# (1 Point) Find The Set Of Solutions For The Linear System 3x1 6x25x25x38x3==910 Use S1, S2, Etc. For

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Introduction

Solving linear systems is a fundamental aspect of algebra that helps us understand relationships between variables and find specific solutions. The problem at hand involves a linear system expressed with multiple variables and coefficients, specifically:

> 3x□ + 6x□ + 5x□ + 25x□ + 25x□ + 38x□ + 3x□ = 910

Our goal is to find the set of all solutions for this system, which involves expressing the solutions in terms of free variables, often called parameters (S□, S□, etc.). This process includes translating the given equation into a simplified form, identifying dependencies among variables, and describing the solution set comprehensively.

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Understanding the Linear System

Before diving into solving, it's important to clarify the structure of the problem.

The Equation

The given linear equation is:

> $3x_1 + 6x_2 + 5x_3 + 25x_4 + 25x_5 + 38x_6 + 3x_7 = 910$

This single linear equation relates seven variables. Since there's only one equation, there are infinitely many solutions, depending on the values assigned to some variables.

Variables and Parameters

- Variables: $x_1, x_2, x_3, x_4, x_5, x_6, x_7$
- Parameters: To express the solution set, we will assign free variables as parameters, commonly denoted as $S_1, S_2$, etc.

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Step-by-Step Solution Approach

The process involves several key steps:

1. Express Variables in Terms of Parameters

Since we have only one linear equation, we can choose some variables as free parameters and express others in terms of these.

2. Identify Free Variables

Some variables can be arbitrarily assigned values (parameters), and others will be expressed in terms of these.

3. Write the General Solution

The solution set will be described as a combination of parameters multiplied by vectors, representing the entire solution space.

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Detailed Solution Process

Step 1: Choose Free Variables

Because there are 7 variables and only 1 equation, we can select any 6 variables as free variables, and solve for the remaining one.

For simplicity, let's choose:

- Free variables: $x_{\square}$, $x_{\square}$, $x_{\square}$, $x_{\square}$, $x_{\square}$, $x_{\square}$

- Dependent variable: $x_{\square}$

Step 2: Express $x_{\square}$ in Terms of Parameters

The original equation:

> $3x_{\square} + 6x_{\square} + 5x_{\square} + 25x_{\square} + 25x_{\square} + 38x_{\square} + 3x_{\square} = 910$

Solve for $x_{\square}$:

> $3x_{\square} = 910 - (6x_{\square} + 5x_{\square} + 25x_{\square} + 25x_{\square} + 38x_{\square} + 3x_{\square})$

Therefore,

> $x_{\square} = [910 - (6x_{\square} + 5x_{\square} + 25x_{\square} + 25x_{\square} + 38x_{\square} + 3x_{\square})] / 3$

Step 3: Define Parameters

Let:

- $S_{\square} = x_{\square}$

- $S_{\square} = x_{\square}$

- $S_{\square} = x_{\square}$

- $S_{\square} = x_{\square}$

- $S_{\square} = x_{\square}$

- $S_{\square} = x_{\square}$

All these parameters are free variables that can take any real value.

Step 4: Write the Solution Set

Expressing $x_{\square}$ in terms of $S_{\square}$ through $S_{\square}$:

> $x_{\square} = [910 - (6S_{\square} + 5S_{\square} + 25S_{\square} + 25S_{\square} + 38S_{\square} + 3S_{\square})] / 3$

The complete solution set:

> $x_{\square} = (910 - 6S_{\square} - 5S_{\square} - 25S_{\square} - 25S_{\square} - 38S_{\square} - 3S_{\square}) / 3$

> $x_{\square} = S_{\square}$, $x_{\square} = S_{\square}$, $x_{\square} = S_{\square}$, $x_{\square} = S_{\square}$, $x_{\square} = S_{\square}$, $x_{\square} = S_{\square}$

This describes all solutions parametrized by six free variables.

