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Understanding the Problem: Finding a Vector Function R(t) with Given Derivative Conditions

When dealing with vector calculus, a common problem involves finding a vector function R(t) given certain conditions on its derivative R'(t). In this particular case, we are asked to find a vector function R(t) such that its derivative R'(t) is specified as (sin 7t, Sin...). Although the problem statement appears incomplete, the core idea remains: given the derivative, we need to determine the original function R(t).

This type of problem is fundamental in calculus, especially when analyzing motion in physics, where R(t) could represent position vectors, or in mathematics when reconstructing functions from their derivatives. The process primarily involves integration, since the derivative is given, and the original function is obtained via indefinite integrals.

In this article, we will explore the detailed steps to find R(t), discuss the concepts of vector differentiation and integration, and provide a comprehensive guide to solving such problems.

Key Concepts in Vector Calculus Relevant to the Problem

1. Vector Functions and Their Derivatives

A vector function R(t) in two or three dimensions can be written as:

$$ R(t) = \langle x(t), y(t) \rangle $$

or in three dimensions:

$$ R(t) = \langle x(t), y(t), z(t) \rangle $$

The derivative R'(t) is then:

$$ R'(t) = \left\langle x'(t), y'(t) \right\rangle $$

(or include z'(t) in three dimensions).

This derivative represents the rate of change of the position vector with respect to the parameter t.

2. Integration of Vector Functions

Given R'(t), to find R(t), we integrate each component separately:
$$ R(t) = \int R'(t) \, dt + C $$
where C is a constant vector determined by initial conditions.

Step-by-Step Solution Approach

Since the problem specifies that:
$$ R'(t) = (\sin 7t, \sin \text{...}) $$
we focus on integrating each component to find R(t). Assuming the second component is $\sin t$ or similar, the process would be analogous; the core idea remains the same.

Step 1: Write Down the Derivative Components Clearly

Suppose the derivative is:
$$ R'(t) = (\sin 7t, \sin t) $$
This is a common form in such problems, but you should adapt depending on the actual components provided.

Step 2: Integrate Each Component

- For the first component:
$$ x(t) = \int \sin 7t \, dt $$
- For the second component:
$$ y(t) = \int \sin t \, dt $$

Step 3: Compute the Integrals

Integral of $\sin 7t$:
$$ \int \sin 7t \, dt = -\frac{1}{7} \cos 7t + C_1 $$

Integral of $\sin t$:
$$ \int \sin t \, dt = -\cos t + C_2 $$

Thus, the vector function R(t) becomes:
$$ R(t) = \left\langle -\frac{1}{7} \cos 7t + C_1, -\cos t + C_2 \right\rangle $$

Step 4: Include Constants of Integration

Constants C_1 and C_2 can be determined if initial conditions are provided, such as R(t0) = R0. Without initial conditions, the most general form includes these arbitrary constants.

Step 5: Write the Final Expression for R(t)

The general solution:
$$R(t) = \left\langle -\frac{1}{7} \cos 7t + C_1, -\cos t + C_2 \right\rangle$$

If initial conditions are given, for example, R(0) = R0, you can substitute t = 0 to find these constants.

Understanding the Role of Constants and Initial Conditions

Constants of integration C_1 and C_2 are essential to fully specify R(t). They represent the initial position of the vector at some initial parameter value t_0.

Example:
Suppose $R(0) = (x_0, y_0)$, then:
$$R(0) = \left\langle -\frac{1}{7} \cos 0 + C_1, -\cos 0 + C_2 \right\rangle = (x_0, y_0)$$
which simplifies to:
$$\left\langle -\frac{1}{7} (1) + C_1, -1 + C_2 \right\rangle = (x_0, y_0)$$
Leading to:
$$C_1 = x_0 + \frac{1}{7}$$
$$C_2 = y_0 + 1$$

Thus, the specific vector function becomes:
$$R(t) = \left\langle -\frac{1}{7} \cos 7t + x_0 + \frac{1}{7}, -\cos t + y_0 + 1 \right\rangle$$

Extensions and Additional Considerations

1. Higher Dimensions

If the problem involves three-dimensional space, R'(t) would have three components, and the integration process would be similar for each component.

2. Non-constant Derivatives

In more complex cases where $R'(t)$ isn't straightforward, techniques such as substitution, integration by parts, or numerical methods may be necessary.

3. Physical Interpretation

In physics, if $R(t)$ represents position, then $R'(t)$ is velocity. Finding $R(t)$ from velocity involves integrating velocity over time, accounting for initial position.

