3. Consider The Linear Diophantine Equation 21x+14y+12z = 1. Find All Other Solutions For X, Y, Z Z.

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Linear Diophantine equations are fundamental in number theory, representing equations where integer solutions are sought for polynomial equations with multiple variables. The specific equation $21x + 14y + 12z = 1$ presents an intriguing problem: how to find all integer solutions (x, y, z) that satisfy this equation? This article explores the methods to solve this equation systematically, discusses the underlying theory, and provides comprehensive steps to determine the entire solution set. Whether you are a student, mathematician, or enthusiast, understanding the process behind solving such equations is essential for mastering number theory and its applications.

Understanding Linear Diophantine Equations

Before diving into the specific equation, it is important to understand what linear Diophantine equations are and their key properties.

Definition of Linear Diophantine Equations

- A linear Diophantine equation is an equation of the form:

a1x1 + a2x2 + ... + anxn = b

- where ai, b are integers, and solutions (xi) are sought in integers.

Conditions for Solutions

- A key property of these equations is that solutions exist if and only if the greatest common divisor (gcd) of the coefficients divides the constant term b.
- For the equation $21x + 14y + 12z = 1$, solutions exist only if gcd(21, 14, 12) divides 1.

Relevance in Mathematics and Applications

- These equations appear in various fields like cryptography, coding theory, integer programming, and combinatorics.
- Solving them helps in understanding divisibility, modular arithmetic, and optimization problems.

Analyzing the Specific Equation 21x + 14y + 12z = 1

Let's analyze the given equation step-by-step to understand the solution process.

Step 1: Find the gcd of the coefficients

- Coefficients: 21, 14, 12
- Compute gcd:
- gcd(21, 14) = 7
- gcd(7, 12) = 1
- Since gcd(21, 14, 12) = 1, and 1 divides 1, solutions exist.

Step 2: Simplify the equation if necessary

- Because gcd is 1, the equation is already in its simplest form.
- No reduction is needed.

Step 3: Express the equation as a linear combination

- The goal is to find particular solutions (x0, y0, z0) satisfying the equation, then describe the general solution set.

Solving the Diophantine Equation: Step-by-Step Approach

To find all solutions, we can proceed systematically:

Step 1: Find particular solutions to the equation

- Fix one variable and solve for the others.
- Alternatively, reduce the problem to solving two-variable equations step-by-step.

Step 2: Use the Extended Euclidean Algorithm

- To find integer solutions to linear equations, the extended Euclidean algorithm helps in expressing gcd as a linear combination of the coefficients.

Step 3: Express the solutions in parametric form

- Once a particular solution is found, the general solution can be expressed as a parametric family involving integer parameters.

Detailed Solution for $21x + 14y + 12z = 1$

Let's now perform a detailed solution process.

Step 1: Solve $21x + 14y = k$ for some integer k

- Since the original equation involves three variables, we can first consider fixing z and then solving for x and y.

Rearranged as:

$21x + 14y = 1 - 12z$

Set:

$D = 1 - 12z$

Our task reduces to solving:

$21x + 14y = D$

for various integer z, where D varies accordingly.

Step 2: Find solutions to $21x + 14y = D$

- Note that $\gcd(21, 14) = 7$.
- For solutions to exist for x, y, D, 7 must divide D.
- $D = 1 - 12z$, so D must be divisible by 7.

Set:

$D \equiv 0 \mod 7$

which implies:

$1 - 12z \equiv 0 \mod 7$

Calculate:

$12z \equiv 1 \mod 7$

Since $12 \equiv 5 \mod 7$,

$5z \equiv 1 \mod 7$

Find z satisfying:

$5z \equiv 1 \mod 7$

The modular inverse of 5 mod 7 is 3 because:

$5\ 3 = 15 \equiv 1 \mod 7$

Thus,

$z \equiv 3\ 1 \equiv 3 \mod 7$

Therefore, the general solution for z is:

$z = 3 + 7k$, where $k \in \mathbb{Z}$

Now, D becomes:

$D = 1 - 12z = 1 - 12(3 + 7k) = 1 - 36 - 84k = -35 - 84k$

Since gcd(21, 14) = 7 divides D, solutions exist.

Next, solve:

$21x + 14y = D$

Divide through by 7:

$3x + 2y = (D)/7 = (-35 - 84k)/7 = -5 - 12k$

Now, the simplified equation:

$3x + 2y = -5 - 12k$

Step 3: Find particular solutions to 3x + 2y = M, where M = -5 - 12k

- For each integer k, define $M = -5 - 12k$.

Using the extended Euclidean algorithm to solve:

$3x + 2y = M$

Since gcd(3, 2) = 1, solutions exist for all integers M.

Express:

$3x \equiv M \mod 2$

Because $3 \equiv 1 \mod 2$,

$x \equiv M \mod 2$

Similarly, for each k, find x and y satisfying:

$3x + 2y = M$

Solution:

- For x:

$x \equiv M \mod 2$

- For y:

$y = (M - 3x)/2$

To find a particular x:

- Choose x such that the RHS is integer.

