2^-$:

$$
A \to \frac{8 - 8}{6} = 0
$$

The area tends to zero, as the region collapses when the interval shrinks to a single point at $x=2$.

- Monotonicity: Since K^3 increases with K, the area A decreases as K increases; moving the left boundary closer to 2 reduces the enclosed area.

Practical Applications and Extensions

1. Visual Verification

Plotting the parabola and shading the region between $x=K$ and $x=2$ can help verify the calculation. This visual approach reinforces understanding of how the integral corresponds to the geometric area.

2. Comparing Areas for Different K

By varying K between 0 and 2, one can analyze how the bounded area changes, which is useful in applications like:

- Physics: Calculating work done over an interval if the curve models a force.
- Economics: Finding the accumulated value over a certain range.

3. Generalization

The method shown can be extended to other curves and regions, emphasizing the power of definite integrals in calculating areas bounded by curves and lines.

Additional Considerations

When the Function Crosses the X-axis

In this particular case, $y = \frac{1}{2} x^2 \ge 0$ for all x, so the integral directly gives the area. However, for functions that cross the x-axis within the interval, the area calculation must account for regions above and below the x-axis, often involving absolute values or splitting the integral into parts.

Limitations and Assumptions

- The calculation assumes the curve is above the x-axis in the interval $[K, 2]$.
- The value of K is restricted to $0 < K < 2$. If K approaches 0 or 2, the area approaches the limits discussed.

Summary of Key Steps

1. Identify the region: bounded by the parabola $y = \frac{1}{2} x^2$, x-axis, and vertical lines $x=K$ and $x=2$.
2. Set up the integral: $A = \int_K^2 y \, dx$.
3. Substitute the function: $A = \frac{1}{2} \int_K^2 x^2 \, dx$.
4. Compute the integral: $A = \frac{1}{6} (8 - K^3)$.
5. Interpret the result: understand how K influences the area.

Final Thoughts

Calculating the area bounded by a parabola, the x-axis, and vertical lines is a classic application of definite integrals. The process involves translating a geometric problem into an integral, evaluating it systematically, and interpreting the result within the context of the problem. Mastery of these steps enhances your ability to solve a wide range of problems involving areas under curves, which are foundational in calculus and many applied fields.

Whether analyzing physical systems, economic models, or geometric configurations, understanding how to determine such bounded areas using integrals is an essential skill for students and professionals alike.

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