Common Mistakes to Avoid

- Forgetting to include constants of integration.
- Confusing indefinite and definite integrals.
- Not applying initial conditions when they are available.
- Misinterpreting the components of the vector function.

Summary

To find a vector function $R(t)$ given its derivative $R'(t)$:

1. Identify each component of $R'(t)$.
2. Integrate each component independently with respect to t.
3. Add constants of integration to account for initial conditions.
4. Use initial conditions, if available, to determine the constants.
5. Express the final form of $R(t)$, which reconstructs the original vector function.

This process emphasizes the fundamental role of integration in reverse-engineering vector functions from their derivatives, a key skill in calculus and applied mathematics.

Conclusion

Understanding how to find $R(t)$ from $R'(t)$ is a critical aspect of vector calculus, with applications across physics, engineering, and mathematics. By carefully integrating each component and considering initial conditions, one can reconstruct the original vector function accurately. Whether dealing with simple sinusoidal derivatives or more complex functions, the method remains consistent—integrate, include constants, and apply initial conditions if available.

Remember: The core steps are always the same, and mastering this process enhances your overall proficiency in calculus and vector analysis.

Frequently Asked Questions

What does the derivative R'(t) = (sin 7t, sin t) tell us about the vector function R(t)?

It indicates that the rate of change of R(t) in the x-direction is sin 7t and in the y-direction is sin t, which helps in determining R(t) through integration.

How do you find the vector function R(t) given its derivative R'(t) = (sin 7t, sin t)?

Integrate each component separately with respect to t: $R(t) = (\int \sin 7t\, dt, \int \sin t\, dt) + C$, where C is a constant vector.

What are the indefinite integrals of sin 7t and sin t?

$\int \sin 7t\, dt = -(1/7) \cos 7t + C_1$, and $\int \sin t\, dt = -\cos t + C_2$.

How do initial conditions affect the determination of the constant vector C in R(t)?

Initial conditions specify the value of R(t) at a specific t, allowing you to solve for the constants C to fully determine R(t).

Can you provide a general form for R(t) based on the given derivative R'(t)?

Yes, $R(t) = (-1/7) \cos 7t + A, (-1) \cos t + B$, where A and B are constants determined by initial conditions.

What is the significance of integrating the derivative R'(t) to find R(t)?

Integrating R'(t) reconstructs the original vector function R(t), representing the position vector based on its rate of change.

Additional Resources

Vector Function Construction: Unraveling the Derivative Condition

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Introduction: The Art of Reconstructing a Vector Function from Its Derivative

In the realm of multivariable calculus, understanding how to reconstruct a vector function $\mathbf{R}(t)$ from a given derivative $\mathbf{R'}(t)$ is akin to piecing together a complex puzzle. Imagine having a detailed map of a journey—the derivatives—yet needing to chart the exact route taken, the position at any given time. This process is fundamental not only for theoretical pursuits but also for practical applications like physics, engineering, and computer graphics.

Today, we explore a specific yet quintessential problem: find a vector function $\mathbf{R}(t)$ such that its derivative matches a given vector function:

$$
\mathbf{R'}(t) = \left(\sin 7t, \sin t \right)
$$

This problem serves as a perfect case study for the integration of vector functions, combining knowledge of calculus with creative problem-solving. Let's dive deep into the methodology, step-by-step, while maintaining an engaging, expert-level tone.

Understanding the Problem: What Is Being Asked?

The problem states: Find a vector function $\mathbf{R}(t)$ such that its derivative $\mathbf{R'}(t)$ equals the given vector function $(\sin 7t, \sin t)$.

In mathematical terms:

$$
\mathbf{R'}(t) = \left(\sin 7t, \sin t \right)
$$

The goal is to determine $\mathbf{R}(t)$, which is the antiderivative (or indefinite integral) of this derivative. But what does this entail?

- Component-wise Integration: Since the vector function has two components, we can think of the problem as two separate scalar integrations:
- The first component: integrate $\sin 7t$
- The second component: integrate $\sin t$

- General Solution: Because indefinite integrals include an arbitrary constant, the most complete form of the solution accounts for these constants, which can be different for each component.

- Physical Interpretation: If this vector function describes a particle's position over time, then the derivative gives its velocity; integrating velocity yields position, up to initial conditions.

