(1 Point) Find The Representation Of (-5, 5, 1) In Each Of The Following Ordered Bases. Your Answers

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Understanding how to find the representation of a vector in different bases is a fundamental skill in linear algebra. When working with vector spaces, the basis provides a coordinate system that allows us to express vectors uniquely. The problem at hand involves determining the coordinates of the vector $(-5, 5, 1)$ relative to various ordered bases. This process involves solving systems of linear equations to express the vector as a linear combination of the basis vectors.

In this article, we will explore the step-by-step methods to find the representations of $(-5, 5, 1)$ in different ordered bases. We will examine the general approach, provide detailed examples, and illustrate the importance of basis transformations in understanding vector spaces.

Understanding Bases and Coordinate Representations

What Is a Basis?

A basis of a vector space is a set of vectors that are:

- Linearly independent
- Span the entire space

In $\mathbb{R}^3$, a basis typically consists of three vectors, say $\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3$. Any vector $\mathbf{v}$ in $\mathbb{R}^3$ can then be expressed uniquely as:

$$
\mathbf{v} = c_1 \mathbf{b}_1 + c_2 \mathbf{b}_2 + c_3 \mathbf{b}_3
$$

where c_1, c_2, c_3 are the coordinates of $\mathbf{v}$ relative to that basis.

Coordinate Representation of a Vector

Given a basis $B = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$, the coordinate representation of $\mathbf{v}$ relative to B is the ordered triple (c_1, c_2, c_3). To find these coordinates:
1. Write $\mathbf{v}$ as a linear combination of the basis vectors.
2. Set up a system of equations based on the components of $\mathbf{v}$ and the basis vectors.
3. Solve for the coefficients c_1, c_2, c_3.

Step-by-Step Method to Find Coordinates

1. Write the basis vectors in matrix form

Create a matrix M, where each column is a basis vector:
$$
M = \begin{bmatrix}
| & | & | \\
\mathbf{b}_1 & \mathbf{b}_2 & \mathbf{b}_3 \\
| & | & |
\end{bmatrix}
$$
This matrix transforms coordinate vectors into the standard basis.

2. Set up the equation $M \mathbf{c} = \mathbf{v}$

Where:
- $\mathbf{c} = (c_1, c_2, c_3)^T$
- $\mathbf{v} = (-5, 5, 1)^T$

Solve for $\mathbf{c}$:
$$
\mathbf{c} = M^{-1} \mathbf{v}
$$
if M is invertible.

3. Compute the inverse of the basis matrix

Using methods such as Gaussian elimination or matrix inversion formulas, find M^{-1}, then multiply by $\mathbf{v}$.

4. Interpret the solution

The resulting vector $\mathbf{c}$ gives the coordinates of $\mathbf{v}$ relative to the basis.

Practical Examples: Finding the Representation in Specific Bases

Below, we’ll demonstrate how to find the coordinate representation of $(-5, 5, 1)$ in different bases.

Example 1: Standard Basis in $\mathbb{R}^3$

The standard basis vectors are:

$$
\mathbf{e}_1 = (1, 0, 0), \quad \mathbf{e}_2 = (0, 1, 0), \quad \mathbf{e}_3 = (0, 0, 1)
$$

The basis matrix is the identity matrix:

$$
M = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
$$

Therefore, the coordinate representation of $\mathbf{v} = (-5, 5, 1)$ is simply $(-5, 5, 1)$.

Example 2: A Custom Basis in $\mathbb{R}^3$

Suppose the basis vectors are:

$$
\mathbf{b}_1 = (1, 2, 0), \quad \mathbf{b}_2 = (0, 1, 1), \quad \mathbf{b}_3 = (1, 0, 1)
$$

Construct the basis matrix:

$$
M = \begin{bmatrix}
1 & 0 & 1 \\
2 & 1 & 0 \\
0 & 1 & 1
\end{bmatrix}
$$

Calculate M^{-1}, then multiply by $\mathbf{v} = (-5, 5, 1)^T$.

Step-by-step:
1. Compute the inverse of M:
- Use Gaussian elimination or a calculator.
2. Multiply M^{-1} by $\mathbf{v}$:

$$
\mathbf{c} = M^{-1} \times \begin{bmatrix} -5 \\ 5 \\ 1 \end{bmatrix}
$$

3. Interpret the resulting $\mathbf{c}$ as the coordinates relative to this basis.

Importance of Basis Selection and Transformation

Choosing different bases can simplify computations, reveal different properties of vectors, or align with specific problem contexts. For example:
- In physics, basis vectors might align with coordinate axes, directions of motion, or other meaningful directions.
- In computer graphics, basis transformations are used for coordinate changes, rotations, and scaling.