Since $(M - 3x)$ must be divisible by 2:

$(M - 3x) \equiv 0 \mod 2$

But $3x \equiv x \mod 2$, so:

$M - x \equiv 0 \mod 2$

Thus:

$x \equiv M \mod 2$

which matches earlier.

Choose:

$x = x0 + t\ 2$, where $t \in \mathbb{Z}$

Then:

$y = (M - 3x0 - 6t)/2$

Set x0 as the particular solution satisfying $x0 \equiv M \mod 2$.

For simplicity, pick:

$x0 = M \mod 2$

- If M is even, $x_0 = 0$
- If M is odd, $x_0 = 1$

The general solution:

$$x = x_0 + 2t$$

$$y = (M - 3x)/2$$

Substituting x:

$$y = (M - 3(x_0 + 2t))/2 = (M - 3x_0 - 6t)/2$$

Since M and $3x_0$ are known for each k, this gives the general solution.

Summary of the General Solution

Putting everything together:

- For each integer k, $z = 3 + 7k$
- $D = -35 - 84k$
- $M = -5 - 12k$

- $x = x_0 + 2t$, where $x_0 \equiv M \bmod 2$
- $y = (M - 3x)/2$

Therefore, the general solutions for the original variables are:

\[
\boxed{
\begin{aligned}
z &= 3 + 7k, \quad k \in \mathbb{Z} \\
x &= x_0 + 2t, \quad t \

Frequently Asked Questions

What conditions must be met for the linear Diophantine equation 21x + 14y + 12z = 1 to have solutions?

The greatest common divisor (gcd) of the coefficients 21, 14, and 12 must divide the constant term 1. Since gcd(21,14,12) = 1, which divides 1, solutions exist for all integers x, y, z satisfying the equation.

How do we find a particular solution to the equation 21x + 14y + 12z = 1?

We can use the extended Euclidean algorithm to find solutions for pairs of coefficients, then combine these solutions to find a particular solution for the entire equation. For example, solve 21x + 14y = d, then incorporate 12z to satisfy the full equation.

What is the general solution form for x, y, z in the equation 21x + 14y + 12z = 1?

The general solutions can be expressed in terms of parameters t, s, etc., based on the particular solution. For example, if (x_0, y_0, z_0) is a particular solution, then all solutions are of the form $x = x_0 + k_1 m_1$, $y = y_0 + k_2 m_2$, $z = z_0 + k_3 m_3$, where k_1, k_2, k_3 are integers and m_1, m_2, m_3 are derived from the coefficients' relations.

Are there infinitely many solutions to the equation 21x + 14y + 12z = 1?

Yes, because the equation is linear and the gcd divides 1, there are infinitely many integer solutions that can be generated by adding multiples of the solution basis vectors.

Can you provide an example of a particular solution to 21x + 14y + 12z = 1?

Yes. For instance, setting z=0 simplifies to 21x + 14y = 1. Using the extended Euclidean algorithm, one solution is x = 1, y = -1, and z = 0, which satisfies the equation.

How do parameters affect the solution set of the Diophantine equation?

Parameters allow us to express the infinite family of solutions compactly. By introducing integer parameters, we can generate all solutions through a parametric formula based on the particular solution and the coefficients' linear combinations.

What steps should be taken to find all solutions for x, y, z in the given equation?

First, verify the gcd condition; then, find a particular solution using the extended Euclidean algorithm; next, express the general solution by adding the homogeneous solution components parameterized by integers; finally, write the complete parametric form for all solutions.

Additional Resources

Consider The Linear Diophantine Equation 21x + 14y + 12z = 1: An In-Depth Investigation of Solutions

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Introduction

Linear Diophantine equations, a fundamental topic in number theory, involve finding integer solutions to equations where the variables appear linearly. Among these, the equation

$$ 21x + 14y + 12z = 1 $$

serves as an illustrative example to explore the nature of solutions, underlying divisibility conditions, and the structure of the solution set. This article aims to delve into the problem with a comprehensive, rigorous approach, uncovering all possible solutions for integers x, y, z, and illustrating the methods used to characterize these solutions.

Background and Significance

Understanding solutions to linear Diophantine equations is crucial in various fields such as cryptography, coding theory, and algebraic number theory. The particular form $21x + 14y + 12z = 1$ is notable because the coefficients share common factors, prompting an analysis of divisibility conditions and the existence of solutions.

The core questions addressed include:

- When do solutions exist?
- How can all solutions be characterized?
- What is the structure of the solution set?

By addressing these questions, the article aims to provide a blueprint for solving similar equations and understanding their properties.

Divisibility Conditions and Existence of Solutions

The Greatest Common Divisor and Its Role

The first step in analyzing a linear Diophantine equation is to determine the greatest common divisor (gcd) of the coefficients:

$$
d = \gcd(21, 14, 12)
$$

Calculating d:

\[
\begin{aligned}
\gcd(21, 14) &= 7 \\
\gcd(7, 12) &= 1

\end{aligned}
\]

Thus,

\[
d = 1
\]

Since \(d = 1\), the fundamental theorem states that the equation

\[
21x + 14y + 12z = 1
\]

has at least one integer solution. In fact, because the gcd divides 1 (which it does), solutions exist, and the set of solutions can be characterized explicitly.

