5. Determine The Area Of The Region That Is Inside Both Of The Curves R = 3 - 2 Sin 0 And R=-3+2 Sin

5. Determine The Area Of The Region That Is Inside Both Of The Curves R = 3 - 2 Sin θ And R = -3 + 2 Sin θ

Understanding the area enclosed by two polar curves is a fundamental problem in calculus, particularly in the study of polar coordinate systems. The curves given—R = 3 - 2 Sin θ and R = -3 + 2 Sin θ—are both sinusoidal in nature and exhibit symmetry about certain axes. In this article, we'll explore a systematic approach to find the area of the region that lies inside both of these curves. This involves analyzing the equations, determining their points of intersection, establishing the appropriate limits of integration, and calculating the area using integral calculus in polar coordinates.

Understanding the Curves and Their Properties

1. The Polar Equations: R = 3 - 2 Sin θ and R = -3 + 2 Sin θ

The given equations are in the form of sinusoidal polar curves. To analyze them effectively, let's examine each:

- Curve 1: R = 3 - 2 Sin θ
- Curve 2: R = -3 + 2 Sin θ

Both involve sinusoidal functions, which suggests symmetry about certain axes and specific behaviors at various angles.

Key observations:

- The first curve, R = 3 - 2 Sin θ, is a type of limacon with an inner loop, given that the coefficient of Sin θ is less than the constant term.
- The second curve, R = -3 + 2 Sin θ, can be rewritten to better understand its shape.

2. Symmetry and Basic Characteristics

Understanding symmetry helps in graphing and analyzing these curves:

- Curve 1 (R = 3 - 2 Sin θ):
- Has a maximum radius when Sin θ = -1, i.e., at θ = 3π/2, R = 3 - 2(-1) = 3 + 2 = 5.
- Has a minimum radius when Sin θ = 1, i.e., at θ = π/2, R = 3 - 2(1) = 3 - 2 = 1.

- Curve 2 (R = -3 + 2 Sin θ):

- Can be rewritten as $R = 2 \sin\theta - 3$.
- Max radius when $\sin\theta = 1$, $R = 2(1) - 3 = -1$.
- Min radius when $\sin\theta = -1$, $R = 2(-1) - 3 = -2 - 3 = -5$.

Because R is negative in some regions, the curves may be reflected about the origin, which affects their graphing and the area calculation.

Finding the Points of Intersection

The key step in determining the common area is to identify where the two curves intersect. To do this, set the two equations equal:

$R = 3 - 2 \sin\theta$
$R = -3 + 2 \sin\theta$

Set equal:

$3 - 2 \sin\theta = -3 + 2 \sin\theta$

Simplify:

$3 + 3 = 2 \sin\theta + 2 \sin\theta$

$6 = 4 \sin\theta$

$\sin\theta = 6/4 = 3/2$

This yields an impossible value since $\sin\theta$ cannot exceed 1 in magnitude.

Implication: The curves do not intersect in the traditional sense within the real values of θ. However, due to the negative R values, the curves may represent the same set of points reflected across the origin, or they may overlap partially in certain regions.

Alternative approach: Instead of solving directly, analyze the curves graphically or analyze their "limacon" nature to understand where they enclose a common region.

Reinterpreting the Curves for Intersection Analysis

Notice that $R = -3 + 2 \sin\theta$ can be rewritten as:

$R = 2 \sin\theta - 3$

Similarly, $R = 3 - 2 \sin\theta$ remains as is.

Because R can be negative, the points corresponding to negative R are plotted in the direction $\theta + \pi$ with positive R. This reflection impacts the region of intersection.

Graphical Analysis and Symmetry Considerations

To proceed, it's helpful to examine the graphs of both curves:

- The first curve ($R = 3 - 2 \sin \theta$) is a limacon with an inner loop, symmetric about the vertical axis.
- The second curve ($R = -3 + 2 \sin \theta$), due to the negative constant term, is a reflection of a similar limacon about the origin.

