The Value Of Yvaries Directly With X. If X= 3, Then Y= 21.What Is The Value Of X When Y= 105?

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Understanding the relationship between two variables is fundamental in mathematics, especially when analyzing how one quantity depends on another. In particular, the concept that "Y varies directly with X" is a common and powerful principle used across various fields such as physics, economics, engineering, and everyday problem-solving. This relationship allows us to predict unknown values once we understand how the variables are connected.

In this article, we will explore the concept of direct variation, analyze the given problem where Y varies directly with X, and determine the value of X when Y equals 105. We will also discuss the mathematical principles involved, provide step-by-step solutions, and examine practical applications of direct variation in real-world scenarios.

Understanding Direct Variation

What Is Direct Variation?

Direct variation describes a relationship between two variables where one variable is proportional to the other. Mathematically, this is expressed as:

$$ Y = kX $$

where:

- Y and X are the variables,
- k is the constant of proportionality (also called the constant of variation).

This means that as X increases or decreases, Y does so at a consistent rate determined by k. Conversely, if X decreases, Y decreases proportionally.

Characteristics of Direct Variation

- The graph of a directly varying relationship is a straight line passing through the origin (0,0).
- The ratio $\frac{Y}{X}$ remains constant for all values of X and Y.
- The constant k can be found if any pair of values (X, Y) is known:

$$k = \frac{Y}{X}$$

- If Y varies directly with X, then knowing one pair of (X, Y) allows us to find the entire relationship.

Analyzing the Given Problem: Y Varies Directly With X

Given Data

- When $X = 3$, $Y = 21$.

- Find X when $Y = 105$.

Step 1: Find the Constant of Proportionality k

Using the known values:

$$Y = kX$$

Substitute:

$$21 = k \times 3$$

Solve for k:

$$k = \frac{21}{3} = 7$$

This means the constant of variation k is 7. The direct variation relationship is:

$$Y = 7X$$

Step 2: Find X when $Y = 105$

Using the relationship $Y = 7X$:

$$105 = 7X$$

Solve for X:

$$X = \frac{105}{7} = 15$$

Answer: When $Y = 105$, the value of X is 15.

Mathematical Principles Behind the Solution

Constant of Variation Calculation

The key step in solving direct variation problems is identifying the constant k. Given a pair of known values, the constant is calculated as:

$$k = \frac{Y}{X}$$

This ratio remains consistent for all pairs of (X, Y) in the relationship.

Applying the Constant to Find Unknowns

Once k is known, the relationship:

$$Y = kX$$

can be rearranged to find X:

$$X = \frac{Y}{k}$$

This process illustrates how direct variation simplifies the process of predicting or calculating unknown quantities.

Practical Applications of Direct Variation

Understanding direct variation is not just an academic exercise; it has numerous real-world applications:

1. Physics: Speed, Distance, and Time

- When speed varies directly with time (assuming constant acceleration), the distance traveled is directly proportional to time.
- Example: If a car travels at 60 km/h for 2 hours, the distance covered is:

$$ \text{Distance} = \text{Speed} \times \text{Time} $$

$$ \text{Distance} = 60 \times 2 = 120 \text{ km} $$

- If the speed increases to 90 km/h, the distance covered in 2 hours becomes:

$$ 90 \times 2 = 180 \text{ km} $$

2. Economics: Cost and Quantity

- The total cost of purchasing items varies directly with the number of items.
- If one item costs $5, then the total cost for n items is:

$$ \text{Cost} = 5 \times n $$

- For 10 items, total cost:

$$5 \times 10 = \$50$$

3. Engineering: Power and Voltage

- In electrical engineering, power often varies directly with voltage and current, following the formula:

$$P = V \times I$$

- If voltage and current are known, the power can be calculated directly, demonstrating a direct relationship.

4. Business and Finance: Revenue and Sales

- Revenue might be directly proportional to the number of units sold, assuming a fixed price per unit.

Additional Examples and Practice Problems

Example 1:

Suppose a recipe requires 2 cups of sugar for 4 servings. How much sugar is needed for 10 servings?

Solution:

- Find the constant of variation k:

$$k = \frac{2}{4} = 0.5 \text{ cups per serving}$$

- For 10 servings:

$$\text{Sugar} = 0.5 \times 10 = 5 \text{ cups}$$

Example 2:

A worker earns $150 for 8 hours of work. What is the hourly wage, and how much would they earn for 12 hours at the same rate?

Solution:

- Find the hourly rate:

$$k = \frac{150}{8} = \$18.75 \text{ per hour}$$

- Earnings for 12 hours:

$$18.75 \times 12 = \$225$$

Common Mistakes to Avoid

- Assuming relationships other than direct variation: Not all relationships are directly proportional; some are inverse or nonlinear.

- Incorrect calculation of \(k \): Always verify the known data points before calculating the constant.
- Misapplying the formula: Remember \(Y = kX \); avoid swapping variables unless justified.
- Ignoring units: Ensure units are consistent throughout calculations to prevent errors.

Conclusion

The problem of determining the value of \(X \) when \(Y \) is known, given that \(Y \) varies directly with \(X \), is straightforward once the constant of proportionality is identified. In the provided example, with \(X=3 \) and \(Y=21 \), the constant \(k \) is 7, leading us to find that when \(Y=105 \), \(X=15 \).