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Expressing the Solution Set in Vector Form

The solution can be written as:

\[
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6 \\
x_7
\end{bmatrix}
=
\begin{bmatrix}
\frac{910}{3} \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
+
S_1
\begin{bmatrix}

-\frac{6}{3} \\

1 \\

0 \\

0 \\

0 \\

0 \\

0

\end{bmatrix}

+

S_2

\begin{bmatrix}

-\frac{5}{3} \\

0 \\

1 \\

0 \\

0 \\

0 \\

0

\end{bmatrix}

+

S_3

\begin{bmatrix}

-\frac{25}{3} \\

0 \\

0 \\

1 \\

0 \\

0 \\

0

\end{bmatrix}

+

S_4

\begin{bmatrix}

-\frac{25}{3} \\

0 \\

0 \\

0 \\

1 \\

0 \\

0

\end{bmatrix}

+

S_5

\begin{bmatrix}

-\frac{38}{3} \\

0 \\

0 \\

0 \\

0 \\

1 \\

0

\end{bmatrix}

+

S_6

\begin{bmatrix}

-\frac{3}{3} \\

0 \\

0 \\

0 \\

0 \\

0 \\

1

\end{bmatrix}

\]

where \( S_1, S_2, S_3, S_4, S_5, S_6 \in \mathbb{R} \).

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Interpretation of the Solution

Infinite Solution Set

Since there are more variables than equations, the solution set forms an affine subspace of \(\mathbb{R}^7\). By assigning specific values to the free parameters (S□ through S□), we can generate infinitely many solutions.

Geometric Perspective

The solution space is a 6-dimensional hyperplane in 7-dimensional space, shifted by the particular solution corresponding to the constant term (910/3 in x□).

Practical Examples

- Example 1: Set all free parameters to zero:

\[

x_2 = x_3 = x_4 = x_5 = x_6 = x_7 = 0

\]

then,

\[

x_1 = \frac{910}{3} \approx 303.33

\]

- Example 2: Choose specific values for the parameters to generate a particular solution.

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Summary

Finding the set of solutions for the linear system:

> $3x_1 + 6x_2 + 5x_3 + 25x_4 + 25x_5 + 38x_6 + 3x_7 = 910$

involves parametrizing the solution space using free variables. By selecting six variables as free parameters ($S_1$ through $S_6$), the dependent variable $x_1$ can be expressed explicitly in terms of these parameters. The resulting solution set is an affine hyperplane in \(\mathbb{R}^7\), characterized by:

\[
\boxed{
\begin{aligned}
x_1 &= \frac{910 - 6S_1 - 5S_2 - 25S_3 - 25S_4 - 38S_5 - 3S_6}{3} \\
x_2 &= S_1 \\
x_3 &= S_2 \\
x_4 &= S_3 \\
x_5 &= S_4 \\
x_6 &= S_5 \\
x_7 &= S_6
\end{aligned}
}
\]

where $S_1, S_2, S_3, S_4, S_5, S_6 \in \mathbb{R}$.

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Additional Tips for Solving Similar Problems

- Always identify which variables to treat as free parameters.
- Express dependent variables in terms of free parameters.
- Write the general solution in vector form for clarity.
- Understand the geometric interpretation to grasp the nature of the solution space.
- Use parameterization to generate specific solutions for practical applications.

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Conclusion

Understanding how to find the set of solutions for a linear system with multiple variables and a single equation is crucial in algebra and various applied fields. By systematically choosing free variables and expressing others accordingly, we can fully describe the solution set. This approach emphasizes the importance of parametrization, linear independence, and the geometric perspective in solving linear systems efficiently and effectively

## Frequently Asked Questions

### What is the first step to solve the linear system 3x1 + 6x2 + 5x3 = 9?

Begin by rewriting the equations clearly and identifying the variables involved, then set up the system in matrix form or use substitution/elimination methods to find solutions.

## How do I interpret the solutions when the system has multiple variables?

The solutions can form a point, a line, or a plane depending on the system's consistency and the number of free variables; express solutions in parametric form when necessary.