Plotting these or examining key points:

θ	$R1 = 3 - 2 \sin \theta$	$R2 = -3 + 2 \sin \theta$
0	$3 - 0 = 3$	$-3 + 0 = -3$
$\pi/2$	$3 - 2(1) = 1$	$-3 + 2(1) = -1$
π	$3 - 0 = 3$	$-3 + 0 = -3$
$3\pi/2$	$3 - 2(-1) = 3 + 2 = 5$	$-3 + 2(-1) = -3 - 2 = -5$

Given these values, the two curves exhibit symmetry and are positioned such that their enclosed regions might overlap in certain angular ranges.

Determining the Enclosed Region

Since the curves are symmetric and involve negative R values, their intersection points, and the common region, are best determined by analyzing their graphs or parametric representations.

Key steps:

1. Find the angles θ where $R1 = R2$ or where their graphs overlap.
2. Use these angles as limits of integration.
3. Calculate the area enclosed by each curve over these limits.
4. Find the intersection of these areas, i.e., the common region.

1. Analyzing R1 and R2 for Overlap

The curves are symmetric with respect to the origin, given the negative R values. The regions of interest are where the magnitude of R1 and R2 are both positive or both negative, considering the direction of plotting.

Since $R1 = 3 - 2 \sin \theta$, R1 is positive when:

$3 - 2 \sin \theta \geq 0 \rightarrow \sin \theta \leq 3/2$

which is always true since $\sin \theta \leq 1$.

Similarly, $R2 = -3 + 2 \sin \theta$ is negative or positive depending on $\sin \theta$:

- For R2 ≥ 0:

$-3 + 2 \sin \theta \geq 0 \rightarrow 2 \sin \theta \geq 3 \rightarrow \sin \theta \geq 3/2$, impossible.

- For R2 ≤ 0:

$2 \sin \theta \leq 3 \rightarrow$ always true; since $\sin \theta \leq 1$, R2 ≤ 0 in the relevant domain.

Therefore, the second curve consistently has negative R in the principal domain, implying that the points are reflected across the origin.

Conclusion: The overlapping region must be analyzed considering the reflection of the second curve.

Calculating the Area of the Overlapping Region

The area enclosed by a polar curve R(θ) between θ = a and θ = b is given by:

$$
A = \frac{1}{2} \int_{a}^{b} R^2 d\theta
$$

Since the curves are reflections, the total overlapping area can be constructed by integrating over the appropriate angular intervals where the curves' graphs intersect or overlap.

Approach:

- Identify the angular intervals where the two curves are inside each other.
- Use the symmetry to simplify calculations.
- Compute the area of the intersection by integrating the minimum of the two functions over these intervals.

1. Establishing the Limits of Integration

Given the nature of the functions, the points of intersection are best found by considering the curves in terms of their reflections.

- For $R = 3 - 2 \sin \theta$, the maximum R occurs at $\theta = 3\pi/2$.
- For $R = -3 + 2 \sin \theta$, the maximum R (less negative) occurs at θ where $\sin \theta = 1$, i.e., at $\theta = \pi/2$.

In the interval [0, 2π], the curves intersect or overlap in regions where the magnitude of R is positive for R1 and negative for R2, but considering the reflection, the actual points of intersection occur where:

$$
3 - 2 \sin \theta = -3 + 2 \sin \theta
$$

which, as shown earlier, yields no solution for real θ. However, the curves might cross when considering the points where their graphs reflect across the origin.

Since the curves are symmetric, the intersection points are located at angles where R1 and R2 represent the same point in the plane, considering the negative R values corresponding to $\theta + \pi$.

Therefore, the relevant angles can be found by solving:

$$
R_1(\theta) = R_2(\theta + \pi)
$$

which implies:

$$
3 - 2 \sin \theta = -3 + 2 \sin (\theta + \pi)
$$

Using the identity:

$$
\sin (\theta + \pi) = - \sin \theta
$$

we get:

$$
3 - 2 \sin \theta = -
$$

Frequently Asked Questions

What is the main goal when determining the area of the region inside both curves R = 3 - 2 Sin θ and R = -3 + 2 Sin θ?

The main goal is to find the intersection points of the two polar curves and then compute the total area enclosed within both curves by integrating over the appropriate angles.

How do you find the points where the two polar curves intersect?

Set the two equations equal to each other: $3 - 2 \sin \theta = -3 + 2 \sin \theta$, and solve for θ to find the intersection points.

What is the significance of the intersection points in calculating the area?

The intersection points determine the limits of integration for calculating the area of the overlapping region between the two curves.

