(5 Points) Use Normal Approximation To Estimate The Probability Of Getting More Than 46 Girls In 100

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When dealing with probability questions involving binomial distributions, especially with large sample sizes, the normal approximation becomes an invaluable tool. In this article, we explore how to use the normal approximation to estimate the probability of getting more than 46 girls in a sample of 100 children. This approach simplifies complex calculations and provides reasonably accurate results, especially when certain conditions are met.

Understanding the Problem

Before delving into the solution, it's essential to clearly understand the problem:

- Sample size (n): 100 children
- Number of girls (X): Random variable representing the count of girls
- Event of interest: X > 46 (more than 46 girls)
- Assumption: Each child has an equal probability of being a girl or a boy, typically 0.5, assuming a fair gender ratio

The goal is to estimate the probability that the number of girls exceeds 46 in this sample.

Why Use Normal Approximation?

The binomial distribution, which models the number of successes (here, girls) in a fixed number of independent Bernoulli trials, can be cumbersome to compute exactly for large n. The binomial probability mass function (PMF) involves factorial calculations that become impractical as n grows.

The normal approximation offers a solution due to the Central Limit Theorem, which states that for sufficiently large n, the sum of independent Bernoulli trials tends to follow a normal distribution. This approximation simplifies calculations and is accurate when:

- n is large (commonly n ≥ 30)
- The probability p is not too close to 0 or 1 (p ≈ 0.5 is ideal)

Step-by-Step Guide to Using Normal Approximation

Let's walk through how to approximate P(X > 46) using the normal distribution.

1. Define the Binomial Distribution Parameters

- Number of trials (n): 100
- Probability of success (p): 0.5 (assuming equal likelihood of girl or boy)

- Expected value (mean):

\[
\mu = n \times p = 100 \times 0.5 = 50
\]

- Standard deviation:

\[
\sigma = \sqrt{n \times p \times (1 - p)} = \sqrt{100 \times 0.5 \times 0.5} = \sqrt{25} = 5
\]

2. Apply Continuity Correction

Since the binomial distribution is discrete and the normal distribution is continuous, we apply a continuity correction to improve accuracy. To find P(X > 46), we consider:

\[
P(X > 46) \approx P\left(X \geq 47\right) \implies P\left(X > 46.5\right)
\]

This adjustment accounts for the fact that the normal curve is continuous, while the binomial is discrete.

3. Convert to the Standard Normal Distribution (Z-Score)

Calculate the z-score corresponding to X = 46.5:

\[
z = \frac{X - \mu}{\sigma} = \frac{46.5 - 50}{5} = \frac{-3.5}{5} = -0.7
\]

Note: Since we're interested in P(X > 46), which after continuity correction is P(X > 46.5), the z-score is -0.7.

4. Find the Corresponding Probability

Using standard normal distribution tables or a calculator, find:

\[

$$
P(Z > -0.7)
$$

From the standard normal table:

$$
P(Z > -0.7) = 1 - P(Z \leq -0.7)
$$

But because the standard normal distribution is symmetric:

$$
P(Z > -0.7) = P(Z < 0.7)
$$

From the table:

$$
P(Z < 0.7) \approx 0.7580
$$

Thus,

$$
P(X > 46) \approx 0.7580
$$

Therefore, the estimated probability of having more than 46 girls in 100 children is approximately 75.8%.

Additional Considerations and Tips

- Check Conditions: The normal approximation is most accurate when n is large and p is not close to 0 or 1. For p = 0.5 and n = 100, these conditions are satisfied.
- Use of Continuity Correction: Always include the continuity correction when approximating discrete distributions with continuous ones to improve accuracy.
- Calculating Exact Probabilities: For small sample sizes or when high precision is needed, consider using the binomial probability formula directly or computational tools.
- Software Tools: Utilize statistical software like R, Python, or calculator functions for quick and accurate probability calculations.

Real-Life Applications

Using the normal approximation for probability estimation has numerous practical applications:

- Quality Control: Estimating defect rates in manufacturing processes

- Epidemiology: Predicting disease prevalence based on sample data
- Polling and Surveys: Estimating proportions based on survey responses
- Education: Calculating probabilities in standardized testing scenarios

Example Problem for Practice

Suppose in a different scenario, the probability p of a child being a girl is 0.55. If a sample size of 100 children is taken, estimate the probability that more than 55 children are girls.