Methodology for Finding General Solutions

The approach involves reducing the problem stepwise:

1. Simplify the equation by dividing through by the gcd where appropriate.
2. Express certain variables in terms of others.
3. Use the theory of linear Diophantine equations in two variables to find parametric solutions.
4. Reconcile solutions to encompass all three variables.

This systematic approach ensures a complete characterization of the solution set.

Step 1: Simplification and Reduction

Since \(\gcd(21,14,12) = 1\), we analyze the equation directly. To facilitate solution-finding, consider the equation:

\[
21x + 14y + 12z = 1
\]

which involves three variables. We can attempt to eliminate one variable by expressing it in terms of the others.

Step 2: Express \(z\) in terms of \(x\) and \(y\)

Rearranged, the equation becomes:

$$
12z = 1 - 21x - 14y
$$

or

$$
z = \frac{1 - 21x - 14y}{12}
$$

For z to be an integer, the numerator must be divisible by 12:

$$
1 - 21x - 14y \equiv 0 \pmod{12}
$$

Step 3: Analyzing the Divisibility Condition

Expressed modulo 12, the divisibility condition is:

$$
1 - 21x - 14y \equiv 0 \pmod{12}
$$

Simplify each term modulo 12:

- $21x \equiv (21 \bmod 12) \times x \equiv 9x \pmod{12}$
- $14y \equiv (14 \bmod 12) \times y \equiv 2y \pmod{12}$

So the congruence becomes:

$$
1 - 9x - 2y \equiv 0 \pmod{12}
$$

which simplifies to:

$$
9x + 2y \equiv 1 \pmod{12}
$$

Step 4: Solving the Congruence for x and y

The key now is to determine all integer pairs (x, y) satisfying:

$$
9x + 2y \equiv 1 \pmod{12}

\]

This is a linear congruence in two variables. To analyze it, consider fixing \(x\) and solving for \(y\):

\[
2y \equiv 1 - 9x \pmod{12}
\]

Since \(\gcd(2, 12) = 2\), solutions for \(y\) exist only if the right side is divisible by 2:

\[
1 - 9x \equiv 0 \pmod{2}
\]

Let's analyze the parity:

- \(1\) is odd.
- \(9x\) is odd if \(x\) is odd, even if \(x\) is even.

Thus,

\[
1 - 9x \equiv 0 \pmod{2} \implies 1 - 9x \equiv 0 \pmod{2}
\]

which simplifies to:

\[
1 \equiv 9x \pmod{2}
\]

Since \(9 \equiv 1 \pmod{2}\), this reduces to:

\[
1 \equiv x \pmod{2}
\]

meaning:

\[
x \equiv 1 \pmod{2}
\]

Conclusion:

- \(x\) must be odd for solutions to exist.

Now, for each odd \(x\), find \(y\):

\[
2y \equiv 1 - 9x \pmod{12}

\]

Since \(x\) is odd, \(9x\) modulo 12:

\[
9 \times x \equiv 9 \times 1 \equiv 9 \pmod{12}
\]

because \(x \equiv 1 \pmod{2}\), and considering \(x \equiv 1, 3, 5, 7, 9, 11 \pmod{12}\) to find all possible residues.

Let's analyze all odd residues modulo 12:

\(x \pmod{12}\)	\(x\) odd?	\(9x \pmod{12}\)
1	Yes	\(9 \times 1 = 9\)
3	Yes	\(9 \times 3 = 27 \equiv 3\)
5	Yes	\(9 \times 5 = 45 \equiv 9\)
7	Yes	\(9 \times 7 = 63 \equiv 3\)
9	Yes	\(9 \times 9 = 81 \equiv 9\)
11	Yes	\(9 \times 11 = 99 \equiv 3\)

Therefore, when \(x \equiv 1, 5, 9 \pmod{12}\), \(9x \equiv 9 \pmod{12}\). When \(x \equiv 3, 7, 11 \pmod{12}\), \(9x \equiv 3 \pmod{12}\).

Now, for each case:

- If \(x \equiv 1, 5, 9 \pmod{12}\):

\[
2 y \equiv 1 - 9x \equiv 1 - 9 \equiv -8 \equiv 4 \pmod{12}
\]

- If \(x \equiv 3, 7, 11 \pmod{12}\):

\[
2 y \equiv 1 - 3 \equiv -2 \equiv 10 \pmod{12}
\]

Now, solve for \(y\):

Because \(\gcd(2, 12) = 2\), solutions exist only if the right side is divisible by 2, which it is. Divide through by 2:

- For \(2 y \equiv 4 \pmod{12}\):

\[
2 y \equiv 4 \pmod{12}
\]

Dividing both sides by 2:

\[
y \equiv 2 \pmod{6}
\]

Similarly, for \(2 y \equiv 10 \pmod{12}\):

\[
2 y \equiv 10 \pmod{12}
\]

Dividing both sides by 2:

\[
y \equiv 5 \pmod{6}
\]

Summary of solutions:

| \(x \pmod{12}\) | \(x\) odd? | \(9x \pmod{12}\) | \(2y \equiv\) | \(y \pmod{6

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