What Is An Equation Of The Line That Is Parallel To Y = 3x - 8 And Passesthrough The Point (4,-5)?

What Is An Equation Of The Line That Is Parallel To Y = 3x - 8 And Passes Through The Point (4, -5)?

Understanding the concepts of lines, slopes, and equations is fundamental in algebra and coordinate geometry. When working with lines, one common problem involves finding a line that is parallel to a given line and passes through a specific point. In this case, we're asked to determine the equation of a line that is parallel to the line y = 3x - 8 and passes through the point (4, -5). This process involves understanding the properties of parallel lines, the slope-intercept form of a line, and how to derive an equation based on a point and slope.

Understanding the Basics: Lines, Slopes, and Equations

Before delving into solving the specific problem, it is essential to grasp some fundamental concepts related to lines in the coordinate plane.

What Is a Line in the Coordinate Plane?

A line in the coordinate plane is a straight, one-dimensional figure extending infinitely in both directions. It is uniquely determined by its slope and a point that it passes through.

What Is the Slope of a Line?

The slope of a line measures its steepness and is represented by the letter 'm'. It indicates how much y changes for a unit change in x. The slope is calculated as:

$$m = (\text{change in } y) / (\text{change in } x) = (y_2 - y_1) / (x_2 - x_1)$$

The slope is fundamental in determining whether lines are parallel, perpendicular, or intersecting.

Equation of a Line in Slope-Intercept Form

The most common form for the equation of a line is the slope-intercept form:

$$y = mx + b$$

Where:
- **m** is the slope
- **b** is the y-intercept (the point where the line crosses the y-axis)

Identifying the Slope of the Given Line

The given line is:

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y = 3x - 8
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From this equation, we can directly identify:

1. The slope (**m**) is 3.
2. The y-intercept (**b**) is -8.

Since the goal is to find a line parallel to this, the key property is that parallel lines have identical slopes.

Understanding Parallel Lines

What Does It Mean for Lines to Be Parallel?

Parallel lines are lines in the same plane that never intersect. They have the same slope but different y-intercepts.

Properties of Parallel Lines

- Same slope: If line 1 has slope m, then line 2 must also have slope m.
- Different y-intercepts: To ensure they are distinct lines, their y-intercepts must differ.
- Equal steepness: Both lines rise or fall at the same rate but are offset from each other.

Applying this understanding, the line we seek will have the same slope as the given line, which is 3.

Finding the Equation of the Parallel Line Passing Through (4, -5)

Now, we can proceed to derive the equation of the required line.

Step 1: Determine the Slope

Since the line must be parallel to y = 3x - 8, its slope is:

```
m = 3
```

Step 2: Use the Point-Slope Form

The point-slope form is useful for finding the equation of a line when you know:
- The slope (**m**)

- A point ((x_1, y_1)) the line passes through

The point-slope form is:

$$y - y_1 = m(x - x_1)$$

Plugging in the known point (4, -5) and the slope 3:

$$y - (-5) = 3(x - 4)$$

Simplify:

$$y + 5 = 3(x - 4)$$

Step 3: Convert to Slope-Intercept Form

Distribute the slope:

$$y + 5 = 3x - 12$$

Subtract 5 from both sides:

$$y = 3x - 12 - 5$$

Simplify:

$$y = 3x - 17$$

Thus, the equation of the line parallel to $y = 3x - 8$ and passing through the point (4, -5) is:

$$y = 3x - 17$$

Summary of the Solution Process

To recap, here are the essential steps for solving such problems:

1. Identify the slope of the given line. For $y = 3x - 8$, the slope is 3.
2. Recognize that parallel lines share the same slope.
3. Use the point-slope form with the given point (4, -5) and the slope 3.
4. Simplify to get the slope-intercept form, yielding $y = 3x - 17$.

This process can be generalized to find the equation of any line parallel to a given line passing through a specific point.