## What does it mean if the system has no solution?

It means the equations are inconsistent—no set of variable values satisfy all equations simultaneously, often indicated by contradictory equations after simplification.

## How can I verify the solutions obtained for the system?

Substitute the solution values back into the original equations to check if all equations are satisfied, confirming the correctness of the solutions.

## What is the significance of using labels like S1, S2, etc., in solutions?

Labeling solutions as S1, S2, etc., helps organize different solution sets, especially when multiple solutions or parametric solutions exist, making reference easier.

## Can this system have infinitely many solutions?

Yes, if the equations are dependent or there are free variables, the system can have infinitely many solutions represented parametrically.

## What tools or methods are most effective for solving this linear system?

Methods such as Gaussian elimination, substitution, or matrix operations like row reduction are effective; using graphing or computational tools can also help visualize solutions.

# Additional Resources

Linear Systems: A Deep Dive into Solving the System 3x + 6x + 25x + 25x + 38x = 910

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Introduction

When it comes to solving systems of linear equations, the process can often seem daunting, especially when multiple variables and complex coefficients come into play. Today, we’ll explore a specific case: the linear system represented by the equation

$$3x_1 + 6x_2 + 25x_3 + 25x_4 + 38x_5 = 910$$

This equation presents an interesting challenge—how to find the set of solutions that satisfy it. We will break down the problem step by step, adopting a systematic approach similar to an expert analysis or a product review, examining the methods, implications, and potential solution sets in detail.

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Understanding the System: The Core Equation

The equation at hand is a linear combination of five variables:

$$3x_1 + 6x_2 + 25x_3 + 25x_4 + 38x_5 = 910$$

This is a single linear equation with five variables, which naturally leads to an infinite set of solutions unless additional constraints are imposed.

Key Observations:

- The coefficients vary significantly, with 3, 6, 25, and 38.

- The right-hand side, 910, acts as a target sum.
- The system is underdetermined: one equation, five variables.

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The Nature of the Solution Set

Before diving into calculations, it's essential to understand the nature of solutions for such an equation:

- Infinite solutions: Since there are more variables than equations, the solution space forms a hyperplane in five-dimensional space.
- Parameterization: To express solutions explicitly, some variables are chosen as free parameters, and others are expressed in terms of these parameters.

The goal is to find a general solution—a set of all possible solutions—that can be expressed parametrically.

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Step 1: Simplify the Equation

The initial step involves simplifying the coefficients where possible or restructuring the equation for easier manipulation.

Given:

$$ 3x_1 + 6x_2 + 25x_3 + 25x_4 + 38x_5 = 910 $$

Notice that the coefficients 25 are the same for $x_3$ and $x_4$, which suggests grouping these terms:

$$ 3x_1 + 6x_2 + 25(x_3 + x_4) + 38x_5 = 910 $$

This reorganization makes the structure clearer and allows us to assign parameters to some variables.

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Step 2: Choose Free Variables

Given the degrees of freedom, we select certain variables as parameters:

- Let's choose $x_2, x_4, x_5$ as free parameters, denoting:

$$
x_2 = s,\quad x_4 = t,\quad x_5 = u
$$

where $s, t, u \in \mathbb{R}$ (or integers, if we assume integrality).

Now, solve for $x_1$ and $x_3$:

$$
3x_1 + 6s + 25(x_3 + t) + 38u = 910
$$

Rearranged:

$$
3x_1 + 25x_3 = 910 - 6s - 25t - 38u
$$

---

Step 3: Expressing Variables

To proceed, express $x_1$ and $x_3$ in terms of the free parameters:

- For $x_3$, choose it as a free variable:

$$
x_3 = v,\quad v \in \mathbb{R}
$$

Then, the equation becomes:

$$
3x_1 + 25v = 910 - 6s - 25t - 38u
$$

Solve for $x_1$:

$$
x_1 = \frac{1}{3} \left( 910 - 6s - 25t - 38u - 25v \right)
$$

---

Summary of the General Solution:

$$
\boxed{
\begin{aligned}
x_1 &= \frac{1}{3} \left( 910 - 6s - 25t - 38u - 25v \right) \\
x_2 &= s \\

x_3 &= v \\

x_4 &= t \\

x_5 &= u

\end{aligned}

}

\]

where \(s, t, u, v \in \mathbb{R}\).

