(4) Find An Equation Of The Line That Passes Through The Point (0,2) And Is Paraliel To 8x+5y=6. (b)

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Understanding how to find the equation of a line that passes through a specific point and is parallel to a given line is a fundamental concept in analytic geometry. In this article, we will explore the step-by-step process to derive such an equation, focusing on the line passing through the point (0, 2) and parallel to the line represented by the equation 8x + 5y = 6. We will cover the essential concepts, the detailed calculations, and provide practical examples to ensure clarity and comprehension.

Understanding the Problem

Before diving into the solution, it's crucial to understand what the problem entails:

- Given Point: (0, 2)
- Given Line: 8x + 5y = 6
- Objective: Find the equation of a line that passes through the point (0, 2) and is parallel to the given line.

Key Concepts and Definitions

Line Equation Forms

- Standard Form: Ax + By = C
- Slope-Intercept Form: y = mx + b

Parallel Lines

- Two lines are parallel if they have the same slope.
- Parallel lines do not intersect and are equidistant from each other at all points.

Slope of a Line

- In the slope-intercept form, $y = mx + b$, m is the slope.
- To find the slope from the standard form $Ax + By = C$, rearranged into slope-intercept form.

Step-by-Step Solution

Step 1: Find the Slope of the Given Line

The given line is:

$8x + 5y = 6$

To find its slope, rewrite it in slope-intercept form:

- Subtract 8x from both sides:

$5y = -8x + 6$

- Divide both sides by 5:

$y = (-8/5)x + 6/5$

The slope (m_1) of the given line is:

$m_1 = -8/5$

Step 2: Determine the Slope of the Parallel Line

Since parallel lines have the same slope:

$m_2 = m_1 = -8/5$

Step 3: Use the Point-Slope Form to Find the Equation

The point the new line passes through is $(x_0, y_0) = (0, 2)$.

The point-slope form of a line:

$y - y_0 = m(x - x_0)$

Plugging in the values:

$y - 2 = (-8/5)(x - 0)$

Simplify:

$y - 2 = (-8/5)x$

Step 4: Write the Equation in Slope-Intercept Form

Add 2 to both sides:

$y = (-8/5)x + 2$

This is the equation of the required line.

Final Equation of the Line

$$
\boxed{
y = -\frac{8}{5}x + 2
}
$$

This line passes through (0, 2) and is parallel to the line $8x + 5y = 6$.

Additional Considerations

Converting to Standard Form

If necessary, convert the slope-intercept form back to standard form:

$y = -\frac{8}{5}x + 2$

Multiply both sides by 5 to clear the denominator:

$5y = -8x + 10$

Rearranged:

$8x + 5y = 10$

This is the standard form of the parallel line.

Summary of the Process

- Find the slope of the given line.
- Use the same slope for the new line (since lines are parallel).
- Use the point-slope formula with the given point.
- Simplify to the desired form (slope-intercept or standard).

Practical Applications

Understanding how to find the equation of a line parallel to a given line through a specific point is useful in various fields:

- Geometry and Trigonometry: Solving problems involving parallel lines and angles.
- Engineering: Designing structures with parallel components.
- Computer Graphics: Calculating parallel lines for rendering and modeling.
- Navigation and Mapping: Plotting parallel routes or boundaries.

Common Mistakes to Avoid

- Confusing slopes: Remember that parallel lines share the same slope but are not necessarily the same line.
- Incorrect algebraic manipulation: Be careful when converting between forms to avoid errors.
- Forgetting to check the point: Always verify that the derived line passes through the given point.

Summary

Finding an equation of a line passing through a specific point and parallel to a given line involves understanding the relationship between slopes and line equations. By following the outlined steps—finding the slope of the original line, applying the same slope, and using the point-slope form—you can derive the desired line equation efficiently. This process is fundamental in coordinate geometry and has versatile applications across scientific and engineering disciplines.

FAQs

Q1: Can the process be used for lines not in standard form?
A: Yes. The key is to determine the slope, which can be obtained directly from any linear equation once it is in slope-intercept form or by rearrangement.

Q2: What if the given line is vertical?
A: Vertical lines have undefined slopes. Parallel lines to a vertical line are also vertical, and their equations are of the form x = constant. The method differs slightly in that case.

Q3: How do I check if the line is correct?
A: Substitute the point (0, 2) into your derived equation to verify it satisfies the equation. Also, confirm that the slope matches the original line.