Solution Approach:

- Calculate mean and standard deviation:

$$
\mu = 100 \times 0.55 = 55
$$

$$
\sigma = \sqrt{100 \times 0.55 \times 0.45} \approx \sqrt{24.75} \approx 4.974
$$

- Use continuity correction for $P(X > 55)$:

$$
P(X > 55) \approx P(X \geq 56) \approx P(X > 55.5)
$$

- Convert to z-score:

$$
z = \frac{55.5 - 55}{4.974} \approx 0.1008
$$

- Find probability:

$$
P(Z > 0.1008) = 1 - P(Z \leq 0.1008) \approx 1 - 0.5400 = 0.4600
$$

Estimated probability: Approximately 46.0% that more than 55 children are girls.

Conclusion

Using the normal approximation to estimate probabilities in binomial contexts provides a powerful and efficient method for handling large sample sizes. By understanding the steps — calculating mean and standard deviation, applying the continuity correction, converting to z-scores, and interpreting the standard normal distribution — you can confidently estimate probabilities such as the likelihood of observing more than a certain number of successes.

In our specific example, estimating the probability of getting more than 46 girls in 100 children yields approximately 75.8%, making it a practical technique for quick decision-making and statistical analysis in various fields.

Remember: Always verify the conditions for using the normal approximation to ensure your estimates are valid. When in doubt, computational tools or exact binomial calculations are excellent alternatives for precision.

Frequently Asked Questions

What is the normal approximation method used to estimate probabilities in binomial distributions?

The normal approximation involves approximating a binomial distribution with a normal distribution when the sample size is large, typically using the mean and standard deviation of the binomial to model the probability.

How do you determine if the normal approximation is appropriate for estimating the probability of more than 46 girls in 100 trials?

The approximation is appropriate when np and n(1-p) are both greater than 5 or 10. For p=0.5 in 100 trials, both are equal to 50, making the normal approximation suitable.

What are the steps to use the normal approximation for P(X > 46) in this problem?

Calculate the mean (np), standard deviation (sqrt(np(1-p))), apply a continuity correction (P(X > 46) becomes P(X ≥ 47)), convert to z-score, and then find the probability using standard normal tables.

What is the mean and standard deviation for the number of girls in 100 trials assuming equal probability?

The mean is 50 (since p=0.5), and the standard deviation is sqrt(100 0.5 0.5) = 5.

How do you apply the continuity correction in this estimation?

Since we're calculating P(X > 46), we use P(X ≥ 47) by subtracting 0.5 from 47, so the z-score is calculated at 46.5 instead of 46.

What is the final probability estimate for getting more than 46 girls using the normal approximation?

Calculate the z-score for X=46.5: z = (46.5 - 50) / 5 = -0.7. Then, find P(Z > -0.7) ≈ 0.76, so the probability is approximately 76%.

Are there any limitations or assumptions to keep in mind when using the normal approximation in this context?

Yes, the approximation assumes the sample size is large and the distribution is symmetric. It may be less accurate if p is close to 0 or 1 or if np or n(1-p) are small. Continuity correction improves accuracy.

Additional Resources

Use Normal Approximation To Estimate The Probability Of Getting More Than 46 Girls In 100

When faced with the problem of determining the probability of an event involving a large number of independent Bernoulli trials, such as the likelihood of having more than 46 girls out of 100 children, the normal approximation to the binomial distribution offers a powerful and practical solution. This approach simplifies the calculations significantly and provides insight into probabilities that would otherwise require complex computations. In this article, we will explore the concept of normal approximation, how it applies to the given scenario, and the methodological steps to estimate the probability accurately.

Understanding the Binomial Distribution

Before delving into the normal approximation, it is essential to understand the binomial distribution's fundamentals. The binomial distribution models the number of successes in a fixed number of independent Bernoulli trials, each with the same probability of success.

Key Characteristics

- Number of trials: n (here, 100)
- Probability of success in each trial: p (assuming p = 0.5 for a fair gender ratio)
- Random variable: X, representing the number of girls

Probability Mass Function (PMF)

The probability of getting exactly k girls is given by:
\[P(X = k) = \binom{n}{k} p^{k} (1 - p)^{n - k} \]
Calculating this directly for large n (like 100) can be computationally intensive, especially when

calculating tail probabilities such as $P(X > 46)$.