Knowing how to convert vector representations between different bases is essential in many fields, including engineering, computer science, and mathematics.

Summary of Key Points

- The coordinate representation of a vector depends on the basis chosen.
- To find the representation, express the vector as a linear combination of basis vectors and solve the resulting system of equations.
- Construct the basis matrix, compute its inverse, and multiply by the vector to find coordinates.
- Different bases can provide different insights and simplify specific computations.

Conclusion

The process of finding the representation of a vector like $(-5, 5, 1)$ in various ordered bases is a fundamental aspect of understanding vector spaces and linear transformations. Whether working with the standard basis or custom bases, the core method involves constructing the basis matrix, inverting it, and solving for the coefficients. Mastery of this technique enables better manipulation of vectors across different coordinate systems, which is invaluable across scientific and engineering disciplines.

By practicing these steps with various bases, students and professionals can deepen their understanding of linear algebra concepts and develop more flexible problem-solving skills.

Frequently Asked Questions

How do I find the coordinate representation of the vector (-5, 5, 1) in a given ordered basis?

To find the coordinate representation, express the vector as a linear combination of the basis vectors and solve for the coefficients. These coefficients are the coordinates relative to the basis.

What is the process to convert a vector's standard coordinates to a different ordered basis?

You need to find the change of basis matrix from the given basis to the standard basis, then multiply its inverse by the vector's standard coordinates to get the new representation.

If given an ordered basis B = {b1, b2, b3}, how do I compute the coordinates of (-5, 5, 1) in B?

Set up the equation (-5, 5, 1) = c1b1 + c2b2 + c3b3 and solve for c1, c2, c3. The solutions are the coordinates of the vector in basis B.

Can you provide an example of finding the representation of (-5, 5, 1) in a specific basis?

Yes. For example, if the basis is B = { (1,0,0), (0,1,0), (0,0,1) }, then the representation is simply (-5, 5, 1). For a different basis, you'd solve accordingly.

What are common mistakes to avoid when converting a vector to a different basis?

Common mistakes include mixing up basis vectors, incorrectly solving the linear system, or not verifying that the basis vectors are linearly independent. Always double-check your calculations.

Additional Resources

Representation of a Vector in Different Bases: An Expert Breakdown

Understanding how vectors are represented in various bases is fundamental in linear algebra, with practical applications spanning computer graphics, physics, engineering, and data science. Today, we delve into the detailed process of expressing the vector (-5, 5, 1) in different ordered bases, providing a comprehensive guide that is both educational and insightful.

Introduction to Vector Representation in Different Bases

At its core, a vector's representation depends on the basis chosen for the vector space. Think of a basis as the coordinate system or the "building blocks" that define the space. When you change the basis, the coordinates of a vector change, but the vector itself remains the same geometrically.

Why is this important? Because different bases can simplify problems, reveal hidden properties, or make computations more efficient. Whether you're transforming coordinates in 3D space, working with eigenvectors, or analyzing signals, understanding basis representations is crucial.

Understanding the Vector and Bases in Question

The vector under consideration is:

$$
\mathbf{v} = (-5, 5, 1)
$$

We are asked to find its coordinate representation relative to different ordered bases. Although the specific bases are not provided in the prompt, typical problems involve bases such as:

- Standard basis: $\mathbf{e}_1 = (1,0,0)$, $\mathbf{e}_2 = (0,1,0)$, $\mathbf{e}_3 = (0,0,1)$

- Custom bases: e.g., $\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3$, where each basis vector is a specific 3D vector.

Important note: For each basis $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$, the vector $\mathbf{v}$ can be expressed as:

$$
\mathbf{v} = c_1 \mathbf{b}_1 + c_2 \mathbf{b}_2 + c_3 \mathbf{b}_3
$$

where \(c_1, c_2, c_3\) are the coordinates of \(\mathbf{v}\) in basis \(\mathcal{B}\).

Step-by-Step Methodology for Finding Coordinates

The general approach involves solving a system of linear equations. Let's break down the process:

1. Express the basis vectors

Suppose the basis vectors are:

\[
\mathbf{b}_1 = (b_{11}, b_{12}, b_{13}), \quad
\mathbf{b}_2 = (b_{21}, b_{22}, b_{23}), \quad
\mathbf{b}_3 = (b_{31}, b_{32}, b_{33})
\]

2. Set up the matrix equation

The vector \(\mathbf{v}\) can be written as:

\[
\begin{bmatrix}
b_{11} & b_{21} & b_{31} \\
b_{12} & b_{22} & b_{32} \\
b_{13} & b_{23} & b_{33}
\end{bmatrix}
\begin{bmatrix}
c_1 \\
c_2 \\
c_3
\end{bmatrix}
=
\begin{bmatrix}
-5 \\
5 \\
1
\end{bmatrix}
\]

This is a matrix equation \(B \mathbf{c} = \mathbf{v}\).

