Which Point Is A Solution Of X^2 + Y^2 > 49 And Y X^2 4 ? A. (8, 1)B. (3, 1)C. (1, 9)D. (0, 6)

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Understanding the Problem: Analyzing the Mathematical Conditions

In this content piece, we aim to determine which of the given points—(8, 1), (3, 1), (1, 9), or (0, 6)—satisfies the system of inequalities:

- $x^2 + y^2 > 49$
- $y \leq x^2 + 4$

This involves exploring both inequalities carefully, understanding their geometric interpretations, and then applying these to the specific points to identify the correct solution.

Breaking Down the Inequalities

1. The Inequality $x^2 + y^2 > 49$

This inequality describes all points (x, y) that lie outside the circle centered at the origin $(0, 0)$ with radius 7, since:

- The general form of a circle centered at $(0, 0)$ with radius r is $x^2 + y^2 = r^2$.
- Here, $r^2 = 49$, so $r=7$.

Therefore:

- Points satisfying $x^2 + y^2 > 49$ are outside or beyond the circle with radius 7.
- Points with $x^2 + y^2 = 49$ lie on the circle.
- Points with $x^2 + y^2 < 49$ are inside the circle.

2. The Inequality $y \leq x^2 + 4$

This inequality describes all points below or on the parabola:

- $y = x^2 + 4$.

The parabola opens upwards, with its vertex at $(0, 4)$. Points satisfying $y \leq x^2 + 4$ lie on or below this parabola.

Geometric Interpretation of the Constraints

Understanding these inequalities in a geometric context helps visualize the solution set:

- The circle $x^2 + y^2 = 49$ divides the plane into interior (inside circle) and exterior (outside circle).
- The inequality $x^2 + y^2 > 49$ restricts us to the exterior region.
- The parabola $y = x^2 + 4$ divides the plane into regions above and below it.
- The inequality $y \leq x^2 + 4$ restricts us to points on or below the parabola.

The solution to the system is the intersection of:

- The exterior of the circle $x^2 + y^2 > 49$,
- And the region on or below the parabola $y \leq x^2 + 4$.

Evaluating Each Point Against the Conditions

Now, we will analyze each of the four points by substituting their (x, y) values into both inequalities.

Point A: (8, 1)

- Check $x^2 + y^2 > 49$:

$$
8^2 + 1^2 = 64 + 1 = 65
$$

Since $65 > 49$, this satisfies the first inequality.

- Check $y \leq x^2 + 4$:

$$
1 \leq 8^2 + 4 = 64 + 4 = 68
$$

Since $1 \leq 68$, this also satisfies the second inequality.

Conclusion: Point (8, 1) satisfies both inequalities.

Point B: (3, 1)

- Check $x^2 + y^2 > 49$:

$$
3^2 + 1^2 = 9 + 1 = 10
$$

Since $10 < 49$, this does not satisfy the first inequality.

- Check $y \leq x^2 + 4$:

$$
1 \leq 3^2 + 4 = 9 + 4 = 13
$$

Even though this is true, the first inequality's condition isn't met, so Point B does not satisfy the system.

Point C: (1, 9)

- Check $x^2 + y^2 > 49$:

$$
1^2 + 9^2 = 1 + 81 = 82
$$

Since $82 > 49$, this satisfies the first inequality.

- Check $y \leq x^2 + 4$:

$$
9 \leq 1^2 + 4 = 1 + 4 = 5
$$

Since $9 \not\leq 5$, this does not satisfy the second inequality.

Conclusion: Point (1, 9) fails the second condition.

Point D: (0, 6)

- Check $x^2 + y^2 > 49$:

$$
0^2 + 6^2 = 0 + 36 = 36
$$

Since $36 < 49$, this does not satisfy the first inequality.

- Check $y \leq x^2 + 4$:

$$
6 \leq 0 + 4 = 4
$$

Since $6 \not\leq 4$, this also fails the second inequality.

Conclusion: Point (0, 6) does not satisfy the system.

Final Selection: Which Point Is a Solution?

Based on the above analysis:

- Point A (8, 1): Satisfies both inequalities.
- Point B (3, 1): Fails the first inequality.
- Point C (1, 9): Fails the second inequality.
- Point D (0, 6): Fails both inequalities.

Therefore, the point (8, 1) is the only among the options that satisfies both $x^2 + y^2 > 49$ and $y \leq x^2 + 4$.

Implications and Additional Insights

Understanding the solution to such inequalities is fundamental in coordinate geometry, especially in graphing and analyzing regions defined by inequalities.

