3. Given That:X(z) = 2 + 3z-1+4z-2 A) Determine The Initial Value Of Corresponding Sequence X(n). B)

Understanding the Z-Transform and Its Application to Sequence Analysis

The Z-transform is a powerful mathematical tool used extensively in signal processing, control systems, and discrete-time systems analysis. It allows engineers and mathematicians to convert complex difference equations into algebraic equations, simplifying the process of analyzing and designing systems. When given a Z-transform expression, such as **$X(z) = 2 + 3z^{-1} + 4z^{-2}$**, one of the key tasks is to determine the corresponding time-domain sequence, particularly its initial value and subsequent terms. This article explores the process of extracting the initial sequence value from a given Z-transform, focusing on the specific example provided: **$X(z) = 2 + 3z^{-1} + 4z^{-2}$**.

Fundamentals of the Z-Transform

Definition of the Z-Transform

The Z-transform of a discrete-time sequence *{x[n]}* is defined as:

$X(z) = \sum_{n=0}$^∞ $x[n]z^{-n}$

where:

- **x[n]** is the sequence in the time domain
- **z** is a complex variable
- The summation runs over all non-negative integers, assuming causality

Inverse Z-Transform

The inverse Z-transform allows us to retrieve the sequence *{x[n]}* from its Z-transform *X(z)*. Methods for inverse conversion include:

- Power series expansion
- Partial fraction decomposition
- Residue method
- Contour integration (complex analysis)

Decoding the Given Z-Transform Expression

Analyzing $X(z) = 2 + 3z^{-1} + 4z^{-2}$

The given Z-transform can be rewritten as a sum of terms, each corresponding to specific sequence elements:

$X(z) = 2 + 3z^{-1} + 4z^{-2}$

Comparing this to the general form of the Z-transform:

$X(z) = x[0] + x[1]z^{-1} + x[2]z^{-2} + ...$

we observe that:

- **x[0] = 2**
- **x[1] = 3**
- **x[2] = 4**

This direct comparison indicates that the sequence *{x[n]}* begins with these initial values, assuming the sequence is causal (i.e., x[n] = 0 for n < 0).

Part A: Determining the Initial Value of the Sequence X(n)

Step-by-Step Process

To explicitly determine the initial value of the sequence *X(n)*, we follow a systematic approach:

1. **Identify the coefficients in the Z-transform**: Recognize the terms corresponding to specific sequence values.
2. **Match terms with sequence elements**: Use the power series expansion to relate each term to *x[n]*.
3. **Extract initial value**: The coefficient of the term independent of z^{-1} (the constant term) corresponds to *x[0]*.

Application to the Given Example

In our example, the Z-transform is:

$X(z) = 2 + 3z^{-1} + 4z^{-2}$

Since the constant term (independent of z^{-1}) is 2, it directly indicates the initial sequence value:

- **x[0] = 2**

This initial value is crucial because it serves as the starting point for reconstructing the entire sequence and analyzing system behavior.

Part B: Reconstructing the Sequence X(n)

Methodology for Sequence Reconstruction

Using the inverse Z-transform, the sequence can be reconstructed by identifying each coefficient in the Z-transform expression:

- The coefficient of z^0 (constant term): **x[0]**
- The coefficient of z^{-1}: **x[1]**
- The coefficient of z^{-2}: **x[2]**

Consequently, the sequence *{x[n]}* corresponding to the given Z-transform is:

- **x[0] = 2**
- **x[1] = 3**
- **x[2] = 4**

Extending the Sequence

Since the Z-transform provided is a polynomial (finite sum), the sequence terminates after the second term. If a more complex or infinite series were involved, methods like partial fraction decomposition or long division would be necessary to find subsequent terms.

Practical Applications of Initial Sequence Values

System Analysis and Design

Knowing the initial value *x[0]* is essential in various engineering applications, including:

- Designing digital filters where initial conditions affect transient response

- Analyzing system stability and response characteristics
- Implementing recursive algorithms that depend on initial states

Signal Processing and Control Systems

Initial sequence values provide insights into the system's history or starting conditions, which are critical in predictive modeling, adaptive filtering, and control system tuning.

