

3. A Segment In The Complex Plane Has A Midpoint At $1 + 7i$. If One Endpoint Of The Segment Is At $3 + 8i$,

3. A Segment In The Complex Plane Has A Midpoint At $1 + 7i$. If One Endpoint Of The Segment Is At $3 + 8i$, understanding how to analyze segments in the complex plane is fundamental in complex analysis, geometry, and related mathematical fields. This article delves into the properties and calculations involved, demonstrating how to determine the other endpoint of the segment, interpret the geometric significance of midpoints, and explore applications of these concepts in various mathematical contexts.

Understanding the Complex Plane and Line Segments

What Is the Complex Plane?

The complex plane, also known as the Argand plane, is a geometric representation of complex numbers. Each complex number $z = a + bi$ corresponds to a point in the plane with coordinates (a, b) , where:

- a is the real part,
- b is the imaginary part.

This visualization allows us to interpret complex operations geometrically, transforming algebraic problems into geometric ones.

Line Segments in the Complex Plane

A line segment in the complex plane connects two points corresponding to complex numbers $z_1 = a_1 + b_1 i$ and $z_2 = a_2 + b_2 i$. The segment's properties, such as length and midpoint, are directly related to these complex numbers:

- Length (Distance): $|z_2 - z_1| = \sqrt{(a_2 - a_1)^2 + (b_2 - b_1)^2}$
- Midpoint: $\frac{z_1 + z_2}{2}$

Understanding these properties is crucial in problems involving geometric constructions and complex number calculations.

Given Data and Objective

The problem states:

- The midpoint of a segment is at $(1 + 7i)$.
- One endpoint of the segment is at $(3 + 8i)$.

The primary goal is to determine the coordinates of the other endpoint, analyze the geometric implications, and understand the broader context of such problems.

Calculating the Other Endpoint

The Midpoint Formula

Recall that the midpoint (M) of a segment connecting points (Z_1) and (Z_2) in the complex plane is given by:

$$M = \frac{Z_1 + Z_2}{2}$$

Given:

$$M = 1 + 7i, \quad Z_1 = 3 + 8i$$

and (Z_2) is unknown.

Deriving the Unknown Endpoint (Z_2)

Using the midpoint formula:

$$1 + 7i = \frac{(3 + 8i) + Z_2}{2}$$

Multiply both sides by 2:

$$2(1 + 7i) = (3 + 8i) + Z_2$$

Simplify:

$$2 + 14i = 3 + 8i + Z_2$$

Solve for (Z_2) :

$$Z_2 = 2 + 14i - (3 + 8i) = (2 - 3) + (14i - 8i) = -1 + 6i$$

Therefore, the other endpoint of the segment is at $(-1 + 6i)$.

Geometric Interpretation of the Results

Visualizing the Segment

Plotting the points:

- Known endpoint: $(Z_1 = 3 + 8i)$
- Unknown endpoint: $(Z_2 = -1 + 6i)$
- Midpoint: $(M = 1 + 7i)$

These points form a line segment in the complex plane with the midpoint at $(1, 7)$.

Calculating the Length of the Segment

The length (L) of the segment is:

$$L = |Z_2 - Z_1| = \sqrt{(-1 - 3)^2 + (6 - 8)^2} = \sqrt{(-4)^2 + (-2)^2} = \sqrt{16 + 4} = \sqrt{20} \approx 4.472$$

This length indicates the distance between the two endpoints and can be useful in geometric and physical applications.

Direction and Orientation

The vector from (Z_1) to (Z_2) :

$$Z_2 - Z_1 = (-1 + 6i) - (3 + 8i) = -4 - 2i$$

This vector points in the direction from (Z_1) to (Z_2) :

- Horizontal component: (-4)
- Vertical component: (-2)

The overall direction can be expressed as an angle (θ) relative to the positive real axis:

$$\theta = \arctan\left(\frac{-2}{-4}\right) = \arctan\left(\frac{1}{2}\right) \approx 26.57^\circ$$

Since both components are negative, the vector points to the third quadrant, and the actual angle from the positive x-axis is:

$$\theta \approx 180^\circ + 26.57^\circ = 206.57^\circ$$

This information can be valuable in vector analysis and physics.

Broader Applications and Significance

Complex Number Applications in Geometry

- Construction of Geometric Figures: Using complex numbers simplifies the process of constructing and analyzing geometric figures like triangles, rectangles, and circles.
- Transformations: Rotations, translations, and scalings are easily represented with complex multiplication and addition.
- Midpoint and Segment Calculations: As demonstrated, the midpoint formula in complex form aids in solving geometric problems efficiently.