Key takeaways:

- The circle $x^2 + y^2 = 49$ acts as a boundary, with solutions outside the circle satisfying $x^2 + y^2 > 49$.
- The parabola $y = x^2 + 4$ acts as an upper boundary for the y-values.
- The feasible region for the solution is the area outside the circle but on or below the parabola.
- The point (8, 1) lies outside the circle (since $x^2 + y^2 = 65 > 49$) and below the parabola ($1 \leq 68$), making it a valid solution.

Summary and Conclusion

In conclusion, after carefully analyzing each option against the inequalities, we find that Option A: (8, 1) is the correct point that satisfies both conditions:

- $x^2 + y^2 > 49$
- $y \leq x^2 + 4$

This detailed exploration underscores the importance of understanding geometric representations of inequalities and applying systematic evaluation to identify solutions efficiently.

SEO Optimization Tips for Similar Content

- Use relevant keywords such as "coordinate geometry inequalities," "solution of inequalities," "graphing inequalities," and "point satisfying inequalities."
- Incorporate long-tail keywords like "how to determine whether a point satisfies inequalities."
- Add internal links to related topics such as "graphing circles and parabolas" or "solving systems of inequalities."
- Use descriptive meta descriptions for better search engine ranking.
- Include visuals like graphs of the circle and parabola for enhanced user engagement.
- Regularly update content with new examples and practice problems to maintain relevance.

If you want more detailed explanations or additional examples on solving inequalities, feel free to ask!

Frequently Asked Questions

Does the point (8, 1) satisfy the inequality $X^2 + Y^2 > 49$?

Yes, because $8^2 + 1^2 = 64 + 1 = 65$, which is greater than 49.

Is the point (8, 1) also satisfying the inequality $Y < X^2 / 4$?

Yes, since $1 < (8)^2 / 4 = 64 / 4 = 16$, the point satisfies the second inequality.

Does the point (3, 1) satisfy both inequalities?

Yes, $3^2 + 1^2 = 9 + 1 = 10$, which is not greater than 49, so it does not satisfy the first inequality.

Is the point (1, 9) a solution to both inequalities?

No, because $1^2 + 9^2 = 1 + 81 = 82 > 49$, so it satisfies the first inequality, but for the second, $9 < (1)^2 / 4 = 0.25$, which is false.

Does the point (0, 6) satisfy the inequality $X^2 + Y^2 > 49$?

No, since $0^2 + 6^2 = 36$, which is not greater than 49.

Is the point (0, 6) a solution to the inequality $Y < X^2 / 4$?

Yes, because $6 < 0 / 4 = 0$, which is false, so it does not satisfy the second inequality.

Which point among the options satisfies both inequalities?

The point (8, 1) satisfies both $X^2 + Y^2 > 49$ and $Y < X^2 / 4$.

How can we verify if a point is a solution to both inequalities?

By substituting the point into both inequalities and checking if both conditions are true.

Additional Resources

Which Point Is A Solution Of X^2 + Y^2 > 49 And Y X^2 4 ? A. (8, 1)B. (3, 1)C. (1, 9)D. (0, 6)

Introduction

In the realm of analytical geometry, understanding the relationship between points and inequalities is fundamental. Today, we explore a problem that involves determining which of four given points satisfies two specific inequalities: \(x^2 + y^2 > 49 \) and \(y \leq x^2 \). The question is presented as follows:

Which point among (8, 1), (3, 1), (1, 9), and (0, 6) is a solution to the inequalities \(x^2 + y^2 > 49 \) and \(y \leq x^2 \)?

This problem requires a step-by-step analysis of each point against the given inequalities. The goal is to understand not only which points satisfy these inequalities but also to grasp the underlying geometric concepts they represent. This article aims to dissect these inequalities, interpret their geometric significance, and methodically evaluate each candidate point.

Understanding the Inequalities

The First Inequality: $x^2 + y^2 > 49$

This inequality describes a region outside a circle with radius 7 centered at the origin (0,0).

- Equation of the circle: $x^2 + y^2 = 49$
- Region satisfying $x^2 + y^2 > 49$: All points outside the circle.
- Region satisfying $x^2 + y^2 < 49$: All points inside the circle.

Thus, for any point (x, y), if the sum of the squares of its coordinates exceeds 49, the point lies outside the circle of radius 7.

The Second Inequality: $y \leq x^2$

This inequality describes all points below or on the parabola $y = x^2$.

- The parabola $y = x^2$: Opens upward, with vertex at (0, 0).

