What Is The Equation Of The Line That In Parallel To Y=-5x-3 And That Pastes Through (2,-12) ?

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Understanding how to find the equation of a line that is parallel to a given line and passes through a specific point is a fundamental skill in algebra and coordinate geometry. In this comprehensive article, we will explore the steps involved in determining such a line, with detailed explanations, formulas, and examples to enhance your understanding. Whether you are a student preparing for exams or someone interested in mastering the concepts of lines and slopes, this guide provides a thorough walkthrough.

Introduction to Line Equations and Parallel Lines

Before diving into the specific problem, it’s essential to understand some basic concepts:

What Is a Line Equation?

A line in the coordinate plane can be represented by an equation. The most common form is the slope-intercept form:

$$ y = mx + b $$

where:
- m is the slope of the line,
- b is the y-intercept, the point where the line crosses the y-axis.

Understanding Slope

The slope m indicates the steepness and direction of the line:
- A positive slope means the line rises as it moves from left to right.
- A negative slope indicates the line falls as it moves from left to right.
- A slope of zero corresponds to a horizontal line.
- An undefined slope (which occurs in vertical lines) means the line is perfectly vertical.

Parallel Lines and Their Slopes

Two lines are parallel if they have:

- the same slope m,
- different y-intercepts to ensure they are distinct lines.

This means that if you know the slope of one line, any line parallel to it will have the same slope.

Analyzing the Given Line: Y = -5x - 3

The original line provided is:

$$y = -5x - 3$$

This is in slope-intercept form:
- Slope $m = -5$
- Y-intercept $b = -3$

Since the slope of the given line is -5, any line parallel to it will also have a slope of -5.

Finding the Equation of a Parallel Line Passing Through a Specific Point

Given:
- The original line: $y = -5x - 3$
- The point through which the new line passes: $(2, -12)$

Our goal:
- Find the equation of the line parallel to the original line
- That passes through $(2, -12)$

Step 1: Recognize the Slope of the New Line

Since the new line is parallel to the original, its slope is:

$$m_{new} = -5$$

Step 2: Use Point-Slope Form

The point-slope form of a line equation is:

$$y - y_1 = m (x - x_1)$$

where:
- (x_1, y_1) is the point the line passes through,

- m is the slope.

Plugging in the point $(2, -12)$ and $m = -5$:

$$y - (-12) = -5 (x - 2)$$

Simplify:

$$y + 12 = -5 (x - 2)$$

Step 3: Simplify to Slope-Intercept Form

Distribute:

$$y + 12 = -5x + 10$$

Subtract 12 from both sides:

$$y = -5x + 10 - 12$$

$$y = -5x - 2$$

This is the equation of the line parallel to $y = -5x - 3$ and passing through $(2, -12)$.

Final Equation of the Line

$$
\boxed{
\textbf{y = -5x - 2}
}
$$

This line has:
- The same slope as the original line (-5),
- Passes through the point $(2, -12)$.

Verifying the Solution

To ensure correctness, verify that the point $(2, -12)$ satisfies the new line's equation:

Plug in $x = 2$:

$$y = -5(2) - 2 = -10 - 2 = -12$$

Which matches the given point, confirming the solution.

Additional Concepts and Tips

Understanding Why Parallel Lines Have the Same Slope

- The key property of parallel lines is that they never intersect.
- For lines in the same plane, this is only possible if their slopes are equal.
- Different y-intercepts ensure the lines are not coincident but remain parallel.

Vertical and Horizontal Lines

- Vertical lines have an undefined slope and are of the form $x = c$.
- Horizontal lines have a slope of zero and are of the form $y = c$.
- If the original line had an undefined slope, the process would differ slightly.

Practice Problems

1. Find the equation of a line parallel to $y = 2x + 7$ passing through $(4, 10)$.
2. Determine the equation of a line perpendicular to $y = -3x + 5$ passing through $(0, 4)$.
3. Given the line $y = \frac{1}{2}x - 4$, find the equation of a line parallel passing through $(-1, 3)$.

