2) Write The Equation Of The Plane Containing The Points (3, 2, 1), (1, 2,-2) And (2,0,1) (Hint: You

2) Write The Equation Of The Plane Containing The Points (3, 2, 1), (1, 2, -2), And (2, 0, 1) (Hint: You

Understanding how to find the equation of a plane given three points is an essential skill in coordinate geometry. This process involves using vector operations to determine the plane's normal vector and then applying the point-normal form of a plane equation. Whether you're a student working through algebra assignments or a professional dealing with 3D modeling, mastering this concept is vital. In this article, we'll walk through the step-by-step process of writing the equation of the plane that contains the points (3, 2, 1), (1, 2, -2), and (2, 0, 1).

Understanding the Fundamentals

Before diving into calculations, it's important to understand the key concepts involved in determining the plane's equation.

What is a Plane in 3D Space?

A plane in three-dimensional space is a flat, two-dimensional surface that extends infinitely in all directions within that space. It can be uniquely identified with a point through which it passes and a normal vector perpendicular to its surface.

Point-Normal Form of a Plane Equation

The standard form of a plane's equation is:

$$ a(x - x_0) + b(y - y_0) + c(z - z_0) = 0 $$

where:

- (x_0, y_0, z_0) is a point on the plane.
- $\vec{n} = \langle a, b, c \rangle$ is the normal vector to the plane.

Alternatively, it can be written as:

$$ a x + b y + c z + d = 0 $$

where d can be found by substituting the point into the equation.

The main task in our problem is to find the normal vector $\vec{n}$ using the given points, then write the plane's equation accordingly.

Step-by-Step Guide to Find the Plane Equation

Let's now proceed with the calculation steps to find the equation of the plane passing through the given points.

Step 1: Identify the Given Points

The points provided are:

- $P_1 = (3, 2, 1)$
- $P_2 = (1, 2, -2)$
- $P_3 = (2, 0, 1)$

These three points are guaranteed to define a unique plane if they are not collinear.

Step 2: Find Two Direction Vectors on the Plane

To determine the plane's normal vector, we need two vectors lying on the plane, which can be obtained by subtracting the position vectors of the given points:

- $\vec{v}_1 = P_2 - P_1$
- $\vec{v}_2 = P_3 - P_1$

Calculations:

$$
\begin{aligned}
\vec{v}_1 &= (1 - 3,\, 2 - 2,\, -2 - 1) = (-2, 0, -3) \\
\vec{v}_2 &= (2 - 3,\, 0 - 2,\, 1 - 1) = (-1, -2, 0)
\end{aligned}
$$

Step 3: Compute the Normal Vector Using Cross Product

The normal vector $\vec{n}$ to the plane can be found by taking the cross product of $\vec{v}_1$ and $\vec{v}_2$:

$$
\vec{n} = \vec{v}_1 \times \vec{v}_2
$$

Recall the cross product formula for vectors in 3D:

$$
\vec{a} \times \vec{b} = (a_2 b_3 - a_3 b_2,\, a_3 b_1 - a_1 b_3,\, a_1 b_2 - a_2 b_1)
$$

Applying this:

$$
\begin{aligned}
a_1 &= -2, \quad a_2 = 0, \quad a_3 = -3 \\
b_1 &= -1, \quad b_2 = -2, \quad b_3 = 0
\end{aligned}
$$

Compute each component:

$$
\begin{aligned}
n_x &= (a_2 \times b_3) - (a_3 \times b_2) = (0 \times 0) - (-3 \times -2) = 0 - 6 = -6 \\
n_y &= (a_3 \times b_1) - (a_1 \times b_3) = (-3 \times -1) - (-2 \times 0) = 3 - 0 = 3 \\
n_z &= (a_1 \times b_2) - (a_2 \times b_1) = (-2 \times -2) - (0 \times -1) = 4 - 0 = 4
\end{aligned}
$$

Therefore, the normal vector is:

$$
\vec{n} = \langle -6, 3, 4 \rangle
$$

This vector is perpendicular to the plane.

Formulating the Equation of the Plane

With the normal vector $\vec{n} = \langle -6, 3, 4 \rangle$ and a point on the plane (say, $P_1 = (3, 2, 1)$), we can write the plane's equation.

Step 4: Write the Equation Using Point-Normal Form

Plugging into the formula:

$$
a(x - x_0) + b(y - y_0) + c(z - z_0) = 0
$$

we get:

$$
-6(x - 3) + 3(y - 2) + 4(z - 1) = 0
$$

Expand:

$$
-6x + 18 + 3y - 6 + 4z - 4 = 0
$$

Combine like terms:

$$
-6x + 3y + 4z + (18 - 6 - 4) = 0
$$

$$
-6x + 3y + 4z + 8 = 0
$$

Multiplying through by -1 to make coefficients positive:

$$
6x - 3y - 4z - 8 = 0
$$

This is the equation of the plane containing the given points.

