Use Integration To Find The Area Of The Figure Having The Given Vertices. (0,0),(1,4),(3,2),(1,3)

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Calculating the area of a polygon with arbitrary vertices can often be a challenging task, especially when the shape is irregular or non-standard. One powerful method to determine the area of such figures is through the use of integration, specifically by integrating along the boundary of the shape. In this comprehensive guide, we will explore how to apply integration to find the area of a polygon with vertices at (0,0), (1,4), (3,2), and (1,3). We will discuss the step-by-step process, the mathematical principles involved, and provide clarity on how to implement this technique effectively.

Understanding the Problem: Vertices and Shape Description

Before delving into the integration process, it's essential to comprehend the figure's structure and outline.

Vertices of the Polygon

The given vertices are:

1. (0, 0)
2. (1, 4)
3. (3, 2)
4. (1, 3)

Plotting these points on a coordinate plane provides a visual representation of the polygon:

- Starting at (0,0), moving to (1,4), then to (3,2), then to (1,3), and finally back to (0,0).

This forms a quadrilateral with a specific shape, which may be irregular.

Shape Analysis

Understanding the shape's nature is crucial:

- The vertices suggest an irregular quadrilateral.
- The shape may be concave or convex; plotting helps determine this.
- For accurate area calculation, the vertices should be ordered either clockwise or counter-clockwise.

Mathematical Foundations for Area Calculation Using Integration

Integration offers an effective approach to find the area enclosed by a polygon, especially when the boundary can be expressed as functions of x or y.

Basic Principles

The key idea involves:

- Breaking down the polygon into regions where the boundary can be described as functions $y=f(x)$ or $x=g(y)$.
- Applying the definite integral of these functions over the appropriate intervals.
- Summing the areas of these regions, considering their orientation and shape.

The Shoelace Formula Versus Integration

While the shoelace formula is a straightforward algebraic method for polygons with known vertices, integration provides an alternative, especially useful when the boundary can be described analytically.

Step-by-Step Guide to Finding the Area Using Integration

Let's break down the process into manageable steps:

1. Arrange the Vertices

Order the vertices either clockwise or counter-clockwise to ensure consistency.

Given points:

- (0,0)
- (1,4)
- (3,2)
- (1,3)

Order them clockwise:

1. (0,0)
2. (1,4)
3. (3,2)
4. (1,3)
5. Back to (0,0)

2. Divide the Polygon Into Regions

Identify sections where the boundary can be expressed as functions $y=f(x)$ or $x=g(y)$. Typically, this involves:

- Splitting the polygon into simpler regions.
- Determining which parts are "above" or "below" each other.

In this case, it may be easier to describe the boundary as functions of x or y.

3. Find Equations of Boundary Lines

Calculate the equations of the lines connecting the vertices:

- Line between (0,0) and (1,4)
- Line between (1,4) and (3,2)
- Line between (3,2) and (1,3)
- Line between (1,3) and (0,0)

Calculate their equations:

- Line (0,0) to (1,4):

Slope $m = (4-0)/(1-0) = 4$

Equation: $y = 4x$

- Line (1,4) to (3,2):

Slope $m = (2-4)/(3-1) = -2/2 = -1$

Equation: $y - 4 = -1(x - 1) \Rightarrow y = -x + 5$

- Line (3,2) to (1,3):

Slope $m = (3-2)/(1-3) = 1/(-2) = -1/2$

Equation: $y - 2 = -\frac{1}{2}(x - 3) \Rightarrow y = -\frac{1}{2}x + \frac{3}{2} + 2 = -\frac{1}{2}x + \frac{7}{2}$

- Line (1,3) to (0,0):

Slope $m = (0-3)/(0-1) = -3/(-1) = 3$

Equation: $y - 3 = 3(x - 1) \Rightarrow y = 3x$

4. Express Boundary Functions

Determine which functions describe the upper and lower boundaries over specific x-intervals.

Identify x-intervals:

- From x=0 to x=1: boundary between (0,0) and (1,4): $y = 4x$
- From x=1 to x=3: boundary between (1,4) and (3,2): $y = -x + 5$
- From x=1 to x=3: boundary between (3,2) and (1,3): $y = -\frac{1}{2}x + \frac{7}{2}$
- From x=0 to x=1: boundary between (1,3) and (0,0): $y = 3x$

Note the overlapping regions; carefully analyze which functions form the upper and lower boundaries over each segment.

Applying Integration to Calculate the Area

Now, we can set up integrals for each segment.

1. Region from x=0 to x=1

Between x=0 and x=1, the boundary is between $y = 4x$ and $y=3x$.

