4. The Perimeter Of The Rectangle Is Represented By 8y Metres And The Area Is Represented By(6y + 3)

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Understanding the relationship between the perimeter and area of a rectangle is fundamental in geometry. When these measurements are expressed algebraically, such as "8y metres" for the perimeter and "(6y + 3)" for the area, it offers a powerful way to analyze and solve problems involving rectangles. This article explores how to interpret these expressions, derive the dimensions of the rectangle, and solve related algebraic equations to find specific measurements based on given conditions.

Understanding the Basic Properties of a Rectangle

Before diving into the algebraic expressions, it is essential to revisit some fundamental properties of rectangles:

The Perimeter of a Rectangle

- The perimeter (P) is the total distance around the rectangle.
- It is calculated as: $P = 2(\text{length} + \text{width})$.
- When expressed algebraically, if length = L and width = W, then $P = 2(L + W)$.

The Area of a Rectangle

- The area (A) is the amount of space enclosed within the rectangle.
- It is calculated as: $A = \text{length} \times \text{width}$.
- Using algebraic expressions, $A = L \times W$.

These basic formulas serve as the foundation for solving problems where perimeter and area are given in terms of a variable, such as y.

Interpreting the Given Expressions

Given:
- Perimeter, $P = 8y$ metres
- Area, $A = 6y + 3$

The goal is to determine the dimensions (length and width) of the rectangle in terms of y, understand the relationship between these dimensions, and solve for y under specific conditions.

Expressing the Perimeter

Using the perimeter formula:
$$ P = 2(L + W) $$
Given that:
$$ P = 8y $$
So:
$$ 2(L + W) = 8y $$
Dividing both sides by 2:
$$ L + W = 4y $$
This equation relates the sum of the length and width to y.

Expressing the Area

Using the area formula:
$$ A = L \times W $$
Given that:
$$ A = 6y + 3 $$
Thus:
$$ L \times W = 6y + 3 $$

Setting Up Equations for the Dimensions

From the relations:
- $L + W = 4y$
- $L \times W = 6y + 3$

We need to find expressions for L and W in terms of y. This can be approached by considering these as the sum and product of two numbers, and applying quadratic equations.

Using the Sum and Product of Roots Method

Let:
- L and W be the roots of a quadratic equation.

Then, the quadratic equation can be written as:
$$ t^2 - (L + W)t + (L \times W) = 0 $$
Substituting the known sum and product:
$$ t^2 - (4y)t + (6y + 3) = 0 $$

The roots $t = L$ and $t = W$ are solutions to this quadratic.

Solving for Dimensions in Terms of y

To find the explicit expressions for L and W, solve the quadratic:

$$ t^2 - 4y t + (6y + 3) = 0 $$

Apply the quadratic formula:

$$ t = \frac{4y \pm \sqrt{(4y)^2 - 4 \times 1 \times (6y + 3)}}{2} $$

Simplify under the square root:

$$ t = \frac{4y \pm \sqrt{16 y^2 - 4(6 y + 3)}}{2} $$
$$ t = \frac{4y \pm \sqrt{16 y^2 - 24 y - 12}}{2} $$

Expressed as:

$$ t = \frac{4y \pm \sqrt{16 y^2 - 24 y - 12}}{2} $$

This provides the dimensions of the rectangle in terms of y, assuming the discriminant (the expression under the square root) is non-negative.

Conditions for Real Dimensions

For the dimensions to be real and meaningful, the discriminant must be greater than or equal to zero:

$$ 16 y^2 - 24 y - 12 \geq 0 $$

Dividing through by 4:

$$ 4 y^2 - 6 y - 3 \geq 0 $$

Solve the quadratic inequality:

$$ 4 y^2 - 6 y - 3 = 0 $$

Using quadratic formula:

$$ y = \frac{6 \pm \sqrt{(-6)^2 - 4 \times 4 \times (-3)}}{2 \times 4} $$
$$ y = \frac{6 \pm \sqrt{36 + 48}}{8} $$
$$ y = \frac{6 \pm \sqrt{84}}{8} $$
$$ y = \frac{6 \pm 2 \sqrt{21}}{8} $$

