# Use The Substitution Formula To Evaluate The Integral. 2r 3 Co X Sin X Dx 7/3 O O 129 1024 776 774

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## Introduction to Integration and Substitution Method

### Understanding the Concept of Integration

Integration is a fundamental concept in calculus that involves finding the integral of a function, which can be interpreted as the area under a curve or the accumulation of quantities. When direct integration is complex or not straightforward, substitution methods are often employed to simplify the process.

### The Purpose of Substitution in Integration

Substitution, also known as u-substitution, transforms a complex integral into a simpler form by replacing an expression with a new variable. This technique leverages the chain rule in reverse and is particularly useful when the integrand contains composite functions.

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## Analyzing the Given Integral

### Statement of the Problem

The integral to evaluate is:

$$\int 2r^3 \cos x \sin x \, dx$$

with additional annotations: "7/3 O O 129 1024 776 774" which seem to be extraneous or possibly referencing specific values or parameters, but for the scope of this article, the focus remains on the integral itself.

### Understanding the Integrand

The integrand is:

$$2r^3 \cos x \sin x$$

which involves a product of a constant multiple $2r^3$ and the trigonometric functions $\cos x$ and $\sin x$.

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## Applying the Substitution Formula

### Step 1: Recognize a Suitable Substitution

Notice that the integrand contains the product $\cos x \sin x$. Recognizing common identities or derivatives can guide the substitution.

Since:

$$
\frac{d}{dx} (\sin^2 x) = 2 \sin x \cos x
$$

we see that:

$$
\sin x \cos x = \frac{1}{2} \frac{d}{dx} (\sin^2 x)
$$

This suggests substituting $u = \sin x$, which simplifies the integral due to the derivative relationship.

### Step 2: Implement the Substitution

Let:

$$
u = \sin x
$$

then:

$$
du = \cos x \, dx
$$

Rearranged:

$$
\cos x \, dx = du
$$

Substituting into the integral:

$$
\int 2r^3 \cos x \sin x \, dx = 2r^3 \int u \, du
$$

because $\sin x = u$ and $\cos x \, dx = du$.

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# Evaluating the Transformed Integral

## Step 3: Perform the Integration

The integral simplifies to:

$$
2r^3 \int u \, du
$$

which is straightforward:

$$
2r^3 \times \frac{u^2}{2} + C = r^3 u^2 + C
$$

where $C$ is the constant of integration.

## Step 4: Back-Substitute to Original Variable

Recall that:

$$
u = \sin x
$$

thus, the solution in terms of $x$ is:

$$
r^3 \sin^2 x + C
$$

---

# Final Result and Interpretation

## Complete Solution

The evaluated integral is:

$$
\boxed{
\int 2r^3 \cos x \sin x \, dx = r^3 \sin^2 x + C
}
$$

This demonstrates the effectiveness of the substitution method in simplifying and solving the integral involving trigonometric functions.

### Additional Considerations

- The constant $r$ is treated as a parameter; if $r$ is variable, further context is necessary.
- The integral's bounds are not specified; if limits are given, the definite integral can be evaluated similarly.

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## Summary of the Substitution Process

- Identify a part of the integrand whose derivative appears elsewhere in the integrand (here, $\sin x$ and $\cos x$).
- Choose a substitution to simplify the integral (here, $u = \sin x$).
- Express $dx$ and other parts of the integrand in terms of $du$ and $u$.
- Perform the resulting simpler integral.
- Back-substitute to express the answer in original variables.

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## Conclusion

The substitution formula is a powerful tool in calculus that streamlines the process of evaluating integrals, especially those involving composite functions or products of derivatives. In the example evaluated, recognizing the derivative of $\sin^2 x$ facilitated an efficient substitution, leading to a straightforward integration. Mastery of this technique enables solving a wide range of integrals with confidence, reinforcing its importance in calculus and mathematical analysis.

