

4. Write The Equation Of The Line That Passes Through The Points (3, 7) And (-1, 2) in: a. Point-slope

4. Write The Equation Of The Line That Passes Through The Points (3, 7) And (-1, 2) in: a. Point-slope

When working with linear equations, one of the fundamental skills is to find the equation of a line that passes through two specific points. In this guide, we will focus on deriving the equation of the line passing through the points (3, 7) and (-1, 2) using the point-slope form. This method is especially useful because it allows you to directly substitute the slope and a point on the line into the equation, resulting in a straightforward way to express the line mathematically.

Understanding how to determine this equation is essential for students and professionals dealing with algebra, geometry, and related fields. Whether you're solving academic problems, working on engineering tasks, or just strengthening your mathematical foundation, mastering the point-slope form is a valuable skill.

Understanding the Basics of Line Equations

Before diving into the steps, it's important to have a clear understanding of the different forms of linear equations and their purposes.

Common Forms of a Line Equation

- **Slope-Intercept Form:** $y = mx + b$
- **Point-Slope Form:** $y - y_1 = m(x - x_1)$
- **Standard Form:** $Ax + By = C$

Each form has its advantages. The slope-intercept form is useful for graphing quickly and understanding the y-intercept, whereas the point-slope form is most convenient when you know a point on the line and its slope. Standard form is often used for algebraic manipulations and solving systems.

For our purpose—finding the equation passing through two points—using the point-slope form offers a direct path once you determine the slope.

Step 1: Find the Slope of the Line

The first critical step is to calculate the slope (m) of the line passing through the two given points.

Understanding the Slope

The slope measures how steep the line is and is calculated as the change in y divided by the change in x between two points:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Where:

- (x_1, y_1) and (x_2, y_2) are the two points.

Calculating the Slope for (3, 7) and (-1, 2)

Given:

- Point 1: (3, 7)
- Point 2: (-1, 2)

Calculating the slope:

$$m = \frac{2 - 7}{-1 - 3} = \frac{-5}{-4} = \frac{5}{4}$$

Therefore, the slope of the line is:

$$m = \frac{5}{4}$$

This positive slope indicates the line rises as x increases.

Step 2: Choose a Point for the Equation

In the point-slope form, you need to pick one point that lies on the line. Either point will work, but it's often simplest to choose the point with smaller numbers for convenience.

Let's select the point (-1, 2) for this purpose. However, using (3, 7) is equally valid.

Step 3: Write the Point-Slope Equation

The point-slope form of a line's equation is:

$$y - y_1 = m(x - x_1)$$

Substituting the chosen point and the slope:

$$y - 2 = \frac{5}{4}(x - (-1))$$

Simplify the expression:

$$y - 2 = \frac{5}{4}(x + 1)$$

This is the point-slope form of the equation passing through the points (3, 7) and (-1, 2).

Step 4: Simplify the Equation (Optional)

While the point-slope form is perfectly valid, you might want to express the equation in slope-intercept form or standard form for easier interpretation or graphing.

Converting to Slope-Intercept Form

Starting from:

$$y - 2 = \frac{5}{4}(x + 1)$$

Distribute the slope:

$$y - 2 = \frac{5}{4}x + \frac{5}{4}$$

Add 2 to both sides:

$$\begin{aligned} y - 2 + 2 &= \frac{5}{4}x + \frac{5}{4} + 2 \\ y &= \frac{5}{4}x + \frac{13}{4} \end{aligned}$$

$$y = \frac{5}{4}x + \frac{5}{4} + 2$$

Express 2 as a fraction with denominator 4:

$$2 = \frac{8}{4}$$

Therefore:

$$y = \frac{5}{4}x + \frac{5}{4} + \frac{8}{4} = \frac{5}{4}x + \frac{13}{4}$$

This gives the slope-intercept form:

$$y = \frac{5}{4}x + \frac{13}{4}$$

Converting to Standard Form

Starting from the slope-intercept form:

$$y = \frac{5}{4}x + \frac{13}{4}$$

Multiply both sides by 4 to clear denominators:

$$4y = 5x + 13$$

Rearranged into standard form:

$$5x - 4y = -13$$

This is the standard form of the line passing through (3, 7) and (-1, 2).