Understanding direct variation is essential in both academic and practical contexts. It allows for quick calculations, predictive modeling, and deeper insights into the relationships between quantities. Whether in physics, economics, engineering, or daily life, recognizing and applying the principle of direct variation simplifies complex problems and enhances problem-solving skills.

By mastering the concept of direct variation, you equip yourself with a valuable tool for analyzing proportional relationships efficiently and accurately.

Frequently Asked Questions

What is the constant of variation between X and Y in the given problem?

The constant of variation, k, can be found using the formula Y = kX. Given X=3 and Y=21, k = 21 / 3 = 7.

How do you find the value of X when Y is 105?

Using the direct variation formula Y = kX and knowing k=7, solve for X: X = Y / k = 105 / 7 = 15.

What is the direct variation formula used in this problem?

The formula is Y = kX, where k is the constant of variation determined from the initial values.

If Y varies directly with X, what does that imply about their relationship?

It implies that Y is directly proportional to X; as X increases or decreases, Y increases or decreases proportionally.

Can you verify the value of X when Y=105 using the proportionality constant?

Yes. Since k=7, X = 105 / 7 = 15, confirming the direct variation relationship.

What are some real-world examples of direct variation?

Examples include the relationship between distance and time at constant speed, or the amount of money earned based on units sold.

How would you express the relationship between X and Y mathematically?

The relationship can be expressed as Y = 7X, with 7 being the constant of variation.

If X increases to 6, what is the new value of Y?

Using Y=7X, when X=6, Y=76=42.

What is the significance of understanding direct variation in mathematical problems?

Understanding direct variation helps in modeling relationships where one quantity changes proportionally with another, facilitating predictions and problem-solving.

Additional Resources

The Value Of Yvaries Directly With X. If X= 3, Then Y= 21. What Is The Value Of X When Y= 105?

Understanding the relationship between variables is fundamental in mathematics, science, and various real-world applications. When we say "The value of Y varies directly with X," we're describing a specific type of proportional relationship that allows us to predict one variable based on the other. In this article, we'll explore what it means for two variables to vary directly, how to model this relationship, and how to determine the value of X when Y reaches a certain value—in this case, 105.

What Does It Mean for Y to Vary Directly With X?

At its core, a direct variation between two variables implies a linear relationship where one variable depends on the other in a proportional manner. The formula for direct variation is:

$Y = kX$

Where:

- Y is the dependent variable,
- X is the independent variable,
- k is the constant of proportionality (also called the constant of variation).

When Y varies directly with X, doubling X will double Y; tripling X will triple Y, and so forth. The key feature of direct variation is the constant ratio between Y and X:

$Y / X = k$

This means that once you determine k, you can find the value of Y for any value of X, and vice versa.

How to Find the Constant of Proportionality (k)

Given the problem statement:
- When X = 3, Y = 21

We can find the constant k by plugging these values into the direct variation formula:

$k = Y / X = 21 / 3 = 7$

Therefore, the direct variation equation becomes:

$Y = 7X$

This formula indicates that Y is always 7 times the value of X.

Applying the Model: Finding X When Y = 105

Now that we have the formula Y = 7X, the question becomes:
What is the value of X when Y = 105?

To find this, substitute Y = 105 into the equation:

105 = 7X

Solve for X:

X = 105 / 7 = 15

Answer: When Y = 105, the value of X is 15.

Step-by-Step Summary of Solving Direct Variation Problems

1. Identify the relationship: Is it direct variation (Y varies directly with X)?
2. Write the general formula: Y = kX
3. Use given values to find k:
- Plug in the known values to compute k.
4. Formulate the specific equation:
- Use the calculated k to write the relationship.
5. Solve for the unknown:
- Substitute the known Y or X value and solve for the unknown variable.

Practical Applications of Direct Variation

Understanding direct variation isn't just an academic exercise; it applies to numerous real-world scenarios:

- Speed and Distance: When speed remains constant, distance varies directly with time (Distance =

Speed x Time).

- Cost and Quantity: The total cost varies directly with the number of items purchased at a fixed price per item.
- Physics: The pressure of a gas varies directly with temperature (for ideal gases under certain conditions).
- Economics: Supply and demand often exhibit direct relationships under specific market conditions.

Common Mistakes to Avoid

While working through direct variation problems, keep in mind these pitfalls:

- Confusing direct and inverse variation: In inverse variation, Y varies inversely with X (Y = k / X). Don't mix these up!
- Incorrectly calculating the constant k: Always double-check your division.
- Assuming the relationship is linear without verification: Confirm the proportionality before applying the formula.
- Using incorrect values: Always verify the known values before plugging into equations.

Practice Problems to Reinforce Understanding

1. If Y varies directly with X and Y = 50 when X = 10, what is Y when X = 25?
2. The cost C varies directly with the number of items n. If C = $60 when n = 12, what is the cost when n = 30?
3. A car travels at a constant speed. If it covers 180 miles in 3 hours, how far will it travel in 7 hours at the same speed?

Final Thoughts

The concept that "The value of Y varies directly with X" is fundamental in understanding proportional relationships. Recognizing these relationships allows us to easily solve for unknowns, predict outcomes, and model real-world systems accurately. In our specific example, knowing that Y = 7X enabled us to find that when Y = 105, X must be 15.

Whether you're tackling algebra problems, analyzing physical phenomena, or managing real-life scenarios, mastering direct variation equips you with a powerful tool for quantitative reasoning. Remember to always establish the constant of proportionality from known data and apply the direct variation formula carefully to find unknown values efficiently.

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