Additional Tips and Common Mistakes

Tips for Solving Similar Problems

- Always identify the slope of the given line first.
- Remember that parallel lines have identical slopes, but different y-intercepts.
- Use the point-slope form to incorporate the specific point into the equation efficiently.
- Convert to slope-intercept form for a clear, standard equation.

Common Mistakes to Avoid

- Confusing the slope of the original line with that of the parallel line—ensure they are the same.
- Forgetting to convert the point-slope form to slope-intercept form if needed.
- Incorrectly plugging in the point coordinates, especially sign errors.
- Mixing up the y-intercept when writing the final equation.

Practical Applications of Finding Parallel Lines

Understanding how to find the equation of a line parallel to another and passing through a given point has numerous real-world applications, including:

1. Designing roads or pathways that run parallel to existing structures.
2. Creating parallel beams or supports in engineering projects.
3. Plotting parallel lines in graphical data analysis.
4. In computer graphics, drawing parallel lines for visual effects.
5. Analyzing financial or scientific data where relationships are modeled using parallel trends.

Conclusion: Mastering the Concept of Parallel Lines and Line Equations

In conclusion, determining the equation of a line parallel to a given line and passing through a specific point is a fundamental skill in algebra and geometry. By understanding the properties of slopes and the various forms of line equations, you can approach and solve these problems with confidence. The key steps involve identifying the slope of the original line, maintaining that slope for the parallel line, and then applying the point-slope form to incorporate the given point. The resulting equation, such as $y = 3x - 17$ in this case, provides a precise mathematical description of the desired line. Mastering these concepts enhances your problem-solving toolkit and prepares you for more advanced topics in mathematics and related fields.

Frequently Asked Questions

What is the slope of the line parallel to $y = 3x - 8$?

The slope is 3, since parallel lines have the same slope.

How do you find the equation of a line parallel to $y = 3x - 8$ that passes through the point $(4, -5)$?

Use the point-slope form $y - y_1 = m(x - x_1)$ with $m = 3$ and $(x_1, y_1) = (4, -5)$, then simplify to slope-intercept form.

What is the equation of the line passing through $(4, -5)$ with slope 3?

Using point-slope form: $y - (-5) = 3(x - 4)$, which simplifies to $y + 5 = 3x - 12$, and then $y = 3x - 17$.

Why do parallel lines have the same slope?

Parallel lines are always equidistant and never intersect, which requires them to have identical slopes to maintain the same direction.

Can the equation $y = 3x - 17$ be rewritten in different forms?

Yes, it can be written in standard form as $3x - y = 17$.

What is the significance of the point $(4, -5)$ in determining the line's equation?

It provides a specific point through which the line passes, allowing us to determine the exact equation of the line parallel to $y = 3x - 8$.

Additional Resources

Understanding the Equation of a Line Parallel to $Y = 3x - 8$ Passing Through a Specific Point

Mathematics, particularly coordinate geometry, offers a fascinating insight into the relationships and properties of lines, points, and slopes. One common problem students encounter involves finding the equation of a line that is parallel to a given line and passes through a specific point. This problem not only tests one's understanding of the fundamental concepts of slopes and equations but also provides a practical application of algebraic techniques. In this article, we delve deeply into the process of deriving the equation of a line parallel to the given line $y = 3x - 8$ that passes through the point $(4, -5)$. We will explore the underlying principles, step-by-step procedures, and broader implications to provide a comprehensive understanding of this topic.

Foundations of Line Equations and Slope-Intercept Form

Understanding the Equation $y = 3x - 8$

The given equation $y = 3x - 8$ is expressed in slope-intercept form, $y = mx + b$, where:

- m is the slope of the line.
- b is the y-intercept, the point where the line crosses the y-axis.

For $y = 3x - 8$:

- The slope (m) is 3.
- The y-intercept (b) is -8.

This equation describes a straight line with a constant slope of 3, meaning that for every unit increase in x, y increases by 3 units.