Summary of the Process

To summarize, here are the key steps to find the equation of a line passing through two points using the point-slope form:

1. **Calculate the slope (m):** Use the formula $m = (y_2 - y_1) / (x_2 - x_1)$.
2. **Select a point:** Pick either of the two points to plug into the point-slope form.
3. **Write the point-slope equation:** Substitute the slope and point into $y - y_1 = m(x - x_1)$.
4. **(Optional) Simplify:** Convert the equation into slope-intercept or standard form as needed.

Practical Applications of the Point-Slope Form

Understanding how to write the equation of a line using the point-slope form has many practical applications:

- **Graphing lines accurately:** Once the equation is known, graphing becomes straightforward.
- **Finding equations from real-world data:** For example, determining the relationship between variables like time and distance.
- **Solving systems of equations:** When multiple lines are involved, understanding their equations helps find intersection points.
- **Engineering and physics:** Calculating trajectories, slopes, and other line-related metrics.

Tips and Common Mistakes to Avoid

1. Always double-check the points and calculations: Small errors in subtraction or sign can lead to incorrect equations.
2. Be mindful of fractions: When working with slopes like $5/4$, ensure proper distribution and simplification.
3. Choose a convenient point: When multiple options are available, select the point that simplifies calculations.

4. Verify the equation: Substitute the other point into your derived equation to confirm its correctness.

Conclusion

Finding the equation of a line passing through two points using the point-slope form is an essential skill in algebra and geometry. By calculating the slope, selecting a point, and substituting into the point-slope formula, you can quickly derive the line's equation. Further simplification into slope-intercept or standard form can enhance understanding and usability. Mastery of this method opens the door to solving a wide array of mathematical problems, from graphing lines to analyzing real-world data.

Whether you're a student, educator, or professional, practicing these steps will enhance your analytical skills and deepen your understanding of linear relationships in mathematics.

Frequently Asked Questions

What is the first step in writing the point-slope form of a line passing through (3, 7) and (-1, 2)?

Calculate the slope (m) using the formula $m = (y_2 - y_1) / (x_2 - x_1)$.

How do you find the slope of the line passing through the points (3, 7) and (-1, 2)?

Slope $m = (2 - 7) / (-1 - 3) = (-5) / (-4) = 5/4$.

What is the point-slope form of the line passing through (3, 7) with slope 5/4?

The point-slope form is $y - y_1 = m(x - x_1)$, so $y - 7 = (5/4)(x - 3)$.

How can you write the equation of the line in point-slope form for these points?

Substitute the point (3,7) and the slope 5/4 into the formula: $y - 7 = (5/4)(x - 3)$.

What is the next step to convert the point-slope form to slope-intercept form?

Distribute the slope on the right side: $y - 7 = (5/4)x - (15/4)$, then add 7 to both sides.

What is the slope-intercept form of the line passing through (3, 7) and (-1, 2)?

$y = (5/4)x - (15/4) + 7$, which simplifies to $y = (5/4)x + (13/4)$.

Why is the point-slope form useful for writing the equation of a line?

Because it allows you to easily write the equation when you know a point on the line and its slope, without needing to find the y-intercept first.

Additional Resources

4. Write The Equation Of The Line That Passes Through The Points (3, 7) And (-1, 2) in: a. Point-slope

Understanding how to derive the equation of a line that passes through two specific points is a fundamental skill in algebra and analytical geometry. Among the various methods available, the point-slope form offers a straightforward and intuitive approach, especially when you already know a point on the line and its slope. In this article, we'll explore how to determine the equation of the line passing through the points (3, 7) and (-1, 2) using the point-slope form, providing a clear, detailed, and reader-friendly guide suitable for students, educators, and anyone interested in mastering this essential concept.

Understanding the Foundations: Coordinates, Slope, and Line Equations

Before diving into the process, let's review some foundational concepts that underpin the point-slope form.