How do you set up the integral to find the area of the region inside both curves?

Identify the relevant θ bounds from the intersection points and set up the integral of (1/2) $[R_1(\theta)^2 - R_2(\theta)^2]\, d\theta$ over those bounds, considering the region of overlap.

Are the curves R = 3 - 2 Sin θ and R = -3 + 2 Sin θ symmetric?

Yes, these curves are symmetric with respect to the origin and exhibit certain symmetries due to the sinusoidal component, which can simplify the area calculation.

What challenges might arise when calculating the area of the overlapping region for these polar curves?

Challenges include accurately finding the intersection points, determining the correct limits of integration, and ensuring the integral accounts for the overlapping region without double counting.

Can these curves be represented as familiar polar shapes like cardioids or limacons?

The given curves resemble limacon-type shapes due to their sinusoidal components, but their specific parameters may create inner loops or dimpled shapes requiring careful analysis.

How does the sign of the R value affect the interpretation of the polar curves?

Negative R values indicate points plotted in the direction opposite to the angle θ, which affects the shape and position of the curves in the polar plane.

What steps are involved after finding the intersection points to compute the area?

After finding the intersection points, divide the region into sections if necessary, set up the integral for each section, and then compute the sum of these integrals to find the total overlapping area.

Why is it important to verify the intersection points before computing the area?

Verifying intersection points ensures the limits of integration are accurate, preventing errors in the calculation and ensuring the area corresponds to the actual overlapping region.

Additional Resources

In-Depth Analysis of the Area of Intersection Between the Curves $R = 3 - 2 \sin \theta$ and $R = -3 + 2 \sin \theta$

Introduction

Understanding the area enclosed by polar curves is a fundamental aspect of calculus, bridging geometric intuition with analytical rigor. When two curves intersect, determining the shared region's area involves identifying the intersection points, establishing the correct bounds, and performing the appropriate integrations. Here, we focus on the curves:

- $R = 3 - 2 \sin \theta$
- $R = -3 + 2 \sin \theta$

The goal is to find the area of the region that lies inside both of these curves, which requires a detailed examination of their shapes, intersection points, and the integration bounds.

Conceptual Foundations

1. Understanding Polar Curves

Polar curves are defined by a radius R as a function of the angle θ. The shape and position of these curves depend on the functional form of $R(\theta)$:

- When $R(\theta)$ is positive, the point lies at a distance R from the origin at angle θ.
- When $R(\theta)$ is negative, the point is plotted in the direction opposite to θ, effectively reflecting the point across the origin.

2. Significance of the Curves

- $R = 3 - 2 \sin \theta$ describes a limacon with an inner loop, depending on the parameters.
- $R = -3 + 2 \sin \theta$ can be interpreted as a transformation of the first curve, with the negative sign indicating a reflection across the origin.

Understanding these behaviors helps in predicting intersections and the nature of the enclosed regions.

Analyzing the Curves Individually

1. The Curve $R = 3 - 2 \sin \theta$

- Shape: This is a limacon, a well-known family of polar curves characterized by the parameters a and b in expressions like $R = a + b \sin \theta$ or $R = a + b \cos \theta$.

- Key features:
- When $a > b$, the limacon is dimpled and does not have an inner loop.
- For $a = 3$ and $b = 2$, since $a > b$, the curve is a dimpled limacon.

- Maximum and minimum radii:
- Maximum at $\theta = \frac{\pi}{2}$:

$$
R_{\max} = 3 - 2 \sin \left(\frac{\pi}{2} \right) = 3 - 2(1) = 1
$$

- Minimum at $\theta = \frac{3\pi}{2}$:

$$
R_{\min} = 3 - 2(-1) = 3 + 2 = 5
$$

Note: The calculations reveal that the maximum and minimum are influenced by the sine term.

2. The Curve $R = -3 + 2 \sin \theta$

- Shape: Similar to the previous, but with a negative constant term, which indicates reflection and translation.

- Key features:
- Because of the negative sign, the curve is reflected across the origin.
- The behavior of the radius:
- At $\theta = \frac{\pi}{2}$:

$$
R = -3 + 2(1) = -3 + 2 = -1
$$

- At $\theta = \frac{3\pi}{2}$:

$$
R = -3 + 2(-1) = -3 - 2 = -5
$$

- Implication:
- Negative radii imply that the points are plotted in the direction opposite to θ, resulting in a curve symmetric with respect to the origin, but with different regions.

