Type The Correct Answer In Each Box. If Necessary, Use / For The Fraction Bar. If 3 + 2i / 2 - 3i

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When working with complex numbers, especially in algebra and calculus, it's essential to understand how to perform operations such as division, addition, subtraction, and multiplication. One common task is simplifying the division of complex numbers like (3 + 2i) / (2 - 3i). This process requires knowing how to rationalize the denominator, which involves multiplying numerator and denominator by the conjugate of the denominator. Properly handling these calculations ensures accurate results and deepens your understanding of complex number operations. This article will guide you step-by-step on how to simplify the division of (3 + 2i) / (2 - 3i), along with tips on solving similar problems and understanding complex conjugates.

Understanding Complex Numbers and Their Conjugates

What Are Complex Numbers?

Complex numbers are numbers that include a real part and an imaginary part, expressed in the form a + bi, where:
- a is the real part
- b is the coefficient of the imaginary part (i)
- i is the imaginary unit, defined as √(-1)

For example, in 3 + 2i:
- 3 is the real part
- 2i is the imaginary part

What Is a Complex Conjugate?

The conjugate of a complex number a + bi is written as a - bi. It has the same real part but the imaginary part is negated. The conjugate of (2 - 3i) is (2 + 3i).

The importance of conjugates lies in their ability to eliminate the imaginary part when multiplying, which simplifies division of complex numbers.

Step-by-Step Guide to Simplify (3 + 2i) / (2 - 3i)

Step 1: Identify the conjugate of the denominator

The denominator is 2 - 3i. Its conjugate is 2 + 3i.

Step 2: Multiply numerator and denominator by the conjugate

This step rationalizes the denominator, removing the imaginary component:
\[
\frac{3 + 2i}{2 - 3i} \times \frac{2 + 3i}{2 + 3i}
\]

Step 3: Expand numerator and denominator

Use the distributive property (FOIL method) to expand both parts:

- Numerator:
\[
(3 + 2i)(2 + 3i) = 3 \times 2 + 3 \times 3i + 2i \times 2 + 2i \times 3i
\]
\[
= 6 + 9i + 4i + 6i^2
\]
Recall that \(i^2 = -1\), so:
\[
= 6 + 13i + 6(-1) = 6 + 13i - 6 = 0 + 13i = 13i
\]

- Denominator:
\[
(2 - 3i)(2 + 3i) = 2 \times 2 + 2 \times 3i - 3i \times 2 - 3i \times 3i
\]
\[
= 4 + 6i - 6i - 9i^2
\]
Since \(i^2 = -1\):
\[
= 4 + 0 + 9 = 13
\]

Step 4: Write the simplified expression

Now, the expression becomes:
\[
\frac{13i}{13}
\]
which simplifies to:
\[
i
\]

Final answer:
\[
\boxed{i}
\]

Additional Tips for Simplifying Complex Numbers

1. Always Rationalize the Denominator

When dividing complex numbers, multiply numerator and denominator by the conjugate of the denominator to eliminate the imaginary part.

2. Remember That \(i^2 = -1\)

This identity is crucial when expanding products involving \(i^2\).

3. Simplify Step-by-Step

Break complex expressions into smaller steps, carefully expanding and combining like terms to avoid errors.

4. Practice with Different Examples

Try similar problems, such as:
- \(\frac{5 + 4i}{1 - 2i}\)
- \(\frac{7 - 3i}{4 + i}\)

Using these methods will improve your confidence and proficiency with complex numbers.

Common Mistakes to Avoid

- **Not multiplying numerator and denominator by the conjugate:** This step is essential for rationalizing the denominator.
- **Forgetting that \(i^2 = -1\):** Always substitute \(i^2\) with -1 when expanding products.
- **Mixing real and imaginary parts:** Carefully combine like terms to simplify the final expression correctly.
- **Overlooking the importance of simplification:** Make sure to reduce fractions to their

simplest form for clarity and accuracy.

Practice Problems for Mastery

To reinforce your understanding, try solving these problems:

1. Simplify \(\frac{4 + 5i}{3 - 2i}\)
2. Find the result of \(\frac{1 + i}{1 - i}\)
3. Calculate \(\frac{6 - 2i}{2 + 3i}\)
4. Simplify \(\frac{8 + 4i}{-2 + i}\)

Work through each problem using the conjugate method described above, and compare your answers with solutions available online or in textbooks to verify your progress.

Conclusion

Mastering the division of complex numbers, such as (3 + 2i) / (2 - 3i), is a fundamental skill in algebra that enhances your problem-solving toolkit. Remember to always identify and multiply by the conjugate to rationalize the denominator, carefully expand, and simplify step-by-step. With consistent practice and attention to detail, you'll build confidence in handling complex number operations, making advanced mathematics more approachable and manageable.

By following these guidelines and practicing regularly, you'll be well on your way to solving complex number problems efficiently and accurately.

Frequently Asked Questions

What is the correct way to write the fraction for 3 + 2i divided by 2 - 3i?

Use (3 + 2i) / (2 - 3i)

How do you rationalize the denominator when dividing

complex numbers like (3 + 2i) / (2 - 3i)?

Multiply numerator and denominator by the conjugate of the denominator: (2 + 3i)

What is the simplified form of (3 + 2i) / (2 - 3i) after rationalizing?

[(3 + 2i)(2 + 3i)] / [(2 - 3i)(2 + 3i)]

In the expression (3 + 2i) / (2 - 3i), where should the fraction bar be placed?

Place the fraction bar between the numerator and denominator: (3 + 2i) / (2 - 3i)

How do you write the division of complex numbers in a format suitable for calculation?

Write as (a + bi) / (c + di), using a slash / for the fraction bar

What is the importance of using parentheses when writing complex fractions like 3 + 2i / 2 - 3i?