Find the area between these curves:

$$
A_1 = \int_{0}^{1} [(4x) - (3x)] \, dx = \int_{0}^{1} x \, dx
$$

Calculate:

$$
A_1 = \left[\frac{x^2}{2} \right]_{0}^{1} = \frac{1}{2} - 0 = \frac{1}{2}
$$

2. Region from x=1 to x=3

Between x=1 and x=3, the boundary curves are:

- Upper boundary: $y = -x + 5$
- Lower boundary: $y = -\frac{1}{2}x + \frac{7}{2}$

Compute the area between these:

$$
A_2 = \int_{1}^{3} [(-x + 5) - (-\frac{1}{2}x + \frac{7}{2})] \, dx
$$

Simplify the integrand:

$$
(-x + 5) - \left(-\frac{1}{2}x + \frac{7}{2} \right) = -x + 5 + \frac{1}{2}x - \frac{7}{2} = \left(-x + \frac{1}{2}x \right) + \left(5 - \frac{7}{2} \right) = -\frac{1}{2}x + \left(5 - 3.5 \right) = -\frac{1}{2}x + 1.5
$$

Integrate:

$$
A_2 = \int_{1}^{3} \left(-\frac{1}{2}x + 1.5 \right) dx
$$

Calculate:

$$
A_2 = \left[-\frac{1}{4}x^2 + 1.5x \right]_{1}^{3}
$$

Evaluate at $x=3$:

$$
- \frac{1}{4} \times 9 + 1.5 \times 3 = - \frac{9}{4} + 4.5 = -2.25 + 4.5 = 2.25
$$

At $x=1$:

$$
- \frac{1}{4} \times 1 + 1.5 \times 1 = -0.25 + 1.5 = 1.25
$$

Subtract:

$$
A_2 = 2.25 - 1.25 = 1
$$

Summing the Areas for Total Polygon Area

Total area:

\[
\text{Area} = A_1 + A_2 = \frac{1}{

Frequently Asked Questions

How can I set up the integral to find the area of the quadrilateral with vertices (0,0), (1,4), (3,2), and (1,3)?

First, plot the points to understand the shape. Then, divide the quadrilateral into two triangles or use the shoelace formula for polygon area. Alternatively, identify the boundary functions and set up integrals accordingly. For example, determine equations of lines between vertices and integrate the difference in x or y over the relevant intervals.

What is the step-by-step method to use integration for finding the area of this quadrilateral?

1. Order the vertices in a consistent manner (clockwise or counterclockwise). 2. Find equations of the lines forming the sides. 3. Determine the x- or y-intervals over which each boundary applies. 4. Set up integrals of the top function minus bottom function (or right minus left) over these intervals. 5. Evaluate the integrals and sum the results to get the total area.

Can I use the shoelace theorem instead of integration for this problem?

Yes, the shoelace (or Gauss's) formula provides a quick way to find the area of a polygon given its vertices in order. It often simplifies calculations without integration. However, understanding integration helps if the boundary functions are complex or if you need to set up areas between curves.

How do I determine the equations of the lines connecting the vertices for setting up the integrals?

Calculate the slope between each pair of vertices using (y2 - y1)/(x2 - x1). Then, write the line equations in point-slope form: y - y1 = m(x - x1). These equations define the boundaries needed for setting up the integrals.

Is it necessary to divide the quadrilateral into smaller regions before integrating?

Often, yes. Dividing complex polygons into simpler regions like triangles or rectangles makes setting up integrals easier and more straightforward. Alternatively, using the shoelace formula can bypass division and direct integration.

What are common pitfalls to avoid when using integration to find the area of a polygon with given vertices?

Common pitfalls include: misordering vertices, errors in calculating line equations, incorrect setting of integration limits, forgetting to take absolute values when necessary, and neglecting the orientation of the boundary. Carefully plotting and verifying each step helps prevent these mistakes.

Additional Resources

Use Integration To Find The Area Of The Figure Having The Given Vertices (0,0), (1,4), (3,2), (1,3)

Introduction

Mathematics, especially calculus and coordinate geometry, offers powerful tools for analyzing and understanding geometric figures. One of the most elegant methods to determine the area of irregular polygons, especially those with vertices given in coordinate form, is through the application of integration. This technique becomes particularly valuable when the shape's boundaries can be expressed as functions, allowing for the calculation of the enclosed area with high precision.

In this article, we delve into an in-depth exploration of how to utilize integration to find the area of a quadrilateral with vertices at (0,0), (1,4), (3,2), and (1,3). These points form a complex, non-standard shape that challenges straightforward geometric formulas such as the shoelace theorem or simple rectangle and triangle area calculations. Consequently, an analytical approach rooted in calculus becomes essential, providing both accuracy and insight into the shape's structure.

Understanding the Vertices and the Shape

Coordinates of the Vertices

The vertices of the figure are:

- A (0, 0)
- B (1, 4)
- C (3, 2)
- D (1, 3)

Plotting these points reveals a quadrilateral that is not necessarily convex or symmetric. To analyze the shape systematically, it's vital to understand its orientation and how the points connect.

Visualizing the Polygon

Visualizing the figure helps identify its boundaries:

- Starting at A (0, 0)
- Moving to B (1, 4)
- Then to C (3, 2)
- And finally to D (1, 3), closing back to A

Connecting these points in order (A → B → C → D → A) creates the polygon in question. The figure appears to be a quadrilateral with vertices ordered such that the shape is not necessarily convex, which complicates area calculation.

The Significance of Integration in Finding Area

Why Use Integration?