Methodology: From Derivative to Function

Step 1: Break Down the Problem

Given:

$$
\mathbf{R'}(t) = \left(\sin 7t, \sin t \right)
$$

We seek:

$$
\mathbf{R}(t) = \left(R_1(t), R_2(t) \right)
$$

where:

$$
R_1(t) = \int \sin 7t\, dt + C_1
$$
$$
R_2(t) = \int \sin t\, dt + C_2
$$

Here, C_1 and C_2 are arbitrary constants reflecting initial position.

Step 2: Integrate Each Component

Let's analyze each integral carefully.

Integrating $\sin 7t$:

Recall the standard integral:

$$
\int \sin(k t) dt = -\frac{1}{k} \cos(k t) + \text{constant}
$$

Applying this to $R_1(t)$:

$$
R_1(t) = -\frac{1}{7} \cos 7t + C_1
$$

Integrating $\sin t$:

Similarly,

$$
R_2(t) = -\cos t + C_2
$$

Step 3: Assemble the Vector Function

Combining the components, the general form of $\mathbf{R}(t)$ is:

$$
\boxed{
\mathbf{R}(t) = \left(-\frac{1}{7} \cos 7t + C_1,\ - \cos t + C_2 \right)
}
$$

where C_1 and C_2 are arbitrary real constants.

Interpreting the Constants: Initial Conditions and Particular Solutions

In calculus applications, the constants C_1 and C_2 often stem from initial conditions—known positions at a specific time t_0. For example, suppose the initial position at $t=0$ is given by:

$$
\mathbf{R}(0) = (x_0, y_0)
$$

then:

$$
R_1(0) = -\frac{1}{7} \cos 0 + C_1 = -\frac{1}{7} (1) + C_1 = -\frac{1}{7} + C_1
$$
$$
R_2(0) = - \cos 0 + C_2 = -1 + C_2
$$

which gives:

$$
C_1 = x_0 + \frac{1}{7}
$$
$$
C_2 = y_0 + 1
$$

Thus, the particular solution tailored to initial conditions becomes:

$$
\boxed{
\mathbf{R}(t) = \left(-\frac{1}{7} \cos 7t + x_0 + \frac{1}{7},\ - \cos t + y_0 + 1 \right)
}
$$

Special Cases and Additional Insights

1. Zero Constants (Origin-based Solution):

Choosing $C_1 = 0$ and $C_2 = 0$ yields a specific antiderivative passing through the origin:

$$
\mathbf{R}(t) = \left(-\frac{1}{7} \cos 7t,\ - \cos t \right)
$$

which can be useful for understanding the underlying motion pattern without initial offsets.

2. Physical or Geometric Interpretation:

- The first component involves $\cos 7t$, which oscillates rapidly with a frequency 7 times that of $\cos t$. This suggests a motion with multiple oscillations in the x-direction relative to the y-direction.
- The constants C_1 and C_2 shift the entire trajectory, akin to setting the initial position.

3. Graphical Visualization:

Plotting these parametric equations reveals complex oscillatory behavior, characteristic of combined harmonic motions with different frequencies—a common feature in wave physics and signal processing.

Extensions and Additional Considerations

While this particular problem is straightforward, it opens the door to several advanced topics:

- Definite Integrals: If initial or boundary data are known at specific points, the constants are determined, leading to a unique solution.
- Higher-Order Derivatives: Extending the approach to second derivatives can model acceleration or curvature.
- Vector Fields: Understanding how such vector functions relate to vector fields and flow lines can be instrumental in fluid dynamics.

Summary and Final Remarks

Constructing a vector function $\mathbf{R}(t)$ from its derivative $\mathbf{R'}(t)$ is a fundamental skill in calculus, blending integration techniques with geometric intuition. In our specific case:

$$
\mathbf{R'}(t) = \left(\sin 7t, \sin t \right)
$$

the indefinite integral yields:

$$
\boxed{
\mathbf{R}(t) = \left(-\frac{1}{7} \cos 7t + C_1,\ - \cos t + C_2 \right)
}
$$

This solution encapsulates the essential process—integrating each component separately, incorporating arbitrary constants, and recognizing the importance of initial conditions for particular solutions.

Whether modeling the oscillatory motion of a particle, analyzing waveforms, or exploring the fundamental properties of vector calculus, this approach exemplifies the powerful simplicity and elegance of calculus techniques. It highlights how, with careful step-by-step reasoning, complex derivative conditions can be unraveled into meaningful, functional forms.

In essence, mastering the art of reconstructing vector functions from derivatives enhances your mathematical toolkit, empowering you to tackle diverse problems in science, engineering, and beyond with confidence and precision.

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