Parentheses clarify numerator and denominator, ensuring correct order of operations

When dividing complex numbers, why is it necessary to multiply numerator and denominator by the conjugate of the denominator?

To eliminate the imaginary part from the denominator, making it real for easier simplification

How would you write the division problem to ensure clarity in a math worksheet?

Use parentheses and a slash: (3 + 2i) / (2 - 3i)

What are common mistakes to avoid when writing complex number divisions with fractions?

Omitting parentheses, incorrect placement of the fraction bar, and not rationalizing the denominator

Additional Resources

Type The Correct Answer In Each Box. If Necessary, Use / For The Fraction Bar. If 3 + 2i / 2 - 3i

In the realm of complex numbers, manipulating and simplifying algebraic expressions involving imaginary units is a fundamental skill. Whether you're a student preparing for exams or a professional working in fields like engineering or physics, understanding how to handle complex fractions is essential. This article aims to guide you through the process of simplifying the complex fraction $\frac{3 + 2i}{2 - 3i}$, emphasizing clarity and precision while maintaining a reader-friendly tone. We will explore the core concepts of complex numbers, the importance of conjugates, and step-by-step methods for simplifying such expressions.

Understanding Complex Numbers and Their Components

Before diving into the specific problem, it's important to grasp the basics of complex numbers.

What Are Complex Numbers?

Complex numbers extend the real number system by introducing the imaginary unit i, defined as:

$$
i^2 = -1
$$

A complex number is generally written in the form:

$$
a + bi
$$

where:
- a is the real part
- b is the imaginary part

For example, in the expression $3 + 2i$:
- The real part is 3
- The imaginary part is 2

Similarly, in $2 - 3i$:
- The real part is 2
- The imaginary part is -3

The Importance of Conjugates

One of the key tools in simplifying complex fractions is the conjugate of a complex number.

- The conjugate of $a + bi$ is $a - bi$

- The conjugate of $a - bi$ is $a + bi$

Multiplying a complex number by its conjugate yields a real number:

$$
(a + bi)(a - bi) = a^2 + b^2
$$

This property is instrumental in rationalizing the denominator of complex fractions.

Step-by-Step Simplification of $\frac{3 + 2i}{2 - 3i}$

Let's explore how to simplify the expression:

$$
\frac{3 + 2i}{2 - 3i}
$$

using a systematic approach.

Step 1: Identify the conjugate of the denominator

The denominator is $2 - 3i$. Its conjugate is:

$$
2 + 3i
$$

Multiplying numerator and denominator by this conjugate will eliminate the imaginary part from the denominator.

Step 2: Multiply numerator and denominator by the conjugate

Expressed mathematically:

$$
\frac{3 + 2i}{2 - 3i} \times \frac{2 + 3i}{2 + 3i}
$$

This operation doesn't change the value of the expression but simplifies the denominator.

Step 3: Expand numerator and denominator

- Numerator:

$$
(3 + 2i)(2 + 3i)
$$

Applying the distributive property (FOIL):

$$
3 \times 2 + 3 \times 3i + 2i \times 2 + 2i \times 3i
$$
$$
= 6 + 9i + 4i + 6i^2
$$

Recall that $i^2 = -1$:

$$
= 6 + 9i + 4i + 6 \times (-1)
$$
$$
= 6 + 13i - 6
$$
$$
= (6 - 6) + 13i = 0 + 13i
$$

- Denominator:

$$
(2 - 3i)(2 + 3i)
$$

Using the difference of squares pattern:

$$
2^2 - (3i)^2 = 4 - 9i^2
$$

Since $i^2 = -1$:

$$
4 - 9 \times (-1) = 4 + 9 = 13
$$

Step 4: Write the simplified form

Now, the entire expression becomes:

$$
\frac{13i}{13}
$$

Dividing numerator and denominator:

$$
\frac{13i}{13} = i
$$

Final Result and Interpretation

The simplified form of $\frac{3 + 2i}{2 - 3i}$ is:

$$
\boxed{i}
$$

This means that the original complex fraction simplifies neatly to the imaginary unit i. Such simplifications are valuable in complex analysis, electrical engineering, and applied mathematics, where complex fractions frequently appear.

Additional Tips for Simplifying Complex Fractions

To ensure mastery over similar problems, consider the following tips:

- Always identify the conjugate of the complex denominator before multiplying.
- Remember that multiplying by the conjugate results in a real number in the denominator, simplifying division.
- Be cautious with signs when expanding products, especially with imaginary units.
- Simplify carefully by substituting $i^2 = -1$ immediately to avoid confusion.
- Express the final answer in standard form $a + bi$, even if the answer reduces to just i or a real number.

Practice Problems to Hone Your Skills

Engaging with similar problems can reinforce understanding. Here are a few practice exercises:

1. Simplify $\frac{5 + 4i}{1 - 2i}$
2. Simplify $\frac{7 - 3i}{4 + i}$
3. Simplify $\frac{2i + 3}{i - 4}$

Work through these problems using the conjugate method, expanding, and simplifying step-by-step as demonstrated.

Conclusion

Handling complex fractions may initially seem daunting, but with a systematic approach grounded in the properties of conjugates and the imaginary unit, it becomes manageable. The key steps involve multiplying numerator and denominator by the conjugate of the denominator, expanding products carefully, substituting $i^2 = -1$, and simplifying to the standard form. Mastery of these techniques not only improves your algebraic proficiency but also prepares you for advanced studies in mathematics, physics, and engineering where complex numbers are indispensable.

Remember, practice makes perfect — so regularly work through similar problems to build confidence and fluency in simplifying complex fractions efficiently and accurately.

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