Physics and Engineering Applications

- Signal Processing: Complex plane analysis assists in understanding waveforms and phasors.
- Electrical Engineering: Impedance and AC circuit analysis often involve complex numbers and their geometric interpretations.
- Robotics and Navigation: Path planning and movement analysis utilize the properties of segments and midpoints.

Mathematical Problem Solving and Education

- Understanding segments and midpoints in the complex plane enhances problem-solving skills, spatial reasoning, and comprehension of complex analysis.

Additional Related Problems and Explorations

Problem 1: Finding the Length and Direction of Other Segments

Given two points in the complex plane, determine:

- The length of the segment.
- The angle of the segment relative to the positive real axis.
- The midpoint.

This practice reinforces understanding of the fundamental formulas and their geometric interpretations.

Problem 2: Reflection and Symmetry

Explore how reflecting points across various axes or lines affects the segment's properties, such as:

- Midpoint location.
- Length.
- Orientation.

Problem 3: Applying the Concept to Polygon Construction

Use segments and midpoints to construct polygons like triangles and quadrilaterals, analyzing their properties in the complex plane.

Conclusion

Understanding how to determine the other endpoint of a segment in the complex plane when the midpoint and one endpoint are known is a foundational skill in complex analysis and geometry. By applying the midpoint formula and interpreting the results geometrically, mathematicians and students can solve a wide array of problems, from simple constructions to complex transformations. The process not only deepens comprehension of complex numbers but also highlights their versatility in representing and analyzing geometric figures effectively.

This exploration emphasizes the importance of the complex plane as a powerful tool for visualizing and solving mathematical problems, bridging algebraic formulas with geometric intuition. Mastery of these concepts paves the way for advanced studies in mathematics, physics, engineering, and related fields, showcasing the elegance and utility of complex analysis in various scientific disciplines.

Frequently Asked Questions

What is the coordinate of the other endpoint of the segment if one endpoint is at $3 + 8i$ and the midpoint is at $1 + 7i$?

The other endpoint is at $-1 + 6i$.

How do you find the missing endpoint of a segment in the complex plane when given the midpoint and one endpoint?

Use the midpoint formula: If midpoint $M = (x_m, y_m)$ and one endpoint $A = (x_1, y_1)$, then the other endpoint $B = (2x_m - x_1, 2y_m - y_1)$.

What is the significance of the midpoint in a complex plane segment problem?

The midpoint helps determine the position of the other endpoint when one endpoint and the midpoint are known, reflecting the symmetry of the segment.

Can the midpoint of a complex segment be outside the segment itself?

No, the midpoint always lies exactly halfway between the two endpoints of the segment.

If the known endpoint is at $3 + 8i$ and the midpoint at $1 + 7i$, what is the length of the segment?

The length of the segment is $2\sqrt{2}$ units, calculated using the distance formula between the endpoints.

How does understanding midpoints in the complex plane help in geometric interpretations?

It allows for visualization of symmetry, bisectors, and helps solve problems related to segment division and geometric constructions.

What is the importance of correctly applying the midpoint formula in complex plane problems?

Accurate application ensures correct determination of unknown endpoints and facilitates solving geometric problems efficiently.

Additional Resources

A Segment in the Complex Plane Has a Midpoint at $1 + 7i$. If One Endpoint is at $3 + 8i$, What Can We Learn?

In the realm of complex analysis and geometric interpretations of complex numbers, understanding the relationships between points, segments, and their midpoints is fundamental. When examining a segment within the complex plane—an essential tool for visualizing complex numbers—the midpoint provides critical information about the segment's length and orientation. This article delves into a specific problem: given that a segment's midpoint is at $1 + 7i$ and one endpoint is at $3 + 8i$, what are the implications? We will explore the mathematical foundations, the geometric significance, and the broader context of such problems, guiding readers through the analytical process with clarity and depth.

Understanding the Complex Plane and Line Segments

The Complex Plane as a Geometric Framework

The complex plane, also known as the Argand plane, is a two-dimensional coordinate system where each complex number $(z = x + yi)$ corresponds to a point with coordinates $((x, y))$. The horizontal axis (the real axis) represents the real part, while the vertical axis (the imaginary axis) represents the imaginary part.

This visualization transforms algebraic operations into geometric transformations:

- Addition corresponds to vector addition.
- Multiplication involves scaling and rotation.
- Subtraction measures the distance between points.

Understanding these geometric interpretations is crucial in analyzing segments and their properties.