Determining the Intersection Points

To find the shared region, the first step is to identify where the two curves intersect. This occurs when:

$$
3 - 2 \sin \theta = -3 + 2 \sin \theta
$$

Solve for θ:

$$

$$3 - 2 \sin \theta = -3 + 2 \sin \theta$$

Bring all terms to one side:

$$3 + 3 = 2 \sin \theta + 2 \sin \theta$$

$$6 = 4 \sin \theta$$

$$\sin \theta = \frac{6}{4} = \frac{3}{2}$$

Since $\sin \theta$ cannot be greater than 1, this indicates no real solutions for θ.

Implication: The curves do not intersect in the conventional sense where their radii are equal at the same θ.

Alternative Approach: Considering the Curves’ Reflection and Symmetry

Given that the direct equality yields no solutions, analyze the curves' behavior to determine the overlapping region:

- Observation: The second curve $R = -3 + 2 \sin \theta$ can be rewritten as:

$$R = -(3 - 2 \sin \theta)$$

which suggests that it is the reflection of $R = 3 - 2 \sin \theta$ across the origin.

Therefore:

- The curves are symmetric with respect to the origin.
- The region inside both curves is the common area between these two limacons, possibly overlapping in a more subtle way.

Visualizing the Curves

Plotting or sketching the curves aids in understanding:

- The first curve, $R = 3 - 2 \sin \theta$, with its maximum and minimum radii, produces a limacon with a dimple.
- The second curve, $R = -3 + 2 \sin \theta$, is a reflected version, producing a similar shape but oriented oppositely.

Key insight: The curves are symmetric in a certain sense, and their intersection points occur where the regions "overlap" in the plane, even if their radii at the same θ do not match exactly.

Establishing the Correct Bounds for Area Calculation

Since the curves do not intersect at the same θ directly, the approach involves:

- Finding the angles θ where the curves bound the shared region.
- Analyzing the curves’ behavior over specific intervals.
- Recognizing that the curves are symmetric, and their intersection regions are best determined by considering the union and intersection in polar coordinates.

Method:

- Use the properties of the curves to find the regions where the inside of both curves overlap.
- For each θ, compare the radii to determine which curve encloses the point.

Calculating the Area of the Overlap

1. General Area Formula in Polar Coordinates

The area enclosed by a curve $R(\theta)$ between $\theta = \alpha$ and $\theta = \beta$ is:

$$
A = \frac{1}{2} \int_{\alpha}^{\beta} R^2(\theta) \, d\theta
$$

For two curves, the overlapping area is obtained by integrating over the region where one curve is "inside" the other.

2. Strategy

- Identify the θ intervals where the curves enclose each other.
- Determine the lower and upper bounds for the integrals.
- Use the difference of the integrals (if applicable) to compute the shared area.

Step-by-Step Calculation

Step 1: Find the intersection points in terms of angles

Since the direct equality yields no solution, analyze the radii:

- For $R = 3 - 2 \sin \theta$, the radius varies between:

\[

\text{Maximum } R = 3 - 2(1) = 1
\]
\[
\text{Minimum } R = 3 - 2(-1) = 3 + 2 = 5
\]

- For \(R = -3 + 2 \sin \theta \), the radius varies between:

\[
\text{Maximum } R = -3 + 2(1) = -3 + 2 = -1
\]
\[
\text{Minimum } R = -3 + 2(-1) = -3 - 2 = -5
\]

Note that:

- When the radius is negative, the point is plotted in the opposite direction.
- The actual positions can be better understood by considering the positive and negative parts.

Step 2: Determine the regions where both curves are "inside" each other

Because the curves are reflections, the overlapping area is primarily where the magnitudes of their radii are comparable, and the points in the plane are shared.

Geometric Approach for the Overlap

Given the symmetry and the difficulty in solving directly, a more effective approach involves:

- Converting the polar equations into Cartesian coordinates to analyze the intersection points.
- Alternatively, considering the parametric forms and plotting the curves.

Conversion to Cartesian Coordinates

For \(R

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