Coordinate Points

A coordinate point in the Cartesian plane is represented as (x, y), where:

- x is the horizontal position (abscissa)
- y is the vertical position (ordinate)

In our case:

- First point: (3, 7)
- Second point: (-1, 2)

Slope of a Line

The slope (m) measures the steepness and direction of a line. It is calculated as the ratio of the change in y to the change in x between two points:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Calculating the slope is a critical step because it informs the orientation of the line in the plane.

Line Equations: Slope-Intercept and Point-Slope Forms

- Slope-Intercept Form: $y = mx + b$, where b is the y-intercept.
- Point-Slope Form: $y - y_1 = m(x - x_1)$, which uses a known point (x_1, y_1) on the line and the slope m .

The point-slope form is particularly useful when you have the slope and a specific point, as in our case.

Step-by-Step: Deriving the Equation Using Point-Slope Form

Let's walk through the process to find the desired line equation step by step.

Step 1: Calculate the Slope (m)

Given the points:

- $(x_1, y_1) = (3, 7)$
- $(x_2, y_2) = (-1, 2)$

Apply the slope formula:

$$m = \frac{2 - 7}{-1 - 3}$$

Calculate numerator and denominator:

$$m = \frac{-5}{-4}$$

Simplify:

$$m = \frac{-5}{-4} = \frac{5}{4}$$

So, the slope of the line is $\frac{5}{4}$.

Step 2: Choose a Point to Use in the Point-Slope Equation

You can use either of the two points; however, for consistency, we'll use the point $(3, 7)$.

Step 3: Write the Point-Slope Equation

Recall the point-slope form:

$$y - y_1 = m(x - x_1)$$

Plugging in the known values:

$$y - 7 = \frac{5}{4}(x - 3)$$

This is the point-slope form of the line passing through (3, 7) with slope $\frac{5}{4}$.

Step 4: Simplify to Slope-Intercept Form (Optional)

While the question emphasizes the point-slope form, converting to slope-intercept form can be helpful for graphing or interpretation.

Distribute the slope:

$$y - 7 = \frac{5}{4}x - \frac{5}{4} \times 3$$

Calculate:

$$y - 7 = \frac{5}{4}x - \frac{15}{4}$$

Add 7 to both sides:

$$y = \frac{5}{4}x - \frac{15}{4} + 7$$

Expressing 7 as a fraction with denominator 4:

$$y = \frac{5}{4}x - \frac{15}{4} + \frac{28}{4}$$

Combine like terms:

$$y = \frac{5}{4}x + \frac{13}{4}$$

While not part of the original request, this form may be useful for context.

Key Takeaways and Practical Tips

- Choose the right point: Using a point that simplifies calculations or is more convenient can streamline the process.
- Calculate the slope carefully: Double-check numerator and denominator for sign errors.
- Always verify: Plug the points into your final equation to confirm correctness.
- Understand the forms: Point-slope form is flexible; converting to slope-intercept or standard form can aid different applications.

Common Pitfalls to Avoid

- Ignoring signs: Remember that subtracting negatives can reverse the slope's sign.
- Miscalculating the slope: Be meticulous in numerator and denominator calculations.
- Using inconsistent points: Always double-check which point you're substituting into the formula.

Conclusion: Mastering the Point-Slope Form for Line Equations

Deriving the equation of a line that passes through two points is a vital skill in algebra, laying the groundwork for understanding more complex concepts in geometry and calculus. The point-slope form provides a direct, elegant approach, especially when you know one point and the slope—a common scenario in problem-solving.

By calculating the slope from the given points and then applying the point-slope formula with a known point, you can quickly arrive at the line's equation. In our specific example, the line passing through (3, 7) and (-1, 2) has a slope of $\frac{5}{4}$, and the equation in point-slope form is:

$$y - 7 = \frac{5}{4}(x - 3)$$

From here, you can convert to other forms as needed, or use this as a basis for graphing and further analysis. Mastering this process enhances your algebraic toolkit, empowering you to approach a wide range of problems with confidence and clarity.

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