Line Segments in the Complex Plane

A line segment in the complex plane is defined by its two endpoints (z_1) and (z_2) , which are complex numbers. The segment is the shortest path connecting these points, with a length given by the distance formula:

$$\begin{aligned} &|z_2 - z_1| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \end{aligned}$$

The midpoint (M) of the segment connecting (z_1) and (z_2) is the average of their coordinates:

$$\begin{aligned} M &= \frac{z_1 + z_2}{2} \end{aligned}$$

This midpoint is a critical concept, as it reflects the center point of the segment and is useful in various geometric and algebraic calculations.

Mathematical Foundations of the Problem

Given Data and Objective

The problem specifies:

- The midpoint of a segment is at $(M = 1 + 7i)$.
- One endpoint of the segment is at $(z_1 = 3 + 8i)$.

The goal is to find the other endpoint (z_2) .

This is a classic problem involving the properties of midpoints and endpoints, which can be approached systematically.

Applying the Midpoint Formula

From the definition of a midpoint:

$$M = \frac{z_1 + z_2}{2}$$

Rearranged to solve for (z_2) :

$$z_2 = 2M - z_1$$

Substituting the known values:

$$z_2 = 2(1 + 7i) - (3 + 8i)$$

Calculating step-by-step:

$$z_2 = (2 + 14i) - (3 + 8i) = (2 - 3) + (14i - 8i) = -1 + 6i$$

Thus, the other endpoint of the segment is at $(-1 + 6i)$.

Geometric Interpretation and Significance

Location of the Endpoints

Plotting the points:

- Known endpoint: $(3, 8)$
- Calculated endpoint: $(-1, 6)$
- Midpoint: $(1, 7)$

The midpoint lies exactly midway between these two points, confirming the correctness of the calculation.

Visualizing the Segment

The segment connects $(3, 8)$ and $(-1, 6)$. The segment's length can be calculated as:

$$\begin{aligned} |z_2 - z_1| &= \sqrt{(-1 - 3)^2 + (6 - 8)^2} = \sqrt{(-4)^2 + (-2)^2} = \\ &= \sqrt{16 + 4} = \sqrt{20} \approx 4.47 \end{aligned}$$

The segment's midpoint at $(1, 7)$ is exactly at the average of the endpoints' coordinates:

$$x_{\text{mid}} = \frac{3 + (-1)}{2} = 1, \quad y_{\text{mid}} = \frac{8 + 6}{2} = 7$$

which matches the given data, confirming the geometric consistency.

Implications of the Segment's Orientation and Length

- Direction: The vector from z_1 to z_2 is:

$$z_2 - z_1 = (-1 + 6i) - (3 + 8i) = (-4 - 2i)$$

- Length: As calculated, approximately 4.47 units.
- Orientation: The segment points from the northeast (since $3 + 8i$) to the southwest ($-1 + 6i$), with a negative real component and a slight

decrease in the imaginary part.

Understanding the segment's orientation helps in visualizing transformations such as rotations or reflections in the complex plane.

Broader Context and Applications

Relevance in Complex Analysis and Geometry

This problem exemplifies fundamental principles in complex analysis, especially in geometric interpretations. It demonstrates how algebraic formulas directly translate into visual and spatial relationships, reinforcing the interconnectedness of algebra and geometry.

Such problems are foundational in areas like:

- Complex transformations: understanding how points move under rotations and translations.
- Fractal geometry: analyzing segments and their subdivisions.
- Signal processing: interpreting complex signals in terms of phase and magnitude.

Educational Significance

Problems like these are invaluable in educational contexts, serving as concrete examples to help students grasp abstract concepts:

- Reinforcing the midpoint formula in a new context.
- Developing spatial reasoning skills.
- Encouraging visualization of complex numbers.

Practical Applications

Beyond theoretical interest, understanding segments in the complex plane has practical implications in fields such as:

- Electrical engineering: analyzing phasors representing AC signals.
- Physics: representing vectors and forces.
- Computer graphics: manipulating points and shapes in the complex plane.

Conclusion

This analytical exploration reveals that, given a midpoint at $(1 + 7i)$ and one endpoint at $(3 + 8i)$, the other endpoint must be at $(-1 + 6i)$. This conclusion is rooted in the fundamental properties of midpoints, which serve as a bridge between algebraic formulas and geometric intuition. The exercise underscores the elegance of complex analysis, where algebra and geometry intertwine seamlessly, offering profound insights into spatial relationships, transformations, and the structure of the complex plane.

By understanding the underlying principles, students and professionals alike can better appreciate the utility of complex numbers not just as algebraic entities but as powerful tools for geometric reasoning and problem-solving. The problem exemplifies how simple formulas can unlock a deeper understanding of the spatial arrangements within the complex plane, enriching our mathematical perspective and broadening the scope of applications across science and engineering.

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