Conclusion

Finding the equation of a line parallel to a given line and passing through a specific point involves understanding the relationship between slope and parallelism. The steps are straightforward:

1. Identify the slope of the original line.
2. Use the point-slope form with the given point and the identified slope.
3. Simplify to the desired form (usually slope-intercept form).

In our case, the line parallel to $y = -5x - 3$ passing through $(2, -12)$ is:

$$y = -5x - 2$$

This process can be applied to various problems involving parallel lines, making it a versatile tool in algebra and coordinate geometry.

Summary

- The slope of the given line $y = -5x - 3$ is -5.

- Any line parallel to it must have the same slope -5.
- Using the point-slope form with point $(2, -12)$ yields the new line's equation.
- The final answer is $y = -5x - 2$.

By understanding these steps, you can confidently determine the equations of lines parallel to any given line through any point, enhancing your problem-solving skills in mathematics.

Frequently Asked Questions

What is the slope of the line parallel to y = -5x - 3?

The slope is -5 because parallel lines have the same slope.

How do I find the equation of a line parallel to y = -5x - 3 passing through (2, -12)?

Use the point-slope form $y - y_1 = m(x - x_1)$, with m = -5 and point (2, -12).

What is the point-slope form of the line passing through (2, -12) with slope -5?

The point-slope form is y + 12 = -5(x - 2).

How do I convert y + 12 = -5(x - 2) to slope-intercept form?

Simplify: y + 12 = -5x + 10, then subtract 12 from both sides: y = -5x - 2.

What is the equation of the line parallel to y = -5x - 3 passing through (2, -12)?

The equation is y = -5x - 2.

Why do parallel lines have the same slope?

Because parallel lines never intersect and share the same steepness, which is determined by their slope.

Can the equation y = -5x - 2 be written in standard form?

Yes, rewriting gives 5x + y = -2.

What is the significance of the point (2, -12) in finding the line's equation?

It provides a specific point through which the new line passes, allowing us to determine its equation.

If the original line is y = -5x - 3, what is the y-intercept of the parallel line passing through (2, -12)?

The y-intercept of the parallel line is -2.

How do I verify that y = -5x - 2 passes through (2, -12)?

Plug x=2 into the equation: y = -5(2) - 2 = -10 - 2 = -12, confirming it passes through (2, -12).

Additional Resources

Equation of the Line That Is Parallel to y = -5x - 3 and Passes Through (2, -12)

When tackling the problem of finding the equation of a line that is parallel to a given line and passes through a specific point, it’s important to understand the fundamental concepts of slope, parallel lines, and point-slope form. In this article, we explore the detailed process of deriving this particular line's equation, providing clarity on each step involved. Whether you're a student preparing for exams or someone interested in understanding the geometry of lines, this comprehensive guide will walk you through the entire process, highlighting important features, common pitfalls, and useful tips.

Understanding the Given Line: y = -5x - 3

Before finding a parallel line, it is crucial to analyze the given line's characteristics:

- Slope (m): The slope of the line y = -5x - 3 is -5.
- Y-intercept: The line crosses the y-axis at -3.
- Line format: The equation is in slope-intercept form (y = mx + b).

Key Point: For two lines to be parallel, they must have the same slope. Therefore, any line parallel to y = -5x - 3 will also have a slope of -5.

Determining the Slope of the Parallel Line

Since the lines are parallel, the slope of our new line must be identical to that of the given line:

- Parallel line slope (m): -5

This is a fundamental property of parallel lines in Euclidean geometry: they never intersect and always share the same inclination.

Using the Point (2, -12) to Find the Equation

The next step involves using the given point through which the new line passes. The point (2, -12) provides the specific location that our line must go through.