Simplify:

$$ y = \frac{3 \pm \sqrt{21}}{4} $$

Since the quadratic opens upwards (positive coefficient of y^2), the inequality $4 y^2 - 6 y - 3 \geq 0$ holds when:

$$ y \leq \frac{3 - \sqrt{21}}{4} \quad \text{or} \quad y \geq \frac{3 + \sqrt{21}}{4} $$

Given that y represents a variable relating to measurable dimensions, typically positive, the relevant solution for y is:

$$ y \geq \frac{3 + \sqrt{21}}{4} $$

Practical Applications and Examples

Understanding these relationships enables us to solve real-world problems involving rectangles, such as designing a garden, a room, or a picture frame, when the perimeter and area are specified algebraically.

Example 1: Find Dimensions for a Specific y

Suppose y = 2. Find the dimensions of the rectangle.

Calculate the discriminant:

$$ 16 (2)^2 - 24 (2) - 12 = 16 \times 4 - 48 - 12 = 64 - 48 - 12 = 4 $$

Since the discriminant is positive, proceed:

$$ t = \frac{4 \times 2 \pm \sqrt{4}}{2} = \frac{8 \pm 2}{2} $$

Calculate both roots:

- $ t = \frac{8 + 2}{2} = \frac{10}{2} = 5 $
- $ t = \frac{8 - 2}{2} = \frac{6}{2} = 3 $

Thus, the dimensions are 3 metres and 5 metres.

Verify:

- Sum: 3 + 5 = 8 metres, matching the perimeter formula $ 2(L + W) = 8y \Rightarrow 2 \times 8 = 16 $. Since y=2, perimeter should be 82=16, which matches.

- Area: 3 × 5 = 15 square metres, which matches $ 6y + 3 = 6 \times 2 + 3 = 12 + 3 = 15 $.

Example 2: Find y for a Given Set of Dimensions

Suppose the rectangle has length 4 metres and width 6 metres.

Check if these satisfy the given expressions:

- Sum: 4 + 6 = 10
- Perimeter: $ 2 \times 10 = 20 $

Given perimeter:

$$ 8 y = 20 \Rightarrow y = \frac{20}{8} = 2.5 $$

Calculate the area:

$$ 6 y + 3 = 6 \times 2.5 + 3 = 15 + 3 = 18 $$

Calculate actual area:

$$ 4 \times 6 = 24 $$

Since the computed area is 24, but the expression yields 18, the dimensions do not satisfy the algebraic area expression, indicating inconsistency unless y is adjusted accordingly.

Advanced Analysis and Optimization

The algebraic framework allows for further exploration, such as:

- Optimizing dimensions: Determining y that maximizes or minimizes the rectangle's area or perimeter.
- Design constraints: Ensuring the dimensions meet specific criteria, such as minimum or maximum length or width.
- Scaling properties: Understanding how changes in y affect the overall size of the rectangle.

Conclusion

Analyzing a rectangle where the perimeter is expressed as $8 y$ metres and the area as $6 y + 3$ provides a rich example of applying algebraic methods to geometric problems. By translating these expressions into quadratic equations, we can determine the possible dimensions of the rectangle for various values of y. Understanding the conditions under which these measurements are valid is

Frequently Asked Questions

How can we express the dimensions of the rectangle using the given perimeter and area formulas?

Let the length be represented by y and the width by (6y + 3). The perimeter is 8y, which relates to 2(length + width). The area is (6y + 3) multiplied by y.

What is the equation relating y to the perimeter and area of the rectangle?

Since perimeter P = 8y, and perimeter of a rectangle is 2(length + width), then 2(y + 6y + 3) = 8y, leading to the equation 2(7y + 3) = 8y.

How do we find the value of y using the given perimeter and area expressions?

Set up equations based on perimeter and area, then solve simultaneously. For example, from perimeter: $2(7y + 3) = 8y$, solve for y. From area: area = $y(6y + 3)$. Equate or use substitution to find y.