This parametric form encapsulates all solutions to the original linear equation.

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Interpreting the Solution Set

The solution space is a 4-dimensional hyperplane in five-dimensional space, parametrized by the free variables \(s, t, u, v\).

- Variables \(s, t, u, v\) are independent parameters, allowing us to generate infinitely many solutions.
- The variable \(x_1\) is expressed as a linear combination of these parameters, scaled by the coefficients.

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Practical Implications and Constraints

In real-world applications, certain constraints might restrict solutions:

- Integer Solutions: If variables are required to be integers, the parameters must be chosen such that \(x_1\) and other variables are integers, leading to a Diophantine problem.
- Non-negativity: If variables must be non-negative, additional inequalities must be enforced.

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Additional Exploration: Special Cases

To illustrate the flexibility, consider specific choices of parameters:

Case 1: Fixing some parameters

- Set $s = 0$, $t = 0$, $u = 0$:

$$
x_1 = \frac{1}{3}(910 - 25v) \implies x_1 = \frac{910 - 25v}{3}
$$

For $x_1$ to be integral:

$$
910 - 25v \equiv 0 \pmod{3}
$$

Since $910 \equiv 1 \pmod{3}$, and $25v \equiv (25 \mod 3) \times v \equiv 1 \times v \equiv v \pmod{3}$,

$$
1 - v \equiv 0 \pmod{3} \implies v \equiv 1 \pmod{3}
$$

Thus, $v$ must be congruent to 1 modulo 3 for $x_1$ to be integral, and $x_1$ will be integral accordingly.

Case 2: Numerical example

Choose $s=0, t=0, u=0, v=1$:

$$
x_1 = \frac{1}{3}(910 - 25 \times 1) = \frac{1}{3}(910 - 25) = \frac{885}{3} = 295
$$

$$
x_2=0,\quad x_3=1,\quad x_4=0,\quad x_5=0
$$

This yields a specific solution:

$$
(x_1, x_2, x_3, x_4, x_5) = (295, 0, 1, 0, 0)
$$

which satisfies the original equation.

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Final Thoughts: The Power of Parametric Solutions

The process illustrated demonstrates the power and flexibility of parametric solutions for underdetermined systems:

- It provides a comprehensive view of all possible solutions.
- It allows for custom constraints to be applied, such as integrality or non-negativity.
- It emphasizes the importance of choosing appropriate free variables to simplify the problem.

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Conclusion: The Set of Solutions

In summary, the set of solutions for the linear system

$$
3x_1 + 6x_2 + 25x_3 + 25x_4 + 38x_5 = 910
$$

can be expressed parametrically as:

$$
\boxed{
\begin{aligned}
x_1 &= \frac{1}{3} \left( 910 - 6s - 25t - 38u - 25v \right) \\
x_2 &= s \\
x_3 &= v \\
x_4 &= t \\
x_5 &= u
\end{aligned}
}
$$

where $s, t, u, v \in \mathbb{R}$ are free parameters.

This solution set embodies the entire solution space, offering an in-depth understanding of the problem's structure, potential special cases, and practical applications—much like a detailed expert review or feature analysis of a product, highlighting its strengths, flexibility, and areas for further exploration.

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Final Remarks

Understanding and solving linear systems with multiple variables is fundamental in mathematics, engineering, computer science, and numerous scientific fields. The methodology showcased here—parameterization, analysis of solution spaces, and special case considerations—is a powerful toolkit for tackling similar problems across disciplines. Whether for theoretical insight or practical application, mastering these techniques opens the door to a deeper comprehension of linear algebra and its vast array of real-world applications.

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