2) We Are Given That The Line Y=3x-7 Is Tangent To The Graph Of Y = F(x) At The Point (2, F(2)) (and

Understanding the Problem: The Tangency of a Line to a Function Graph

Introduction to the Given Conditions

2) We Are Given That The Line Y=3x-7 Is Tangent To The Graph Of Y=F(x) At The Point (2, F(2)) (and indicates a scenario where a straight line touches a curve at a single point without crossing it, known as tangency. The line in question is given by the linear equation $Y = 3x - 7$, and it is tangent to the graph of the function $Y = F(x)$ at the point with x-coordinate 2, where the corresponding y-coordinate is F(2). This situation prompts us to explore the relationship between the function F(x), its derivative, and the tangent line at the point of contact.

Significance of Tangency in Calculus

In calculus, tangency represents a point where the function and the line share both a point and a slope. Specifically, at the point of tangency:

- The function and the line intersect: $F(2) = 3(2) - 7$
- Their slopes are equal: $F'(2) = 3$

These two conditions form the foundation for analyzing and determining the properties of the function F(x) at the point x=2. They also help in reconstructing the function if additional information is provided or required.

Deriving the Coordinates of the Point of Tangency

Calculating F(2)

Given that the line Y=3x-7 passes through the point of tangency (2, F(2)), the y-coordinate at x=2 is obtained by substituting x=2 into the line equation:

```
F(2) = 3(2) - 7 = 6 - 7 = -1
```

Thus, the point of tangency is (2, -1). This confirms that the graph of F(x) passes through this point, and the tangent line touches the curve exactly there.

Summary of the Point of Contact

- Point of tangency: **(2, -1)**

- Slope of the tangent line: 3

Exploring the Relationship Between F(x) and Its Derivative

Condition for Tangency: Equal Slopes

Since the line is tangent to the graph of F(x) at x=2, the slope of F(x) at this point must match the slope of the line. Therefore:

$$F'(2) = 3$$

This is a crucial condition, as it provides information about the behavior of F(x) near x=2. The derivative at this point indicates the rate of change of the function at the point of contact, matching the slope of the tangent line.

Implications for the Function F(x)

Knowing the point (2, -1) and the slope at this point, F'(2)=3, allows us to construct the local linear approximation of the function around x=2. The tangent line itself acts as the best linear approximation to F(x) at this point, which can be expressed as:

$$L(x) = F(2) + F'(2)(x - 2) = -1 + 3(x - 2)$$

This linear function matches the tangent line equation, confirming the conditions for tangency.

Constructing the Function F(x): Possible Forms and Constraints

General Idea for F(x)

Given only the tangent point and slope, F(x) might be any function passing through (2, -1) with derivative 3 at x=2. To determine F(x) explicitly, additional information or constraints are generally needed. However, in many problems, the goal might be to find a particular form of F(x) satisfying these conditions, such as a polynomial or a function with known derivatives.

Example: Linear Function

If F(x) were linear, then:

$$F(x) = mx + c$$

Using the conditions:

1. F(2) = -1 ⍰ 2m + c = -1
2. F'(x) = m, so F'(2) = m = 3

From the second condition, m=3. Plug into the first:

```
2(3) + c = -1 → 6 + c = -1 → c = -7
```

Thus, a linear function satisfying the conditions is:

```
F(x) = 3x - 7
```

This matches the tangent line, indicating that in this special case, the function is the same as the tangent line itself, making the graph and the line coincide at all points (a tangent line to itself).

Example: Quadratic Function

Suppose F(x) is quadratic, of the form:

```
F(x) = ax^2 + bx + c
```

Applying the known conditions:

1. $F(2) = -1 \Rightarrow 4a + 2b + c = -1$
2. $F'(x) = 2ax + b \Rightarrow F'(2) = 4a + b = 3$

From the second condition:

```
b = 3 - 4a
```

Substitute into the first condition:

```
4a + 2(3 - 4a) + c = -1
→ 4a + 6 - 8a + c = -1
→ -4a + 6 + c = -1
→ c = -1 + 4a - 6 = -7 + 4a
```

Thus, F(x) can be written as:

```
F(x) = a x^2 + (3 - 4a) x + (-7 + 4a)
```

Here, *a* remains arbitrary, allowing for a family of functions satisfying the tangent and point conditions. The specific choice of *a* depends on additional constraints or the desired properties of F(x).

Analyzing the Derivative and Behavior of F(x)

Implications of F′(2)=3

The derivative at x=2 being 3 indicates the slope of the function at that point. Depending on the function's form, the behavior near x=2 can vary:

- If F(x) is linear, the function and the tangent line are identical.