- Region satisfying $y \leq x^2$: All points on or beneath the parabola.

Visualizing the Region

Understanding the combined constraints requires visualizing the two regions:

- Outside the circle $x^2 + y^2 = 49$: The area outside the circle, which includes points with large magnitude of x or y.

- Below or on the parabola $y = x^2$: All points with y less than or equal to x^2.

The solution set to the system of inequalities is the intersection of these two regions:

- Points outside the circle: large $|x|$ and $|y|$.

- Points below the parabola: where y is less than or equal to x^2.

This intersection creates a specific region in the coordinate plane. Our task is to determine which of the candidate points lies within this region.

Evaluating Each Candidate Point

Let's analyze each point systematically by substituting their coordinates into both inequalities.

Point A: (8, 1)

Step 1: Check $x^2 + y^2 > 49$

- $x^2 = 8^2 = 64$
- $y^2 = 1^2 = 1$
- Sum: $64 + 1 = 65$

Since $65 > 49$, the point (8, 1) satisfies the first inequality.

Step 2: Check $y \leq x^2$

- $y = 1$
- $x^2 = 64$
- Is $1 \leq 64$? Yes.

Conclusion: Point (8, 1) satisfies both inequalities, making it a solution.

Point B: (3, 1)

Step 1: Check $x^2 + y^2 > 49$

- $3^2 = 9$
- $1^2 = 1$
- Sum: $9 + 1 = 10$

Since 10 is not greater than 49, the point (3, 1) does not satisfy the first inequality.

Conclusion: No need to check the second inequality; this point is not part of the solution set.

Point C: (1, 9)

Step 1: Check $x^2 + y^2 > 49$

- $1^2 = 1$
- $9^2 = 81$
- Sum: $1 + 81 = 82$

Since $82 > 49$, the point (1, 9) satisfies the first inequality.

Step 2: Check $y \leq x^2$

- $y = 9$
- $x^2 = 1$
- Is $9 \leq 1$? No.

Because y exceeds x^2, the point does not satisfy the second inequality.

Conclusion: (1, 9) is outside the circle but not on or below the parabola; thus, not a solution.

Point D: (0, 6)

Step 1: Check $x^2 + y^2 > 49$

- $0^2 = 0$
- $6^2 = 36$
- Sum: $0 + 36 = 36$

Since 36 is less than 49, the point (0, 6) does not satisfy the first inequality.

Conclusion: No need to check the second inequality; this point is not part of the solution.

Final Determination

Based on the step-by-step evaluation:

- Point A (8, 1): Satisfies both inequalities.
- Point B (3, 1): Fails the first inequality.
- Point C (1, 9): Satisfies the first but fails the second.
- Point D (0, 6): Fails the first inequality.

Therefore, the only point among the options that satisfies both inequalities is (8, 1).

Geometric Interpretation

Understanding why (8, 1) is a valid solution provides insight into the geometry of the problem:

- The point (8, 1) lies outside the circle of radius 7 because the sum $x^2 + y^2 = 65$ exceeds 49.

- It also lies below the parabola $y = x^2$ because $y = 1$ is less than 64, the value of x^2.

Visualizing this, (8, 1) appears to be in the region outside the circle but below the parabola, satisfying the combined inequalities.

Additional Considerations

Why is understanding the boundary important?

- The inequalities involve strict inequalities ($>$ and $\leq$), which define open and closed regions.

- The point (8, 1) is strictly outside the circle, not on it, and below or on the parabola, maintaining the strictness.

How does this help in graphing?

- Graphing the circle $x^2 + y^2 = 49$ provides a visual boundary.

- Plotting the parabola $y = x^2$ shows the region below it.

- The solution region is outside the circle and below the parabola.

Summary

This exploration reveals that among the four options, only the point (8, 1) satisfies both inequalities:

- $x^2 + y^2 > 49$ (outside the circle of radius 7).

- $y \leq x^2$ (below or on the parabola).

Understanding the geometric regions these inequalities represent helps in visualizing and solving similar problems. Recognizing that the region of interest is the intersection of the outside of a circle and below a parabola simplifies the process of identifying solutions.

Final Remarks

Such problems are fundamental in analytical geometry, combining algebraic reasoning with geometric visualization. They demonstrate the importance of systematically verifying each candidate point against the inequalities and appreciate the spatial regions they define. Mastery of these concepts enables students and enthusiasts to analyze complex systems of inequalities with confidence, paving the way for more advanced studies in mathematics, physics, and engineering disciplines.

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