## Frequently Asked Questions

### What is the substitution formula used to evaluate integrals like $\int 2r^3 \cos x \sin x \, dx$?

The substitution formula involves choosing a substitution that simplifies the integrand, such as setting $u = \sin x$ or $u = \cos x$, and then rewriting the integral in terms of $u$ to facilitate easier integration.

### How do you apply substitution to evaluate $\int 2r^3 \cos x \sin x \, dx$?

You can let $u = \sin x$, which gives $du = \cos x \, dx$. Then, express the integral in terms of $u$, simplifying it to $\int 2r^3 u \, du$ and integrating accordingly.

### What is the significance of the constants 7/3, 129, 1024, 776, 774 in the context of this integral?

These constants might represent specific values or bounds related to the integral or a particular problem setup. In the context of substitution, constants like 7/3 could be coefficients or limits used in the evaluated result.

### Can the substitution method be used to evaluate integrals involving trigonometric functions like cos x sin x?

Yes, substitution is particularly useful for integrals involving products of trigonometric functions, as it can convert the integrand into a polynomial form in terms of a single variable, simplifying the integration process.

### What are common substitution choices for integrals involving cos x sin x?

Common substitutions include setting u = sin x or u = cos x, depending on the integral's structure. These choices often simplify the integrand and make the integral straightforward to evaluate.

### How does understanding the substitution formula help in evaluating complex integrals like the one given?

Understanding the substitution formula allows you to systematically simplify complex integrals by transforming them into more manageable forms, making it easier to find antiderivatives and evaluate the integral accurately.

## Additional Resources

Use The Substitution Formula To Evaluate The Integral: An In-Depth Analysis

In the realm of calculus, the technique of substitution stands as one of the most powerful and versatile tools for evaluating integrals. When faced with complex or seemingly intractable integrals, substitution often simplifies the problem into a more manageable form, allowing for straightforward integration. This article delves into the application of the substitution formula to evaluate a specific integral: $\int 2r^3 \cos x \sin x \, dx$, and explores its broader significance in calculus, including detailed step-by-step procedures, interpretations, and practical implications.

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## Understanding the Substitution Method in Integral Calculus

## What Is the Substitution Method?

The substitution method, also known as u-substitution, is a technique where a complex integral involving a composite function is simplified by changing variables. The core idea is to identify a part of the integrand that, when substituted with a new variable $u$, transforms the integral into a standard form that's easier to evaluate.

Mathematically, if an integral involves a function $f(g(x))$ multiplied by $g'(x)$, then setting $u = g(x)$ leads to $du = g'(x) dx$, and the integral becomes:

$$
\int f(g(x)) g'(x) dx = \int f(u) du
$$

This transformation often turns a complicated integral into a basic polynomial, exponential, or trigonometric integral.

## Why Use Substitution?

- Simplifies integrals involving composite functions.
- Converts difficult integrals into basic forms.
- Provides insight into the structure of the integrand.
- Facilitates the evaluation of definite integrals with variable limits.

## Common Scenarios for Substitution

- Integrals involving composite functions, such as $\sin^2(x)$, $\sqrt{1 - x^2}$, or $e^{x^2}$.
- Trigonometric integrals where substitution simplifies trigonometric identities.
- Functions involving products of functions and their derivatives.

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# Evaluating $\int 2r^3 \cos x \sin x \, dx$ Using Substitution

Before proceeding with the substitution, it's essential to clarify the integral's form and potential variables involved. The expression appears to be:

$$
\int 2r^3 \cos x \sin x \, dx
$$

However, as written, it suggests the presence of a variable $r$ alongside $x$. Typically, in calculus, $r$ might be a constant or a parameter, and the integral is over $x$. Assuming $r$ is a constant, the integral reduces to:

$$
2 r^3 \int \cos x \sin x \, dx
$$

The constant $2 r^3$ can be factored out:

$$
2 r^3 \int \cos x \sin x \, dx
$$

This simplifies the problem to evaluating:

$$
I = \int \cos x \sin x \, dx
$$

Once this integral is computed, multiplying the result by $2 r^3$ gives the final answer.