By mastering these concepts and steps, you'll be well-equipped to handle similar problems involving parallel lines and point-line relationships in coordinate geometry.

Frequently Asked Questions

How do you find the equation of a line passing through (0, 2) and parallel to the line 8x + 5y = 6?

First, find the slope of the given line by rewriting it in slope-intercept form. Since the line is 8x + 5y = 6, then 5y = -8x + 6, so y = (-8/5)x + 6/5. The slope is -8/5. Because parallel lines have the same slope, the new line passing through (0, 2) has slope -8/5. Using point-slope form: y - 2 = (-8/5)(x - 0). Simplify to get the equation: y = (-8/5)x + 2.

What is the slope of the line parallel to 8x + 5y = 6?

The slope of the line 8x + 5y = 6 is -8/5. Therefore, any line parallel to it also has a slope of -8/5.

How do I write the equation of a line passing through (0, 2) with slope -8/5?

Use the point-slope form: $y - y_1 = m(x - x_1)$. Plugging in the point (0, 2) and slope -8/5: y - 2 = (-8/5)(x - 0), which simplifies to y = (-8/5)x + 2.

Can you verify the equation of the line passing through (0, 2) and parallel to 8x + 5y = 6?

Yes. The equation is y = (-8/5)x + 2. To verify, check that it passes through (0, 2): when x=0, y=2, which matches the point. Also, the slope is -8/5, same as the original line, confirming parallelism.

What is the general form of the line passing through (0, 2) and parallel to 8x + 5y = 6?

The equation in slope-intercept form is y = (-8/5)x + 2. In standard form, it can be written as 8x + 5y = 10.

How does the point (0, 2) help in determining the line's equation?

Since the line passes through (0, 2), substituting x=0 into the line's equation gives y=2, confirming the intercept and aiding in deriving the specific equation once the slope is known.

If the original line is 8x + 5y = 6, what is the equation of the line passing through (0, 2) and parallel to it in standard form?

The slope is -8/5. Using point (0, 2), the line's y-intercept is 2. Convert to standard form: 8x + 5y = 10.

Why do parallel lines have the same slope, and how does that relate to this problem?

Parallel lines are always equidistant and never intersect, which means they share the same slope. In this problem, the line parallel to 8x + 5y = 6 has the same slope of -8/5.

What is the step-by-step process to find the equation of a parallel line through a specific point?

1. Find the slope of the given line. 2. Use the same slope for the new line. 3. Use the point-slope form with the given point. 4. Simplify to slope-intercept or standard form as needed. For this problem: slope = -8/5, point (0, 2), so y = (-8/5)x + 2.

Additional Resources

Finding the Equation of a Line Parallel to a Given Line and Passing Through a Specific Point

Understanding the process of deriving the equation of a line that is parallel to a given line and passes through a specific point is fundamental in coordinate geometry. This task combines knowledge of line equations, slopes, and point-slope forms to arrive at the precise mathematical representation. In this comprehensive review, we will explore each step involved in solving this problem, focusing on the specific example: finding the equation of a line passing through (0,2) and parallel to the line 8x + 5y = 6.

Understanding the Problem Statement

Before diving into the solution, it is essential to interpret the problem accurately:

- Given Point: (0, 2)
- Given Line: $8x + 5y = 6$
- Objective: Find the equation of the line passing through (0, 2) and parallel to the given line.

Key Concepts to Recall:

- Parallel Lines: Lines that have the same slope.
- Line Equations: Standard form ($Ax + By = C$), slope-intercept form ($y = mx + b$), and point-slope form.
- Finding the Slope: From the given line, by rewriting in slope-intercept form or directly calculating.

Step 1: Convert the Given Line to Slope-Intercept Form

The first step is to understand the slope of the given line, which will be used to determine the slope of the parallel line.

Equation: $8x + 5y = 6$

Rearrangement to slope-intercept form:

- Solve for y:

$5y = -8x + 6$

$y = (-8/5)x + 6/5$

Result:

- Slope (m): -8/5

This slope indicates the steepness and direction of the line. Since parallel lines share the same slope, the line we seek will also have a slope of -8/5.

Step 2: Recognize the Slope of the Parallel Line

- Parallelism: The new line must have the same slope as the original line.

- Therefore:

$m_{new} = -8/5$

This key point simplifies the process, as the only remaining unknown is the line's y-intercept or the specific equation passing through the point (0, 2).