The most straightforward method to find the equation is to use the point-slope form:

$$ y - y_1 = m(x - x_1) $$

where:

- $(x_1, y_1) = (2, -12)$
- $m = -5$

Plugging these into the equation:

$$ y - (-12) = -5(x - 2) $$

which simplifies to:

$$ y + 12 = -5(x - 2) $$

Deriving the Equation in Slope-Intercept Form

Expanding and simplifying:

$$ y + 12 = -5x + 10 $$

Subtract 12 from both sides:

$$ y = -5x + 10 - 12 $$
$$ y = -5x - 2 $$

Result: The equation of the line parallel to y = -5x - 3 and passing through (2, -12) is:

$$ y = -5x - 2 $$

Features and Properties of the Derived Line

- Slope: -5, identical to the original line, confirming parallelism.
- Y-intercept: -2, indicating where the line crosses the y-axis.
- Passes through: (2, -12), satisfying the given point constraint.
- Equivalence with point-slope form: $y + 12 = -5(x - 2)$

Visualizing the Line

Graphing this line helps to understand its position relative to the original line:

- The original line $y = -5x - 3$ crosses the y-axis at -3.
- The new line $y = -5x - 2$ crosses at -2.
- Both lines have the same slope, thus are parallel but have different y-intercepts, so they are distinct lines.

Plotting these lines on a coordinate plane reveals:

- The original line is slightly lower on the y-axis.
- The new line passes through (2, -12), which lies below the original line at the same x-value.

Tip: Visualizing helps reinforce the concept of parallelism and how different y-intercepts shift the line vertically.

Common Mistakes and How to Avoid Them

When solving such problems, students often encounter common pitfalls:

- Using the wrong slope: Remember, the parallel line must have the same slope.
- Incorrect point substitution: Ensure the point (2, -12) is correctly substituted into the point-slope formula.
- Mixing forms: Be comfortable converting between slope-intercept form, point-slope form, and standard form.
- Sign errors: Pay attention to signs during algebraic manipulation to avoid errors.

Pro Tip: Always double-check calculations, especially signs and arithmetic, to prevent mistakes.

Alternate Methods for Deriving the Equation

While the point-slope form is most straightforward here, other methods include:

- Using slope-intercept form directly: Since the slope is known, substitute the point into $y = mx + b$ and solve for b:

$$-12 = -5(2) + b$$
$$-12 = -10 + b$$
$$b = -12 + 10 = -2$$

Thus:

$$y = -5x - 2$$

- Standard form: Rearrange the slope-intercept form into standard form if needed:

$$y + 5x + 2 = 0$$

Summary of the Solution Process

To recap, here are the key steps involved:

1. Recognize that the slope of the original line y = -5x - 3 is -5.
2. Understand that parallel lines share the same slope.
3. Use the point-slope form with the point (2, -12) and slope -5.
4. Simplify to obtain y = -5x - 2.
5. Verify that the line passes through the given point.

This systematic approach ensures accuracy and clarity in solving line equations related to parallelism and specific points.

Final Remarks and Practical Applications

Understanding how to find the equation of a line parallel to a given line and passing through a particular point is fundamental in analytical geometry, engineering, physics, and computer graphics. Such skills enable professionals and students to model real-world scenarios, analyze geometric relationships, and solve complex problems involving lines and their intersections.

Features:

- Simplicity: Straightforward algebraic manipulation.
- Versatility: Can be applied to various line equations and points.
- Educational value: Reinforces core concepts like slope, point-slope form, and line equations.

Pros:

- Clear method for deriving the equation.
- Visual understanding through plotting.
- Reinforces algebraic skills.

Cons:

- Might be confusing for beginners unfamiliar with different forms of line equations.
- Requires careful attention to signs and algebraic steps.

In conclusion, the equation of the line parallel to y = -5x - 3 passing through (2, -12) is y = -5x - 2. Mastery of this problem enhances understanding of line equations, parallelism, and the application of algebraic methods in geometry. By following systematic steps and verifying each stage, learners can confidently solve similar problems with accuracy and efficiency.

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