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## Step 1: Recognize the Integral Structure

The integral $\int \cos x \sin x \, dx$ involves a product of sine and cosine functions. This is a classic scenario where substitution can be very effective.

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## Step 2: Choose the Appropriate Substitution

Observe that $\sin x$ and $\cos x$ are derivatives of each other:

$$
\frac{d}{dx} \sin x = \cos x
$$
$$
\frac{d}{dx} \cos x = - \sin x
$$

Given this, a natural choice for substitution is:

$$
u = \sin x \quad \Rightarrow \quad du = \cos x \, dx
$$

This substitution transforms the integral into:

$$
I = \int u \, du
$$

which is straightforward to evaluate.

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## Step 3: Perform the Substitution and Integrate

Rewriting the integral:

$$
I = \int \cos x \sin x \, dx = \int u \, du
$$

Integrating:

$$
I = \frac{1}{2} u^2 + C
$$

Substituting back $u = \sin x$:

$$
I = \frac{1}{2} \sin^2 x + C
$$

---

## Step 4: Include the Constant and Final Answer

Recall the original integral had a coefficient $2 r^3$:

$$
\int 2 r^3 \cos x \sin x \, dx = 2 r^3 \times \frac{1}{2} \sin^2 x + C
$$

Simplifying:

$$
= r^3 \sin^2 x + C
$$

Final Result:

$$
\boxed{
\int 2 r^3 \cos x \sin x \, dx = r^3 \sin^2 x + C
}
$$

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# Broader Significance and Applications of the Substitution Method

## Why This Technique Matters

The substitution method is foundational in calculus because it bridges the gap between complex, composite functions and their simpler counterparts. Through this technique, mathematicians and scientists can evaluate integrals that are otherwise intractable, enabling advancements across physics, engineering, and other quantitative sciences.

## Applications in Various Fields

- Physics: Calculating work, energy, or probability distributions often involves integrals where substitution simplifies the process.
- Engineering: Signal processing, control systems, and circuit analysis frequently employ substitution techniques to evaluate integrals related to frequency responses or energy calculations.
- Probability and Statistics: Computing expected values or cumulative distribution functions involves integrals that benefit from substitution to handle complicated probability density functions.

## Limitations and Considerations

While substitution is powerful, it requires identifying an appropriate substitution variable. Misapplication can lead to errors or overly complicated transformations. Also, for definite integrals, changing limits according to the substitution is essential, whereas indefinite integrals include an arbitrary constant.

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# Advanced Techniques and Variations

## Integration by Parts and Substitution

Sometimes, substitution complements other techniques like integration by parts, especially when an integral involves products of functions with different types of derivatives.

### Trigonometric Substitutions

For integrals involving $\sqrt{a^2 - x^2}$, $\sqrt{a^2 + x^2}$, or $\sqrt{x^2 - a^2}$, specialized substitutions such as $x = a \sin \theta$, $x = a \tan \theta$, or $x = a \sec \theta$ are used.

### Numerical Substitution Techniques

In cases where analytical substitution is complex or infeasible, numerical methods like Simpson's rule or Gaussian quadrature are employed, often utilizing substitution principles to improve accuracy.

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## Conclusion: The Power of Substitution in Integral Calculus

The evaluation of the integral $\int 2 r^3 \cos x \sin x \, dx$ exemplifies the elegance and efficiency of substitution in calculus. By recognizing the structure of the integrand and choosing an appropriate substitution—here, $u = \sin x$—we transformed a potentially complicated product into a simple polynomial integral. The resulting expression, $r^3 \sin^2 x + C$, underscores how strategic substitutions can demystify complex integrals.

Beyond this specific example, the substitution formula remains a cornerstone technique in calculus, enabling practitioners to solve a broad spectrum of integrals across scientific disciplines. Its versatility and conceptual clarity make it an indispensable tool, fostering deeper understanding and facilitating problem-solving in mathematical analysis.

As calculus continues to underpin technological and scientific advancements, mastery of substitution techniques ensures that students, researchers, and professionals can tackle diverse challenges with confidence and precision.

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