Final Equation of the Plane

$$
\boxed{
6x - 3y - 4z = 8
}
$$

This equation fully describes the plane passing through the points $(3, 2, 1)$, $(1, 2, -2)$, and $(2, 0, 1)$.

Additional Tips for Writing Plane Equations

- Verify the points: Always confirm the points are not collinear; otherwise, they do not define a unique plane.
- Use vector operations: Cross products are powerful tools to find the perpendicular direction (normal vector) to the plane.
- Simplify equations: You can multiply the entire equation by a scalar to make coefficients cleaner or

to eliminate fractions.
- Check your work: Substitute the original points into the final equation to ensure they satisfy it.

Applications of Plane Equations in Real Life

Understanding how to write the equation of a plane has practical applications across various fields:

- Computer Graphics & 3D Modeling: Defining surfaces, clipping, and rendering.
- Engineering: Analyzing structural components and surfaces.
- Geography & Cartography: Modeling terrains and land features.
- Physics: Describing planes of motion or interaction between objects.

Summary

In this article, we've demonstrated how to find the equation of a plane given three points in 3D space. The process involves:

- Calculating vectors between points.
- Computing the cross product to find the normal vector.
- Applying the point-normal form to derive the plane's equation.

For the specific points provided, the final plane equation is:

$$
6x - 3y - 4z = 8
$$

Mastering these steps enhances your understanding of spatial relationships and prepares you for tackling more complex geometry problems. Whether you’re solving homework problems or working on advanced engineering projects, knowing how to derive and interpret plane equations is a vital skill in the realm of three-dimensional geometry.

Frequently Asked Questions

How do I find the equation of a plane passing through three points?

To find the equation of the plane through points A, B, and C, first find two vectors on the plane, such as AB and AC. Then compute their cross product to get the normal vector. Finally, use the point-normal form of the plane equation: $n \cdot (r - r0) = 0$, where n is the normal vector and r0 is a point on the plane.

What are the steps to write the equation of a plane given three points?

Step 1: Find two vectors on the plane by subtracting coordinates of the points. Step 2: Compute the cross product of these vectors to find the normal vector. Step 3: Use one of the points and the normal vector in the plane equation formula to write the equation.

How do I compute the normal vector of the plane from three points?

Subtract the coordinates of the points to create two vectors, then compute their cross product. The resulting vector is perpendicular to the plane and serves as the normal vector.

Can you show an example of finding the plane equation with points (3, 2, 1), (1, 2, -2), and (2, 0, 1)?

Yes. First, find vectors: AB = (1-3, 2-2, -2-1) = (-2, 0, -3); AC = (2-3, 0-2, 1-1) = (-1, -2, 0). Next, compute the cross product of AB and AC to get the normal vector n. Then, plug n and one point into the plane equation format to find the explicit equation.

What is the formula for the plane passing through a point with a given normal vector?

The plane equation is $n \cdot (r - r0) = 0$, where n is the normal vector, r is the position vector (x, y, z), and r0 is a point on the plane. Expanding this gives the general equation of the plane.

How do I verify if a point lies on the plane once I have its equation?

Substitute the point's coordinates into the plane equation. If the equation holds true (left side equals right side), the point lies on the plane.

What common mistakes should I avoid when writing the plane equation from three points?

Ensure accurate calculation of vectors, double-check the cross product to get the correct normal vector, and carefully substitute the point coordinates to avoid sign errors or algebraic mistakes.

How can I simplify the equation of a plane once I have it in standard form?

Factor out common factors and write the equation in the form $Ax + By + Cz + D = 0$ for clarity and simplicity.

Additional Resources

Write The Equation Of The Plane Containing The Points (3, 2, 1), (1, 2, -2), and (2, 0, 1) is a fundamental problem in three-dimensional geometry that demonstrates the practical application of vector algebra and coordinate geometry. Understanding how to derive the equation of a plane passing through three non-collinear points is essential for students and professionals working in fields such as engineering, computer graphics, physics, and mathematics. This article provides a comprehensive, step-by-step guide to solving this problem, including the underlying concepts, detailed procedures, and critical insights to master this topic effectively.

Understanding the Concept of a Plane in Space

Before diving into the calculations, it is crucial to understand what a plane is in three-dimensional space. A plane can be thought of as a flat, two-dimensional surface that extends infinitely in all directions. The equation of a plane typically takes the form:

$$ ax + by + cz + d = 0 $$

where (a, b, c) are the coefficients representing the normal vector to the plane, and d is a constant.

Key points:
- The normal vector $\vec{n} = (a, b, c)$ is perpendicular to the plane.
- To find the equation of a plane, we need:
- A point on the plane
- The normal vector to the plane

In our scenario, the three points provided are guaranteed to define a unique plane, assuming they are not collinear.