Significance of the Slope in Line Equations

The slope indicates the steepness and direction of a line:

- A positive slope (like 3) indicates the line ascends from left to right.
- A negative slope indicates a descent.
- A zero slope signifies a horizontal line.
- An undefined slope corresponds to vertical lines.

In the context of parallel lines, the key property is that they share the same slope.

What Does It Mean for Lines to Be Parallel?

Characteristics of Parallel Lines

Parallel lines are lines in the same plane that never intersect, regardless of how far they are extended. Their defining property is:

- Equal slopes: Parallel lines have identical slopes but different y-intercepts.

Mathematically, if two lines are parallel:

- Both can be written in the form $y = mx + b1$ and $y = mx + b2$ with $b1 \neq b2$.

Implications for Equation Derivation

Given a line $y = 3x - 8$, any line parallel to it must:

- Have a slope (m) of 3.
- Pass through a given point $(4, -5)$.

The goal is to find the specific equation of this new line, which requires applying the point-slope form of a line equation.

Deriving the Equation of the Parallel Line

Step 1: Recognize the Slope of the Parallel Line

Since the original line $y = 3x - 8$ has a slope of 3, the parallel line must also have:

- $m = 3$

This is fundamental because parallel lines maintain the same slope.

Step 2: Use the Point-Slope Form

The point-slope form of a line's equation is:

- $y - y_1 = m(x - x_1)$

where:

- (x_1, y_1) is a point on the line.
- m is the slope.

Given the point $(4, -5)$ and the slope $m = 3$:

- $y - (-5) = 3(x - 4)$

which simplifies to:

- $y + 5 = 3(x - 4)$

Step 3: Simplify to Slope-Intercept Form

Distribute the slope:

- $y + 5 = 3x - 12$

Subtract 5 from both sides:

- $y = 3x - 12 - 5$

- $y = 3x - 17$

This is the equation of the line parallel to $y = 3x - 8$ passing through $(4, -5)$.

Analyzing the Resulting Equation and Its Properties

Equation in Slope-Intercept Form

The final equation:

- $y = 3x - 17$

has several notable features:

- Same slope (3) as the original line, confirming parallelism.
- Different y-intercept (-17) from the original line's y-intercept (-8), ensuring the lines do not intersect.

Graphical Interpretation

Plotting both lines:

- The original line $y = 3x - 8$ crosses the y-axis at $(0, -8)$.
- The new line $y = 3x - 17$ crosses at $(0, -17)$.
- Both lines have identical slopes, so they are equally steep but offset vertically.

Verifying the Passage Through the Point (4, -5)

Plugging $x = 4$ into the derived line:

- $y = 3(4) - 17 = 12 - 17 = -5$

which matches the point $(4, -5)$, confirming the correctness.

Broader Context and Applications

Relevance in Real-World Scenarios

Understanding how to derive equations of lines parallel to a given line passing through a specific point has practical applications:

- Navigation and mapping: Adjusting routes while maintaining a specific heading.
- Engineering: Designing components that must be parallel to existing parts.
- Economics: Modeling parallel cost or revenue lines.

Extending to Other Line Equations

The same principles apply when:

- The original line is in different forms (standard, point-slope).
- The point through which the line passes varies.
- The line is perpendicular, involving negative reciprocal slopes.

Conclusion: Mastering the Art of Line Equation Derivation

The process of finding the equation of a line parallel to a given line and passing through a specific point exemplifies core algebraic techniques and geometric principles. By recognizing the importance of slope, employing the point-slope form, and simplifying to standard forms, students and practitioners can confidently analyze and construct lines with desired properties. This skill is fundamental not only in academic contexts but also in real-world applications where spatial relationships and linear models are essential. The derivation of $y = 3x - 17$, in this case, underscores the elegance and utility of algebra in solving geometric problems, reinforcing the importance of a solid understanding of slope, forms of line equations, and the properties of parallel lines.

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