Why Use Normal Approximation?

Calculating binomial probabilities directly becomes cumbersome when n is large due to the factorial computations involved. The normal approximation offers a simplified method by approximating the binomial distribution with a normal distribution, which is continuous and easier to handle analytically.

Conditions for Applying Normal Approximation

The approximation is generally considered appropriate when:
- $np \geq 5$
- $n(1 - p) \geq 5$

In our scenario, assuming $p = 0.5$:
- np = 100 0.5 = 50
- $n(1 - p) = 50$

Since both are greater than 5, the normal approximation is suitable.

Applying the Normal Approximation

To approximate the probability $P(X > 46)$, we follow a systematic process involving the mean, standard deviation, and continuity correction.

Step 1: Calculate the Mean and Standard Deviation

- Mean (μ):
$$ \mu = np = 100 \times 0.5 = 50 $$
- Standard deviation (σ):
$$ \sigma = \sqrt{np(1 - p)} = \sqrt{100 \times 0.5 \times 0.5} = \sqrt{25} = 5 $$

Step 2: Apply Continuity Correction

Since the binomial distribution is discrete and the normal distribution is continuous, a continuity correction is applied to improve approximation accuracy. To find $P(X > 46)$, we consider:
$$ P(X > 46) \approx P(X > 46.5) $$
This correction shifts the boundary by 0.5.

Step 3: Standardize the Variable

Convert the discrete variable to a standard normal variable (Z):
$$ Z = \frac{X - \mu}{\sigma} $$
For X = 46.5:
$$ Z = \frac{46.5 - 50}{5} = \frac{-3.5}{5} = -0.7 $$

Step 4: Find the Probability

Since we're interested in $P(X > 46)$, corresponding to $Z > -0.7$:
$$ P(X > 46) \approx P(Z > -0.7) $$

Using standard normal distribution tables or a calculator:
$$ P(Z > -0.7) = 1 - P(Z \leq -0.7) $$
But because the standard normal distribution is symmetric:
$$ P(Z > -0.7) = P(Z < 0.7) $$

From tables:
$$ P(Z < 0.7) \approx 0.7580 $$

Therefore:
$$ P(X > 46) \approx 0.7580 $$

Interpreting the Results

The approximation indicates that there is approximately a 75.8% chance of observing more than 46 girls in a sample of 100 children, assuming a fair gender ratio ($p = 0.5$). This high probability aligns with intuitive expectations; since the mean is 50, the probability of getting more than 46 is quite substantial.

Pros and Cons of Using Normal Approximation

Pros:
- Simplification: Transforms a complex binomial calculation into a straightforward normal probability calculation.
- Efficiency: Reduces computational load, especially for large n.
- Intuitive understanding: Leverages properties of the normal distribution, which is well-understood.

Cons:
- Approximate nature: It provides an estimate, not the exact probability.
- Edge cases: Less accurate when p is near 0 or 1 or when np and n(1 - p) are small.
- Continuity correction dependence: Slight differences in correction can impact results.

Additional Considerations

While the normal approximation is quite reliable under the given conditions, for complete accuracy, especially in critical applications, one might consider alternative methods such as the exact binomial calculation using statistical software or tables.

Alternative approaches:
- Exact binomial probability calculations
- Using software packages like R or Python's SciPy library
- Monte Carlo simulations for empirical probability estimation

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Conclusion

Using the normal approximation to estimate the probability of getting more than 46 girls in a sample of 100 is a practical and efficient method, especially under conditions where np and n(1 - p) are sufficiently large. The process involves calculating the mean and standard deviation, applying a continuity correction, standardizing, and then referring to the standard normal distribution. In our scenario with p = 0.5, the probability is approximately 75.8%, indicating a high likelihood that the number of girls exceeds 46 in such a sample. Despite its approximate nature, the normal approximation remains a cornerstone technique in statistical analysis for large sample sizes, providing a balance between computational simplicity and reasonable accuracy.

Key takeaways:
- The normal approximation simplifies binomial probability calculations significantly.
- Proper application involves checking conditions, calculating parameters, and applying continuity correction.
- Always consider the context and required precision when choosing between approximation and exact calculation.

By mastering the use of normal approximation, statisticians and data analysts can efficiently handle large binomial problems, gaining quick insights without the need for extensive computation.

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