What Is The Diameter Of The Circle Whose Center Is At (6,0) And Passes Through The Point (2,-3)

What Is The Diameter Of The Circle Whose Center Is At (6,0) And Passes Through The Point (2,-3) is a classic question in coordinate geometry that involves understanding the relationship between a circle's center, a point on its circumference, and the resulting diameter. To find the diameter of such a circle, we need to first determine the radius, which is the distance from the center to any point on the circle, and then double that value. This problem provides us with the center point at (6, 0) and a point on the circle at (2, -3), making it straightforward to apply the distance formula to find the radius.

Understanding the Basic Concepts

Before jumping into the calculations, it's essential to understand some fundamental concepts related to circles in coordinate geometry.

What Is a Circle?

A circle is a set of all points in a plane that are equidistant from a fixed point called the center. The fixed distance from the center to any point on the circle is known as the radius. The diameter, on the other hand, is twice the radius and passes through the center, connecting two points on the circle.

The Significance of the Center and a Point on the Circle

Knowing the center and at least one point on the circle allows us to determine the radius directly. Since the radius is the distance from the center to that point, calculating this distance is the key step in solving the problem.

Calculating the Radius of the Circle

Given:
- Center of the circle: $C(6, 0)$
- Point on the circle: $P(2, -3)$

The radius r is the distance between C and P.

Applying the Distance Formula

The distance d between two points (x_1, y_1) and (x_2, y_2) in the coordinate plane is given by:

$$
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
$$

Substituting the coordinates:

$$
r = \sqrt{(2 - 6)^2 + (-3 - 0)^2}
$$

Calculating each component:

- $(2 - 6) = -4$
- $(-3 - 0) = -3$

So,

$$
r = \sqrt{(-4)^2 + (-3)^2} = \sqrt{16 + 9} = \sqrt{25}
$$

$$
r = 5
$$

This means the radius of the circle is 5 units.

Determining the Diameter of the Circle

The diameter D of a circle is twice its radius:

$$
D = 2r
$$

Given that $r = 5$, the diameter is:

$$
D = 2 \times 5 = 10
$$

Therefore, the diameter of the circle is 10 units.

Summary of the Solution Process

Let's recap the steps for clarity:

1. Identify the center $(6, 0)$ and a point on the circle $(2, -3)$.
2. Use the distance formula to find the radius:
    - Calculate the differences in coordinates: -4 and -3.
    - Square these differences: 16 and 9.
    - Sum the squares: 25.
    - Take the square root: 5.
3. Calculate the diameter as twice the radius: $2 \times 5 = 10$.

Additional Insights and Related Concepts

Understanding this problem also provides insights into broader topics in geometry and algebra.

Using the Distance Formula in Coordinate Geometry

The distance formula is fundamental in coordinate geometry, enabling the calculation of distances between points, which is essential for defining geometric shapes.

Equation of the Circle

Once the radius and center are known, the equation of the circle can be written as:

$$
(x - h)^2 + (y - k)^2 = r^2
$$

Where (h, k) is the center. For our circle:

$$
(x - 6)^2 + (y - 0)^2 = 25
$$

This can be useful for graphing or further geometric analysis.

Applications of Circle Geometry

Knowledge of circles and their properties is useful in various fields, including engineering, physics, computer graphics, and navigation.

FAQs About Circle Diameter and Properties

Q1: Can the diameter be less than or greater than 10 in this case?

No. Since the radius is precisely 5 units, the diameter, which is twice the radius, must be 10 units. It cannot be less or more unless the circle's radius changes.

Q2: What if the given point was closer or farther from the center?

The distance calculation would directly adjust the radius. For points farther from the center, the radius increases, and the diameter increases accordingly.

Q3: How does this relate to the equation of the circle?

Knowing the center and radius helps you write the circle's equation, which is useful for plotting and solving related problems.

Conclusion

In conclusion, the diameter of the circle with its center at (6, 0) passing through the point (2, -3) is 10 units. This result was obtained by calculating the radius using the distance formula and then doubling it to find the diameter. Understanding how to determine the radius from the center and a point on the circle is a fundamental skill in coordinate geometry and has broad applications across mathematics and science. Whether you're solving geometry problems, graphing circles, or analyzing geometric relationships, mastering these concepts is essential for a solid mathematical foundation.

Frequently Asked Questions

How do I find the diameter of a circle given its center at (6,0) and a point (2,-3) on its circumference?

First, calculate the radius by finding the distance between the center (6,0) and the point (2,-3) using the distance formula. Then, multiply the radius by 2 to get the diameter.

What is the formula to determine the radius of a circle with center at (6,0) passing through (2,-3)?

The radius r is calculated as $r = \sqrt{[(x2 - x1)^2 + (y2 - y1)^2]}$, where (x1, y1) is the center and (x2, y2) is the point on the circle. Here, $r = \sqrt{[(2 - 6)^2 + (-3 - 0)^2]}$.