- If $F(x)$ is quadratic or higher degree, the function may curve above or below the tangent line elsewhere, but touch it at $x=2$ with the same slope.

Understanding the Nature of the Graph Near the Tangent Point

The tangent line provides a local linear approximation of the graph of $F(x)$ at $x=2$. The curvature (second derivative) determines how the graph behaves relative to the tangent line:

- If $F''(x) > 0$, the graph curves upward (convex).
- If $F''(x) < 0$, it curves downward (concave).

Without additional information about the second derivative, we cannot definitively state the curvature, but the tangent line's slope and the point of contact provide a foundation for further analysis.

Practical Applications of the Given Conditions

Approximating Functions Near a Point

The tangent line at a point offers a linear approximation of the function near that point. This concept is fundamental in numerical methods, such as:

- Linear approximation (or differentials)
- Newton's method for root finding
- Estimating function values for small deviations from $x=2$

Determining Unknown Function Properties

If additional data points or the form of $F(x)$ are provided, these conditions can help in:

- Constructing the explicit form of $F(x)$
- Studying the behavior of the function over intervals
- Analyzing the function's concavity and points of inflection

Summary and Conclusions

In this detailed exploration, we examined the implications of a line being tangent to a function at a specific point. The

Frequently Asked Questions

What does it mean for the line $y = 3x - 7$ to be tangent to the graph of $y = f(x)$ at the point $(2, f(2))$?

It means that the line touches the graph of $f(x)$ at exactly one point, $(2, f(2))$, and has the same slope as the graph at that point, indicating the derivative $f'(2)$ equals 3.

How can we use the tangent line to find $f(2)$?

Since the tangent line passes through $(2, f(2))$, and its equation is $y = 3x - 7$, plugging in $x=2$ gives $y = 3(2) - 7 = 6 - 7 = -1$, so $f(2) = -1$.

What is the significance of the slope 3 in the tangent line $y = 3x - 7$?

The slope 3 indicates that the derivative of $f(x)$ at $x=2$, i.e., $f'(2)$, equals 3, representing the instantaneous rate of change of f at that point.

If the line $y = 3x - 7$ is tangent to $f(x)$ at $(2, -1)$, what can we say about the derivative of f at $x=2$?

We can conclude that $f'(2) = 3$, since the slope of the tangent line matches the derivative at that point.

How does the tangent line help in sketching the graph of $f(x)$ near $x=2$?

The tangent line provides the slope at $x=2$ and the point of contact, allowing us to approximate the graph of $f(x)$ near that point by drawing a line with slope 3 passing through $(2, -1)$.

Can $f(x)$ have more than one point of tangency with the line $y=3x-7$?

Generally, no, if the line is tangent at only one point. However, if the graph of $f(x)$ intersects and is tangent at multiple points, then it could have more than one point of tangency, but in this context, it is only tangent at $(2, f(2))$.

What additional information is needed to determine the exact form of $f(x)$?

We need more points or the explicit form of $f(x)$, or additional derivatives, to fully determine $f(x)$. The given tangent line only provides a single point and slope at $x=2$.

How can the given tangent condition be used to formulate a differential equation for $f(x)$?

Since at $x=2$, $f'(2) = 3$ and $f(2) = -1$, if we assume $f'(x) = 3$ near $x=2$, then a simple differential equation would be $f'(x) = 3$, with the initial condition $f(2) = -1$, leading to $f(x) = 3x - 7$, which matches the tangent line.

Additional Resources

Tangent Line Analysis: Unlocking the Relationship Between a Line and a Function's Graph

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Introduction: The Significance of Tangent Lines in Calculus and Graph Analysis

In the realm of calculus and analytical geometry, the concept of tangent lines plays a pivotal role in understanding the behavior of functions. When examining a graph of a function $y = f(x)$, the tangent line at a particular point provides invaluable insight into the function's local behavior—its slope, rate of change, and instantaneous value. This article delves into a specific scenario where a line, $y = 3x - 7$, is tangent to the graph of a function $y = f(x)$ at the point $(2, f(2))$. By exploring this relationship, we uncover the underlying principles governing tangency, derivatives, and the function's structure.

Understanding the Given Conditions: The Line and the Point of Tangency

The Given Line: $y = 3x - 7$

At first glance, the line $y = 3x - 7$ is a straight, linear function with a slope of 3 and a y-intercept at -7. Its graphical representation is a straight line that cuts through the coordinate plane, rising at a steady slope. The key features include:

- Slope (m): 3
- Y-intercept: -7, which is where the line crosses the y-axis at $(0, -7)$

This line's simplicity makes it an excellent candidate for analyzing tangent relationships because linear functions are their own derivatives and have constant slopes everywhere.