Traditional methods like the shoelace theorem work well for polygons with known vertices, but they can become cumbersome when the shape's sides are not aligned with axes or have irregular slopes. Integration provides a flexible approach by allowing us to:

- Express the boundaries of the figure as functions (or piecewise functions)
- Integrate these functions over specified intervals to find the total area
- Handle complex shapes with non-uniform boundaries more effectively

Integration vs. Geometric Formulas

While geometric formulas are straightforward for regular polygons, irregular shapes require more nuanced approaches. Integration:

- Handles irregular boundaries effectively
- Accommodates non-convex shapes
- Provides a method to verify results obtained via the shoelace theorem or decomposition

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Step-by-Step Approach to Calculating the Area

1. Dividing the Polygon into Simpler Regions

The first crucial step is to partition the quadrilateral into regions where the boundary can be expressed as functions of x or y. For this shape, a typical approach involves:

- Ordering vertices to understand the boundary lines
- Drawing the figure or plotting points to identify which segments can be described as functions

- Decomposing the polygon into two or more regions, each with boundaries represented by functions

2. Establishing the Boundary Functions

To apply integration, we need equations of the lines forming the sides of the polygon. Let's find the equations of the lines connecting the vertices.

Deriving Equations of the Boundary Lines

Line AB: between (0,0) and (1,4)

- Slope $m_{AB} = \frac{4 - 0}{1 - 0} = 4$
- Equation: $y = 4x$

Line BC: between (1,4) and (3,2)

- Slope $m_{BC} = \frac{2 - 4}{3 - 1} = -1$
- Using point-slope form with point (1,4):

$y - 4 = -1(x - 1)$

$y = -x + 5$

Line CD: between (3,2) and (1,3)

- Slope $m_{CD} = \frac{3 - 2}{1 - 3} = -\frac{1}{2}$
- Using point (3,2):

$y - 2 = -\frac{1}{2}(x - 3)$

$y = -\frac{1}{2}x + \frac{3}{2} + 2 = -\frac{1}{2}x + \frac{7}{2}$

Line DA: between (1,3) and (0,0)

- Slope $m_{DA} = \frac{0 - 3}{0 - 1} = \frac{-3}{-1} = 3$
- Using point (1,3):

$y - 3 = 3(x - 1)$

$y = 3x - 3 + 3 = 3x$

Establishing the Integration Regions

From the derived lines, the boundary segments are:

- AB: $y = 4x$, for $0 \leq x \leq 1$
- BC: $y = -x + 5$, for $1 \leq x \leq 3$
- CD: $y = -\frac{1}{2}x + \frac{7}{2}$, for $1 \leq x \leq 3$
- DA: $y = 3x$, for $0 \leq x \leq 1$

The shape can be partitioned into two regions along the x-axis:

Region 1: $0 \leq x \leq 1$

- Upper boundary: $y = 4x$ (AB)
- Lower boundary: $y = 3x$ (DA)

Region 2: $1 \leq x \leq 3$

- Upper boundary: $y = -x + 5$ (BC)
- Lower boundary: $y = -\frac{1}{2}x + \frac{7}{2}$ (CD)

Calculating the Area Using Integration

The overall area is the sum of the areas of these two regions:

$$
\text{Area} = \int_{0}^{1} [\text{upper} - \text{lower}] \, dx + \int_{1}^{3} [\text{upper} - \text{lower}] \, dx
$$

Region 1: $0 \leq x \leq 1$

- Upper boundary: $y = 4x$
- Lower boundary: $y = 3x$

$$
A_1 = \int_0^1 (4x - 3x) \, dx = \int_0^1 x \, dx
$$

Calculating:

$$
A_1 = \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2}
$$

Region 2: $1 \leq x \leq 3$

- Upper boundary: $y = -x + 5$
- Lower boundary: $y = -\frac{1}{2}x + \frac{7}{2}$

$$
A_2 = \int_1^3 \left[(-x + 5) - \left(-\frac{1}{2}x + \frac{7}{2}\right) \right] dx
$$

Simplify the integrand:

$$
(-x + 5) - \left(-\frac{1}{2}x + \frac{7}{2}\right) = -x + 5 + \frac{1}{2}x - \frac{7}{2} = \left(-x + \frac{1}{2}x \right) + \left(5 - \frac{7}{2}\right)
$$

$$
= -\frac{1}{2}x + \frac{3}{2}
$$

Now, compute:

$$
A_2 = \int_1^3 \left(-\frac{1}{2}x + \frac{3}{2} \right) dx
$$

Integrate term by term:

$$
A_2 = \left[-\frac{1}{4}x^2 + \frac{3}{2}x \right]_1^3
$$

Evaluate at the bounds:

At $x = 3$:

$$

-\frac{1}{4} \times 9 + \frac{3}{2} \times 3 = -\frac{9}{4} + \frac{9}{2} = -2.25 + 4.5 = 2.25
\]

At \(x = 1 \):

\[
-\frac{1}{4} \times 1 + \frac{3}{2} \times 1 = -\frac{1}{4} + \frac{3

Use Integration To Find The Area Of The Figure Having The Given Vertices 0 0 1 4 3 2 1 3

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