Summary and Key Takeaways

- The given Z-transform $\mathbf{X(z) = 2 + 3z^{-1} + 4z^{-2}}$ directly indicates the sequence *{x[n]}* via coefficient matching.
- The initial sequence value *x[0]* is 2, derived from the constant term in the Z-transform.
- The subsequent sequence terms are $x[1] = 3$ and $x[2] = 4$.
- Understanding how to invert Z-transforms is vital for system analysis, especially when dealing with more complex or infinite sequences.

Conclusion

The process of determining the initial value of a sequence from its Z-transform is straightforward when the Z-transform is expressed as a polynomial in terms of z^{-1}. Recognizing the coefficients and their correspondence to sequence elements allows engineers and mathematicians to reconstruct the original sequence efficiently. The example $\mathbf{X(z) = 2 + 3z^{-1} + 4z^{-2}}$ clearly demonstrates this approach, revealing that the sequence begins with $x[0] = 2$. This foundational understanding is essential in the analysis and design of discrete-time systems across various engineering disciplines.

Frequently Asked Questions

How do you find the initial value of the sequence X(n) given its Z-transform X(z) = 2 + 3z^{-1} + 4z^{-2}?

The initial value X(0) is obtained by evaluating the inverse Z-transform at n=0, which corresponds to the coefficient of z^{0} in the Z-transform. In this case, X(0) = 2.

What is the significance of the terms 3z^{-1} and 4z^{-2} in the Z-transform X(z)?

These terms represent the contributions of the sequence at time steps n=1 and n=2, respectively. Specifically, 3z^{-1} corresponds to X(1), and 4z^{-2} corresponds to X(2).

How can we determine the sequence X(n) from its Z-transform X(z) = 2 + 3z^{-1} + 4z^{-2}?

By recognizing that the Z-transform is a sum of terms corresponding to the sequence elements, we can directly read off X(0)=2, X(1)=3, and X(2)=4. For n > 2, the sequence elements are zero if no additional terms are present.

If the Z-transform includes only finite terms such as 2 + 3z^{-1} + 4z^{-2}, what is the corresponding time-domain sequence?

The sequence is finite and given by X(0)=2, X(1)=3, X(2)=4, and X(n)=0 for all n > 2.

What is the general approach to determine the initial value of a sequence from its Z-transform expression?

The initial value X(0) can be found by evaluating the inverse Z-transform at n=0, which often involves taking the coefficient of the z^{0} term in the Z-transform or using the limit as z approaches infinity of z times X(z).

Additional Resources

In-Depth Analysis of the Sequence Defined by X(z) = 2 + 3z^{-1} + 4z^{-2}

Understanding the Problem Statement

The problem provides a Z-transform expression:

$$X(z) = 2 + 3z^{-1} + 4z^{-2}$$

and asks two main questions:

- A) Determine the initial value of the corresponding sequence $x(n)$, i.e., $x(0)$.
- B) (The question for part B isn't fully provided in the initial prompt, but typically such problems involve finding the sequence $x(n)$ or other properties based on the Z-transform. We will focus primarily on part A, with insights into how to proceed for part B if needed.)

Key terms to understand:

- Z-transform: A powerful tool in signal processing and discrete-time systems, converting a sequence $x(n)$ into a complex frequency domain representation $X(z)$.
- Initial value theorem: Allows us to find $x(0)$ directly from $X(z)$.

Fundamentals of the Z-Transform and Its Inverse

Definition of Z-Transform

The Z-transform of a discrete sequence $x(n)$ is given by:

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n}$$

where:

- $x(n)$ is the sequence in the time domain.
- z is a complex variable.

Inverse Z-Transform

To recover $x(n)$ from $X(z)$, we can use various methods such as:

- Power series expansion
- Partial fraction decomposition
- Residue calculus
- Applying the initial and final value theorems

Deciphering the Given $X(z)$

The given function:

$$
X(z) = 2 + 3z^{-1} + 4z^{-2}
$$

is expressed explicitly as a sum of powers of z^{-1}. This form suggests that the sequence $x(n)$ can be directly deduced by matching terms.

Expressing $X(z)$ in a standard power series form

Recall that:

$$
X(z) = \sum_{n=0}^{\infty} x(n) z^{-n}
$$

Matching the terms:

- The constant term 2 corresponds to $x(0)$.
- The coefficient of z^{-1} corresponds to $x(1)$.
- The coefficient of z^{-2} corresponds to $x(2)$.