What is the distance between the points (6,0) and (2,-3)?

The distance is $\sqrt{[(2 - 6)^2 + (-3 - 0)^2]} = \sqrt{[(-4)^2 + (-3)^2]} = \sqrt{[16 + 9]} = \sqrt{25} = 5$ units.

What is the diameter of the circle passing through (2,-3) with center at (6,0)?

Since the radius is 5 units, the diameter is $2 \times 5 = 10$ units.

Can you explain the steps to find the diameter of this circle?

Yes. First, find the radius by calculating the distance between the center (6,0) and the point (2,-3), which is 5 units. Then, multiply the radius by 2 to get the diameter, which is 10 units.

Is the diameter of the circle with center at (6,0) passing through (2,-3) always 10 units?

Yes, because the distance from the center to the point (2,-3) is 5 units, so the diameter, being twice the radius, is always 10 units.

Additional Resources

What Is The Diameter Of The Circle Whose Center Is At (6,0) And Passes Through The Point (2,-3)?

Understanding the diameter of the circle whose center is at (6,0) and passes through the point (2,-3) is a fundamental problem in coordinate geometry. This question combines the concepts of the circle's center, radius, and diameter, providing an excellent opportunity to explore how algebra and geometry intertwine to solve real-world mathematical problems. Whether you're a student brushing up on your geometry skills or a professional seeking a clear explanation, this guide aims to walk you through the entire process step-by-step.

Introduction to Circles in Coordinate Geometry

Before diving into the specific problem, it's important to understand the basic elements of a circle in the coordinate plane:

- Center: The fixed point around which all points on the circle are equidistant. Denoted as (h, k).
- Radius (r): The distance from the center to any point on the circle.
- Diameter (d): The longest distance across the circle, passing through the center. It is twice the radius, i.e., d = 2r.

In the problem at hand, the center is given as (6, 0), and the circle passes through (2, -3). Our goal is to find the diameter of this circle.

Step 1: Understanding the Given Data

Let's clarify the key pieces of information:

- Center of the circle, C: (6, 0)
- A point on the circle, P: (2, -3)

Since the circle passes through point P, the distance from C to P is the radius (r) of the circle.

Step 2: Finding the Radius

The radius is the distance between the center (6, 0) and the point (2, -3). To find this, we employ the distance formula in coordinate geometry:

$$
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
$$

Plugging in the coordinates:

$$
r = \sqrt{(2 - 6)^2 + (-3 - 0)^2}
$$

Calculating each component:

- $x_2 - x_1 = 2 - 6 = -4$
- $y_2 - y_1 = -3 - 0 = -3$

Substitute into the formula:

$$
r = \sqrt{(-4)^2 + (-3)^2} = \sqrt{16 + 9} = \sqrt{25}
$$

$$
r = 5
$$

So, the radius of the circle is 5 units.

Step 3: Calculating the Diameter

Recall that the diameter of a circle is twice the radius:

$$
d = 2r
$$

Substitute the radius value:

$$
d = 2 \times 5 = 10
$$

Therefore, the diameter of the circle is 10 units.

Additional Insights: Confirming the Circle's Equation

While not necessary to find the diameter, understanding the full equation of the circle can provide deeper insights:

The standard form of a circle's equation is:

$$
(x - h)^2 + (y - k)^2 = r^2
$$

Plugging in the center (6, 0) and radius 5:

$$
(x - 6)^2 + (y - 0)^2 = 25
$$

which simplifies to:

$$
(x - 6)^2 + y^2 = 25
$$

This equation represents the circle with the given center and passing through the point (2, -3), confirming our earlier calculations.

Practical Applications and Related Concepts

Understanding how to compute the diameter based on a center point and a point on the circle has numerous applications:

- Design and Engineering: Calculating the size of circular components based on specific points.
- Navigation and Mapping: Determining distances and coverage areas.
- Physics: Analyzing circular motion where the radius is critical for calculating speed or acceleration.

Summary of the Process

Here's a quick recap of the steps to determine the diameter:

1. Identify the center point: (6, 0)
2. Identify a point on the circle: (2, -3)
3. Calculate the radius using the distance formula:

$$
r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
$$

4. Compute the diameter: $d = 2r$

Using the data from above:

$$
r = 5 \quad \Rightarrow \quad d = 10
$$

Final Thoughts

The problem of finding the diameter of a circle given its center and a point on it exemplifies the elegance of coordinate geometry. By translating geometric concepts into algebraic formulas, such problems become straightforward to solve with precision. Mastery of these fundamental techniques not only provides confidence in tackling similar questions but also lays the groundwork for more complex analyses in mathematics, physics, and engineering.

If you have any further questions about circle properties, coordinate geometry, or related topics, feel free to explore additional resources or seek guidance from math educators and professionals.

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