What steps are involved in verifying if the given perimeter and area are consistent for a rectangle?

First, express both perimeter and area formulas in terms of y. Then, solve the equations to find y. Verify that the obtained y value satisfies both formulas, ensuring the dimensions are consistent.

Can you find the actual dimensions of the rectangle given the perimeter and area formulas?

Yes. By solving the equations derived from the perimeter and area, you can find the value of y, then determine the length as y and width as $(6y + 3)$. Substitute y back into these expressions to get the actual dimensions.

Additional Resources

Perimeter and Area of a Rectangle: An In-Depth Analytical Review

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Introduction: Understanding the Fundamentals of Rectangle Geometry

In the realm of geometry, rectangles are among the most fundamental and widely studied shapes, acting as essential building blocks for various mathematical applications, architectural designs, and real-world problem-solving scenarios. When analyzing a rectangle, two critical properties often come into focus: its perimeter and its area. These measurements offer insights into the shape's dimensions and spatial properties, and understanding how they relate through algebraic expressions forms a cornerstone of geometric comprehension.

In this detailed review, we examine a specific problem where the perimeter of a rectangle is expressed as 8y metres and the area is given as $(6y + 3)$. By dissecting these expressions, exploring their implications, and deriving the key dimensions involved, we aim to provide an exhaustive understanding suitable for students, educators, and enthusiasts seeking a deeper grasp of the concepts at play.

Section 1: Breaking Down the Given Data

Perimeter of the Rectangle – 8y Metres

The perimeter (P) of a rectangle is the total distance around its boundary. For a rectangle with length (L) and breadth (B), the perimeter is calculated as:

$$ P = 2(L + B) $$

Given that the perimeter is 8y metres, we have:

$$ 2(L + B) = 8y $$

Dividing both sides by 2:

$$ L + B = 4y $$

This equation reveals that the combined sum of the length and breadth of the rectangle is directly proportional to the variable y, scaled by a factor of 4.

Area of the Rectangle – (6y + 3)

The area (A) of a rectangle is the measure of the surface enclosed within its boundaries, calculated as:

$$ A = L \times B $$

Given that the area is (6y + 3), we have:

$$ L \times B = 6y + 3 $$

This algebraic expression indicates that the product of the length and breadth depends linearly on y, with an additional constant term.

Section 2: Expressing Dimensions in Terms of y

To fully understand the rectangle's dimensions, it’s essential to express L and B explicitly in terms of y. The two key equations are:

1. $L + B = 4y$
2. $L \times B = 6y + 3$

This pair of equations resembles the sum and product of two variables, similar to quadratic roots, enabling us to find explicit expressions for L and B.

Deriving the Dimensions

Suppose L and B are the roots of a quadratic equation:

$$ x^2 - (L + B) x + L B = 0 $$

Replacing with known values:

$$ x^2 - 4y x + (6y + 3) = 0 $$

This quadratic equation can be solved for x (representing either length or breadth):

$$ x = \frac{4y \pm \sqrt{(4y)^2 - 4 \times 1 \times (6y + 3)}}{2} $$

Calculating the discriminant:

$$ \Delta = (4y)^2 - 4 \times 1 \times (6y + 3) = 16 y^2 - 4(6 y + 3) $$

$$ \Delta = 16 y^2 - 24 y - 12 $$

For the dimensions to be real and meaningful, the discriminant must be non-negative:

$$ 16 y^2 - 24 y - 12 \geq 0 $$

Section 3: Conditions for Real Dimensions

To analyze the feasible values of y, we solve the quadratic inequality:

$$ 16 y^2 - 24 y - 12 \geq 0 $$

Dividing through by 4 for simplicity:

$$ 4 y^2 - 6 y - 3 \geq 0 $$

Now, solve the quadratic equation:

$$ 4 y^2 - 6 y - 3 = 0 $$

Using the quadratic formula:

$$ y = \frac{6 \pm \sqrt{(-6)^2 - 4 \times 4 \times (-3)}}{2 \times 4} $$

$$ y = \frac{6 \pm \sqrt{36 + 48}}{8} $$

$$ y = \frac{6 \pm \sqrt{84}}{8} $$

$$ y = \frac{6 \pm 2 \sqrt{21}}{8} $$

Simplify:

$$ y = \frac{3 \pm \sqrt{21}}{4} $$

Since we're dealing with length dimensions, y should be non-negative, and the inequality:

$$ 4 y^2 - 6 y - 3 \geq 0 $$

holds for:

$$ y \leq \frac{3 - \sqrt{21}}{4} \quad \text{or} \quad y \geq \frac{3 + \sqrt{21}}{4} $$

Given that the negative root yields a negative y (which isn't meaningful in length measurement), the practical solution is:

$$ y \geq \frac{3 + \sqrt{21}}{4} $$

Calculating approximately:

$$ \sqrt{21} \approx 4.583 $$

$$ y \geq \frac{3 + 4.583}{4} = \frac{7.583}{4} \approx 1.896 $$

Thus, for the rectangle to have real, positive dimensions, y must be at least approximately 1.896.

Section 4: Explicit Expressions for Length and Breadth

Now, returning to the quadratic roots:

$$ x = \frac{4 y \pm \sqrt{16 y^2 - 24 y - 12}}{2} $$

we see that the dimensions L and B are:

$$ L, B = \frac{4 y \pm \sqrt{16 y^2 - 24 y - 12}}{2} $$

or simplified:

$$ L, B = 2 y \pm \frac{1}{2} \sqrt{16 y^2 - 24 y - 12} $$

Given the earlier analysis, these are valid only when the discriminant is non-negative, i.e., when y satisfies the above inequality.

Note: The $\pm$ indicates that one of the dimensions is larger, the other smaller, depending on the sign chosen.

Section 5: Practical Examples and Visualizations

To better understand the implications, consider specific values of y within the permissible range.

Example 1: y = 2

Calculate the discriminant:

$$ \Delta = 16(2)^2 - 24(2) - 12 = 16 \times 4 - 48 - 12 = 64 - 48 - 12 = 4 $$

Square root:

$$ \sqrt{4} = 2 $$

Calculate dimensions:

$$ L, B = 2 \times 2 \pm \frac{1}{2} \times 2 = 4 \pm 1 $$

Thus:

- Length = 4 + 1 = 5 metres
- Breadth = 4 - 1 = 3 metres

Verify the perimeter:

$$ 2(5 + 3) = 2 \times 8 = 16 \text{ metres} $$

which matches the earlier perimeter expression:

$$ 8 y = 8 \times 2 = 16 \text{ metres} $$

Verify the area:

$$ 5 \times 3 = 15 $$

and the area expression:

$$ 6 y + 3 = 6 \times 2 + 3 = 12 + 3 = 15 $$

which confirms our calculations.

Example 2: y = 3

Discriminant:

$$ 16 \times 9 - 24 \times 3 - 12 = 144 - 72 - 12 = 60 $$

Square root:

$$ \sqrt{60} \approx 7.746 $$

Dimensions:

$$ L, B = 2 \times 3 \pm \frac{1}{2} \times 7.746 \approx 6 \pm 3.873 $$

Thus:

- Larger dimension ≈ 6 + 3.873 ≈ 9.873 metres
- Smaller dimension ≈ 6 - 3.873 ≈ 2.127 metres

Verify perimeter:

$$ 2(9.873 + 2.127) = 2 \times 12 = 24 \text{ metres} $$

Check with the perimeter expression:

$$ 8 y = 8 \times 3 = 24 \text{ metres} $$

Calculate area:

$$ 9.873 \times 2.127 \approx 21 $$

Compare with area expression:

$$ 6 \times 3 + 3 = 18 + 3 = 21 $$

Again, the calculations align well, confirming the validity of the approach.

Section 6: Summary of Key Findings and Insights

- The perimeter of the rectangle is directly

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