Given that:

$$
X(z) = 2 + 3z^{-1} + 4z^{-2}
$$

we can directly extract:

$$
x(0) = 2
$$
$$
x(1) = 3
$$

$$
x(2) = 4
$$

and for $n \geq 3$:

$$
x(n) = 0
$$

since no higher-order terms are present.

Part A: Determining the Initial Value $x(0)$

Applying the Initial Value Theorem

The initial value theorem states:

$$
x(0) = \lim_{z \to \infty} X(z)
$$

which is valid when the sequence $x(n)$ is causal and absolutely summable. For rational Z-transforms expressed in terms of z^{-1}, the value of $x(0)$ can be obtained directly from the constant term in $X(z)$.

In this case:

$$
X(z) = 2 + 3z^{-1} + 4z^{-2}
$$

The constant term is 2, which directly gives:

$$
x(0) = 2
$$

Conclusion for Part A:

$$
\boxed{
x(0) = 2
}

\]

This confirms that the initial value of the sequence \(x(n) \) at \(n=0 \) is 2.

Part B: Finding the Entire Sequence \(x(n)\)

Although not explicitly asked, understanding how to find the full sequence helps cement the concepts.

Method 1: Direct Term Matching

Given the explicit form of \(X(z) \):

\[
X(z) = 2 + 3z^{-1} + 4z^{-2}
\]

we can write:

\[
x(n) = \text{coefficient of } z^{-n}
\]

which yields:

\[
x(0) = 2
\]
\[
x(1) = 3
\]
\[
x(2) = 4
\]
\[
x(n) = 0 \quad \text{for } n \geq 3
\]

Resulting sequence:

\[
x(n) =
\begin{cases}
2, & n=0 \\

$$
3, & n=1 \\
4, & n=2 \\
0, & n \geq 3
\end{cases}
$$

Method 2: Using Inverse Z-Transform

Another approach involves recognizing the structure as a finite sum of scaled unit impulses shifted in time.

- The terms $3z^{-1}$ and $4z^{-2}$ imply sequences that are delayed impulses:

$$
x(n) = 2\delta(n) + 3\delta(n-1) + 4\delta(n-2)
$$

where $\delta(n)$ is the Kronecker delta function.

Thus:

$$
x(n) = 2\delta(n) + 3\delta(n-1) + 4\delta(n-2)
$$

which confirms the earlier direct extraction.

Implications and Generalizations

Understanding the Sequence Behavior

- Since all non-zero values are confined to $n=0,1,2$, the sequence is finite and causal.
- The initial value of $x(0)$ is 2, which aligns with the constant term in the Z-transform.

Relation to System Response

Suppose this sequence represents the response of a discrete-time system:

- The initial value indicates the system's response at the start.
- The finite duration suggests an impulsive or finite impulse response system.

Extending to More Complex Cases

In more complex scenarios, \(X(z) \) may involve rational functions with denominators, leading to more intricate sequences. Partial fraction expansion and residue calculus are then employed to find \(x(n) \), especially when dealing with poles.

Summary and Final Remarks

- The given \(X(z) \) is a polynomial in \(z^{-1} \), indicative of a finite sequence.
- The initial value \(x(0) \) corresponds directly to the constant term, which is 2.
- The subsequent terms are read directly from the coefficients of \(z^{-1} \) and \(z^{-2} \).

Final result:

\[
\boxed{
\textbf{A) } \quad x(0) = 2
}
\]

and the complete sequence:

\[
x(n) = \begin{cases}
2, & n=0 \\
3, & n=1 \\
4, & n=2 \\
0, & n \geq 3
\end{cases}
\]

Additional Considerations for Further Study

- Inverse Z-transform techniques: For more complex \(X(z) \), partial fraction decomposition becomes essential.
- Stability considerations: If \(X(z) \) involved denominators, analyzing pole locations would be necessary.
- System interpretation: Understanding whether the sequence is an impulse response, step response, or a general signal.

In conclusion, this problem exemplifies how the structure of a Z-transform directly reveals the sequence it represents, especially when expressed explicitly as a sum of powers of \(z^{-1} \). Recognizing the correspondence simplifies the process of extracting initial values and understanding the underlying discrete-time signal.

3 Given That X Z 2 3z 1 4z 2 A Determine The Initial Value Of Corresponding Sequence X N B

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