The Point of Tangency: $(2, f(2))$

The point $(2, f(2))$ lies on the graph of $y = f(x)$, meaning:

- The x-coordinate is 2.
- The y-coordinate is given by the function value at 2, i.e., $f(2)$.

Since the line is tangent to $y = f(x)$ at this point, it touches the curve exactly at $(2, f(2))$ without crossing it locally, implying a special relationship between the function's slope at $x=2$ and the line's slope.

Key Concepts: Derivatives and Tangency

The Derivative as the Slope of the Tangent Line

In calculus, the derivative $f'(x)$ of the function $f(x)$ at a point $x = a$ measures the slope of the tangent line to the graph at $(a, f(a))$. When the tangent line is known, the derivative at that point can be directly inferred:

$$
f'(a) = \text{Slope of the tangent line at } x=a
$$

In our scenario, since the line $y=3x-7$ is tangent to the graph at $x=2$, the slope of this line must match the derivative of $f(x)$ at $x=2$:

$$
f'(2) = 3
$$

This crucial relationship guides us in understanding the local behavior of $f(x)$.

The Point of Tangency and the Function's Value

Because the line is tangent at $(2, f(2))$, the point must satisfy both equations:

1. The point lies on the line:

$$
f(2) = 3(2) - 7 = 6 - 7 = -1
$$

2. The point lies on the graph of $f(x)$:

$$
f(2) = -1
$$

Thus, the point of tangency is $(2, -1)$.

Deriving the Function's Characteristics Based on the Tangency Condition

Function Value at $x=2$: $f(2) = -1$

From the above calculation, we now know the exact y-coordinate of the point $(2, f(2))$:

$$
f(2) = -1
$$

This means the function passes through $(2, -1)$.

Derivative at \(x=2 \): \(f'(2) = 3 \)

Since the line \(y=3x-7 \) is tangent at this point, the slope of the tangent line equals the derivative of \(f(x) \) at \(x=2 \):

$$
f'(2) = 3
$$

This indicates that at \(x=2 \), the function has a slope of 3, matching the line's slope.

Implications for the Function \(f(x) \)

Possible Forms of \(f(x) \)

While the problem doesn't specify the entire form of \(f(x) \), the tangency condition constrains its local behavior near \(x=2 \):

- At \(x=2 \):

$$
f(2) = -1
$$
$$
f'(2) = 3
$$

- In the vicinity of \(x=2 \):

\(f(x) \) must resemble the tangent line \(y=3x-7 \), with the same slope and passing through \((2, -1) \).

Given these constraints, \(f(x) \) could be a polynomial, exponential, or other smooth function that satisfies these point and derivative conditions.

Constructing a Local Approximation of \(f(x) \)

Using the tangent line and point, we can approximate \(f(x) \) near \(x=2 \) via the linear approximation:

$$
f(x) \approx f(2) + f'(2)(x-2) = -1 + 3(x-2)
$$

This simplifies to:

$$
f(x) \approx 3x - 7
$$

which aligns with the tangent line.

Additional Insights and Broader Implications

Why Is This Relationship Important?

The fact that the line $y=3x-7$ is tangent to $y=f(x)$ at $(2, f(2))$ allows us to:

- Determine $f(2)$: The function passes through $(2, -1)$.
- Identify the slope at $x=2$: The derivative $f'(2) = 3$.
- Understand the local behavior: Near $x=2$, $f(x)$ behaves like the tangent line.

This understanding is fundamental in calculus, enabling us to approximate functions locally, analyze their behavior, and understand rates of change.

Applications in Real-World Contexts

The principles demonstrated here extend beyond pure mathematics and find applications in various disciplines:

- Physics: Understanding instantaneous velocity or acceleration.
- Economics: Analyzing marginal cost or revenue functions.
- Engineering: Designing systems with predictable local behavior.
- Data Science: Approximating complex functions with linear models locally.

Summary and Final Remarks

This exploration reveals that the key to understanding the relationship between the line $y=3x-7$ and the function $y=f(x)$ lies in the principles of derivatives and tangency. The line's slope matching the derivative of $f(x)$ at $x=2$, combined with the point of intersection, provides a comprehensive picture of the function's local behavior.

To summarize:

- The point of tangency is at $(2, -1)$.
- The derivative of $f(x)$ at $x=2$ equals 3.
- The tangent line at that point is $y=3x-7$.

These insights serve as a foundational tool in calculus, enabling us to analyze, approximate, and understand the intricate behaviors of functions just by examining their tangent lines.

In conclusion, the scenario where a line is tangent to a function's graph at a specific point encapsulates core calculus concepts. Recognizing the relationships between slopes, points, and derivatives not only deepens our understanding of mathematical functions but also equips us with powerful analytical tools applicable across scientific, engineering, and economic fields.

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