Step 1: Confirm the Points are Non-Collinear

Before proceeding, verify that the points are not collinear; otherwise, they do not define a unique plane.

Given points:
- $P_1 = (3, 2, 1)$
- $P_2 = (1, 2, -2)$
- $P_3 = (2, 0, 1)$

Calculate vectors:
- $\vec{P_1P_2} = P_2 - P_1 = (1 - 3, 2 - 2, -2 - 1) = (-2, 0, -3)$

- $\vec{P_1P_3} = P_3 - P_1 = (2 - 3, 0 - 2, 1 - 1) = (-1, -2, 0)$

Compute the cross product of these vectors:

$$
\vec{P_1P_2} \times \vec{P_1P_3} =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
-2 & 0 & -3 \\
-1 & -2 & 0
\end{vmatrix}
$$

Calculating the determinant:
- $\mathbf{i}$: $(0)(0) - (-3)(-2) = 0 - 6 = -6$
- $\mathbf{j}$: $- [(-2)(0) - (-3)(-1)] = - [0 - 3] = -(-3) = 3$
- $\mathbf{k}$: $(-2)(-2) - (0)(-1) = 4 - 0 = 4$

Normal vector:

$$
\vec{n} = (-6, 3, 4)
$$

Since $\vec{n} \neq \vec{0}$, the points are non-collinear, and a unique plane exists passing through these points.

Step 2: Find the Normal Vector of the Plane

The normal vector $\vec{n}$ is obtained from the cross product:

$$
\vec{n} = (-6, 3, 4)
$$

It can be simplified by dividing by common factors if desired, but for the purpose of the plane equation, it’s fine to keep as is.

Note: The normal vector indicates the orientation of the plane and is essential in deriving the plane's equation.

Step 3: Use the Point-Normal Form to Write the Plane Equation

The point-normal form of a plane's equation is:

$$
a(x - x_0) + b(y - y_0) + c(z - z_0) = 0
$$
where (x_0, y_0, z_0) is a point on the plane, and $\vec{n} = (a, b, c)$ is the normal vector.

Choose $P_1 = (3, 2, 1)$ for simplicity:
$$
-6(x - 3) + 3(y - 2) + 4(z - 1) = 0
$$

Step 4: Expand and Simplify the Equation

Carry out the expansions:
$$
-6x + 18 + 3y - 6 + 4z - 4 = 0
$$

Combine like terms:
$$
-6x + 3y + 4z + (18 - 6 - 4) = 0
$$
$$
-6x + 3y + 4z + 8 = 0
$$

To express the equation more neatly, multiply through by -1 to avoid negative leading coefficients:
$$
6x - 3y - 4z - 8 = 0
$$

Alternatively, the standard form is:
$$
6x - 3y - 4z = 8
$$

This is the equation of the plane passing through the three given points.

Features and Insights of the Derived Plane Equation

Features:
- The coefficients directly relate to the normal vector $\vec{n} = (-6, 3, 4)$.
- The plane passes through all three points, as verified by substituting their coordinates into the

equation.

Pros:
- The method is systematic and relies on vector operations, which are straightforward with practice.
- It can be extended to find the equation of a plane through any three non-collinear points.
- The normal vector provides insights into the plane’s orientation.

Cons:
- Requires understanding of vector cross products and their properties.
- Errors in calculations of the cross product or substitution can lead to incorrect plane equations.
- Not always the simplest form; sometimes further simplification is needed for specific applications.

Additional Tips and Common Pitfalls

- Always verify that the points are non-collinear before attempting to find the plane equation.
- When calculating the cross product, carefully perform the determinant expansion.
- You can select any of the three points for the point-normal form, but sticking with the original point used in the expansion simplifies calculations.
- After obtaining the equation, verify by substituting the other points to ensure they satisfy the plane equation.

Conclusion

Writing the equation of a plane containing three points involves a clear understanding of vector operations, specifically the cross product, and the application of the point-normal form of a plane's equation. This process, demonstrated with points $(3, 2, 1)$, $(1, 2, -2)$, and $(2, 0, 1)$, highlights the importance of verifying the points' positions, calculating the normal vector accurately, and carefully expanding and simplifying the resulting equations. Mastering these steps equips students and practitioners with a vital mathematical tool applicable in various scientific and engineering domains, allowing them to model and analyze three-dimensional geometrical structures with confidence.

In summary:
- Confirm points are non-collinear.
- Calculate vectors between points and their cross product to find the normal vector.
- Use the point-normal form to derive the plane equation.
- Simplify and verify the equation.

By following this structured approach, deriving the equation of a plane becomes a manageable and insightful process, reinforcing core concepts in vector calculus and geometry.

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