Step 3: Use the Point-Slope Form to Write the Equation

The point-slope form of a line's equation is:

$$ y - y_1 = m(x - x_1) $$

Where:

- (x_1, y_1): a point on the line
- m: slope of the line

Given point: (0, 2)

Using the slope: -8/5

Substituting:

$$ y - 2 = -\frac{8}{5}(x - 0) $$

$$ y - 2 = -\frac{8}{5}x $$

Step 4: Write the Equation in Slope-Intercept Form

Add 2 to both sides to express in $y = mx + b$ form:

$$ y = -\frac{8}{5}x + 2 $$

This is the equation of the line passing through (0, 2) and parallel to $8x + 5y = 6$.

Final Equation:

$$ y = -\frac{8}{5}x + 2 $$

Step 5: Alternative Forms and Verification

While the slope-intercept form is often most straightforward, sometimes the problem requires an answer in standard form or another representation.

Standard form:

- Starting from $y = -8/5 x + 2$

- Multiply both sides by 5 to clear fractions:

$$5y = -8x + 10$$

- Rearrange:

$$8x + 5y = 10$$

This is the standard form of the line's equation.

Verification:

- The line passes through (0, 2):

Substitute $x=0$:

$8(0) + 5(2) = 0 + 10 = 10$, which matches the right side of the standard form equation.

- The line is parallel to the original:

Original line slope: $-8/5$

New line slope: from $y = -8/5 x + 2$, slope is $-8/5$, confirming parallelism.

Deep Dive into the Concepts and Methods

To fully appreciate the process, let's explore some foundational concepts and alternative methods involved:

Understanding the Slope-Intercept and Standard Forms

- Slope-Intercept Form ($y = mx + b$): Useful for quickly identifying the slope (m) and y-intercept (b). It simplifies the process of graphing and understanding the line's behavior.

- Standard Form ($Ax + By = C$): Often preferred for algebraic manipulations and when dealing with multiple lines, especially when checking for perpendicularity or parallelism.

Why is the Slope Critical?

- The core reason for converting to slope-intercept form is to identify the slope easily.

- Since parallel lines have identical slopes, this is the primary property used to find the new line's equation.

Handling Different Forms and Calculations

- When given an equation in standard form, always consider converting it into slope-intercept form to find the slope.

- Conversely, if the problem requires the answer in standard form, convert back after deriving the slope-intercept form.

Working with Points and Slopes

- The point-slope form is particularly useful when you have a point and a slope, as it directly yields the line's equation.

- This method minimizes algebraic errors and is straightforward for many problems.

Practical Applications and Variations of the Problem

Understanding how to find a line parallel to a given one passing through a specific point has broad applications in various fields:

- Engineering and Design: Ensuring components or pathways run parallel for safety or aesthetic reasons.

- Navigation: Plotting routes that maintain a consistent bearing relative to a known path.

- Economics: Analyzing relationships that require parallel constraints.

Variations to consider:

- Finding a perpendicular line: Involves using the negative reciprocal of the original slope.

- Finding a line passing through a point and perpendicular to a given line: Useful for optimization problems.

- Multiple points: When passing through more than one point, the process involves solving systems of equations.

Summary of the Process

To encapsulate, the systematic approach to find the equation of a line passing through a given point and parallel to a given line involves:

1. Identify the slope of the given line: Convert to slope-intercept form.

2. Use the same slope for the new line: Parallel lines share the same slope.

3. Apply the point-slope form: Incorporate the given point into the equation.

4. Simplify to preferred form: Slope-intercept or standard form.

5. Verify: Check whether the point satisfies the equation and whether the slopes match.

Conclusion & Final Remarks

Finding the equation of a line passing through a specific point and parallel to a given line is a fundamental task in algebra and coordinate geometry. It emphasizes the importance of understanding different forms of line equations, the significance of slope, and how geometric properties translate into algebraic expressions.

The example detailed here—finding the line passing through (0, 2) and parallel to 8x + 5y = 6—illustrates the process step-by-step, from extracting the slope to formulating the final equation. Mastery of these skills enables solving more complex problems involving line relationships, intersections, and geometric configurations.

In essence: Recognize the slope, use point-slope form, and convert to the desired form for clarity and application. This approach provides a robust framework for tackling similar problems across mathematics, science, and engineering disciplines.

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