3. Solve for $\mathbf{c}$

To find the coordinate vector $\mathbf{c}$, solve:

$$
\mathbf{c} = B^{-1} \mathbf{v}
$$

where B^{-1} is the inverse of the basis matrix B.

Note: The basis vectors must be linearly independent for the inverse to exist.

Applying the Method to Specific Bases

Since the prompt mentions "each of the following ordered bases" but does not specify them, let's consider some common bases for illustration.

Case 1: Standard Basis

Standard basis vectors:

$$
\mathbf{e}_1 = (1,0,0), \quad
\mathbf{e}_2 = (0,1,0), \quad
\mathbf{e}_3 = (0,0,1)
$$

Representation:

Expressing $\mathbf{v}$ in the standard basis is trivial:

$$
\boxed{
\mathbf{v} = -5 \mathbf{e}_1 + 5 \mathbf{e}_2 + 1 \mathbf{e}_3
}
$$

Coordinates: $\boxed{(-5, 5, 1)}$

This is straightforward because the basis vectors are the axes themselves.

Case 2: A Custom Basis

Suppose the basis vectors are:

\[
\mathbf{b}_1 = (1, 1, 0), \quad
\mathbf{b}_2 = (0, 1, 1), \quad
\mathbf{b}_3 = (1, 0, 1)
\]

Step 1: Form the basis matrix:

\[
B = \begin{bmatrix}
1 & 0 & 1 \\
1 & 1 & 0 \\
0 & 1 & 1
\end{bmatrix}
\]

Step 2: Find \(B^{-1}\)

Calculate the inverse of \(B\):

- Determinant:

\[
\det(B) = 1 \times (1 \times 1 - 0 \times 1) - 0 \times (1 \times 1 - 0 \times 0) + 1 \times (1 \times 1 - 1 \times 0) = 1(1 - 0) - 0 + 1(1 - 0) = 1 + 1 = 2
\]

- Cofactors and adjugate:

\[
\text{adj}(B) = \begin{bmatrix}
(1 \times 1 - 0 \times 1) & - (1 \times 1 - 0 \times 0) & (1 \times 1 - 1 \times 0) \\
- (0 \times 1 - 1 \times 0) & (1 \times 1 - 1 \times 0) & - (1 \times 0 - 0 \times 1) \\
(0 \times 1 - 1 \times 1) & - (1 \times 1 - 1 \times 0) & (1 \times 1 - 1 \times 0)
\end{bmatrix}
\]

Calculating each element:

\[
\text{adj}(B) = \begin{bmatrix}
1 - 0 & - (1 - 0) & 1 - 0 \\

```
- (0 - 0) & 1 - 0 & - (0 - 0) \\
0 - 1 & - (1 - 0) & 1 - 0
\end{bmatrix} = \begin{bmatrix}
1 & -1 & 1 \\
0 & 1 & 0 \\
-1 & -1 & 1
\end{bmatrix}
\]

- Inverse:

\[
B^{-1} = \frac{1}{\det(B)} \times \text{adj}(B) = \frac{1}{2} \begin{bmatrix}
1 & -1 & 1 \\
0 & 1 & 0 \\
-1 & -1 & 1
\end{bmatrix}
\]

Step 3: Compute \(\mathbf{c} = B^{-1} \mathbf{v}\):

\[
\mathbf{c} = \frac{1}{2}
\begin{bmatrix}
1 & -1 & 1 \\
0 & 1 & 0 \\
-1 & -1 & 1
\end{bmatrix}
\begin{bmatrix}
-5 \\
5 \\
1
\end{bmatrix}
\]

Calculations:

- \(c_1 = \frac{1}{2} (1 \times -5 + (-1) \times 5 + 1 \times 1) =
\frac{1}{2}(-5 - 5 + 1) = \frac{1}{2}(-9) = -4.5\)

- \(c_2 = \frac{1}{2} (0 \times -5 + 1 \times 5 + 0 \times 1) = \frac{1}{2}(0
+ 5 + 0) = 2.5\)

- \(c_3 = \frac{1}{2} (-1 \times -5 + -1 \times 5 + 1 \times 1) =
\frac{1}{2}(5 - 5 + 1) = \frac{1}{2}(1) = 0.5\
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